Grade 12 Calculus and Vectors Practice Final Exam

This document was composed and formatted by Annie Bernatchez.

The course material originated from www.jensenmath.ca

Practice Exam – Full Course

Instructions: There is a 3 hour maximum time limit for writing this exam. All questions are to be answered ON the exam paper.

Calculators are permitted. Use RADIAN mode for the Calculus portion and DEGREE mode for the Vectors portion.

Communicate your solutions clearly and **box all final answers**. Use exact values wherever possible. Round to 3 decimal places if you give an approximate answer.

Knowledge and Understanding	Application	Thinking	Communication
(30%)	(30%)	(20%)	(20%)

Sections	Points
Part 1: Derivatives and Applications	25
Part 2: Curve Sketching and Optimization	25
Part 3: Geometric and Algebraic Vectors	30
Part 4: Lines and Planes	20
Total	**/100**

Part 1: Derivatives and Applications [25]

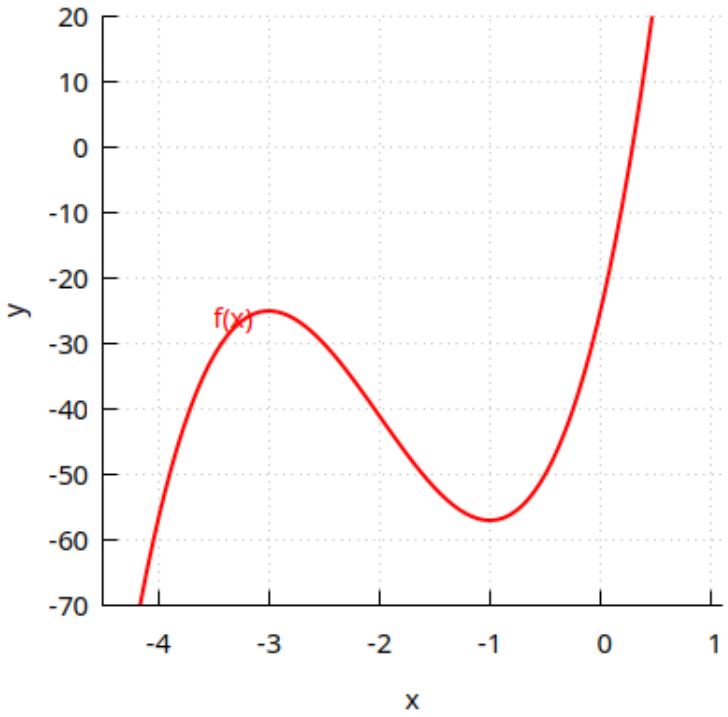
1) Differentiate each of the following with respect to x. Fully simplify your answers. **[16]**

a) $y = 4x^3 - \frac{12}{x} + 71$	**b)** $y = \frac{3x}{2x^2 + 4x - 1}$
c) $y = (x-2)^3(x^2+9)^4$	**d)** $y = \sqrt{3x+4}$
e) $y = 2(6^x)$	**f)** $y = 3\tan x + 2$
g) $f(x) = [\sin(2x)]e^{3x}$	**h)** $f(x) = \cos^2(4x)$

2) Determine the equation of the tangent to the curve $f(x) = \left(\frac{3x}{x-1}\right)^2$ at $x = -2$. **[3]**

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3) Given the graph of $f(x)$, sketch a possible graph of $f'(x)$. **[1]**



4) Given the graph of $f'(x)$, sketch a possible graph of $f(x)$ **[1]**



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5) Bob is on a diet. His original mass was 102 kg. His diet allows him to lose mass exponentially, so his mass is modelled by $W(t) = W_0e^{kt}$, where W_0 is his original mass, t is time in weeks, and k is a constant. After 3 weeks, his mass is 97 kg. **[4]**

a) Determine the value of k to five decimal places.	**b)** After 10 weeks, at what rate will his mass be changing?

Part 2: Curve Sketching and Optimization [25]

6) The function $f(x) = x^3 + bx^2 + c$ has a critical point at $(-2, 6)$ **[5]**

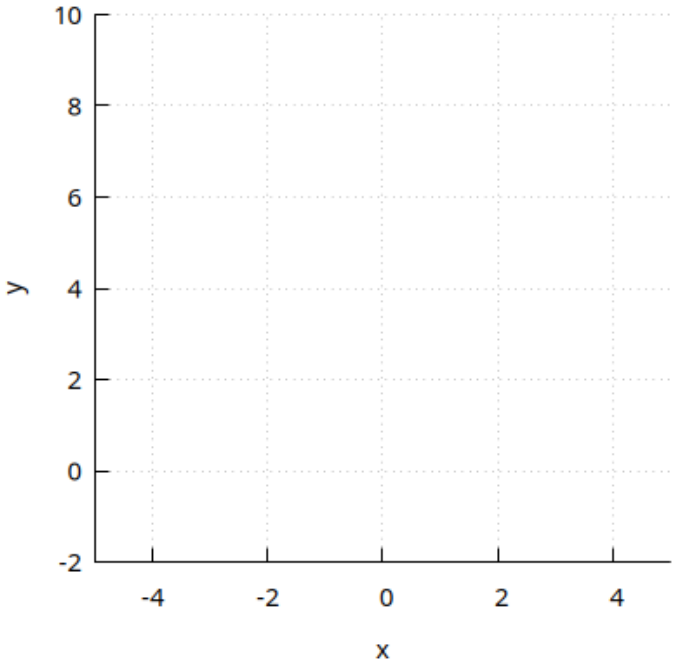
a) Find the constants b and c	**b)** Determine ALL of the critical points for $f(x)$ and classify them using the second derivative test.

7) For the function $y = x^3(2)^x$, find the critical point(s), and then use a derivative test to classify it/them as local max, local min, or neither. **[4]**

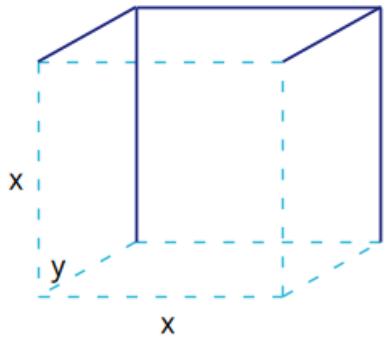
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8) Sketch a continuous curve $f(x)$ with the following properties **[2]**

- $f(-2) = 8,\ f(0) = 4,\ f(2) = 0$
- $f'(2) = f'(-2) = 0$
- $f'(x) > 0$ for $x > 2$ or $x < -2$
- $f'(x) < 0$ for $-2 < x < 2$
- $f''(x) < 0$ for $x < 0$
- $f''(x) > 0$ for $x > 0$



9) A net enclosure for practising golf shots is open at one end, as shown. Find the dimensions that will minimize the amount of netting needed and give a volume of 144 m^3. Netting is required only on the sides, the top, and the far end (not the bottom or front). **[4]**



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10) Use the algorithm for curve sketching to sketch the graph of $f(x) = x^3 + 6x^2 + 9x$. Clearly label key points. **[10]**

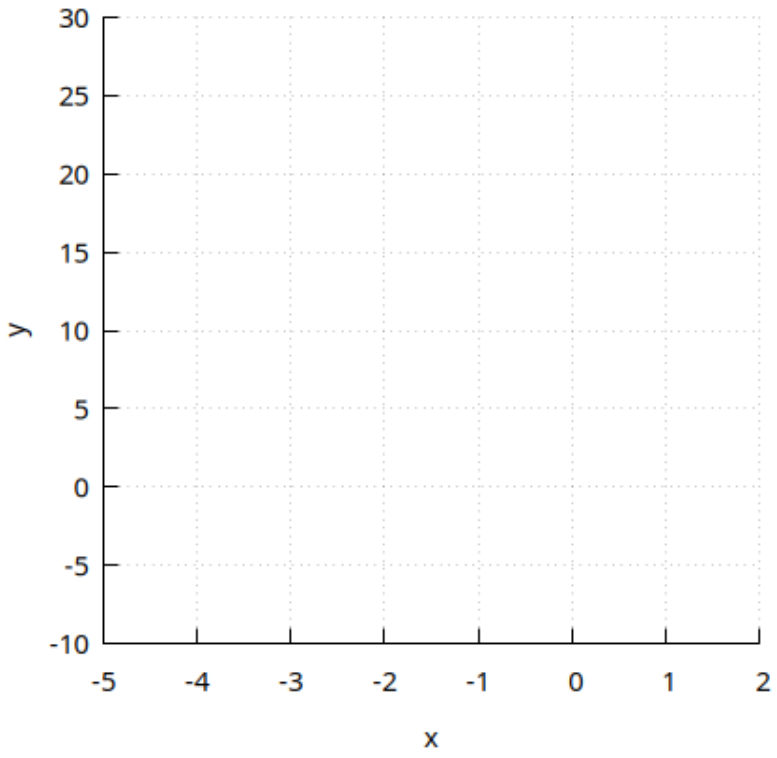
1) Asymptotes or holes:

2) Intercepts: x-intercepts: y-intercept:	

3) Critical points:	**4) Points of inflection:**

5/6/7) Sign chart that uses critical #'s, POI's, and VA's to analyze intervals of increase/decrease and intervals of concavity. Clearly identify local extrema and POI's.

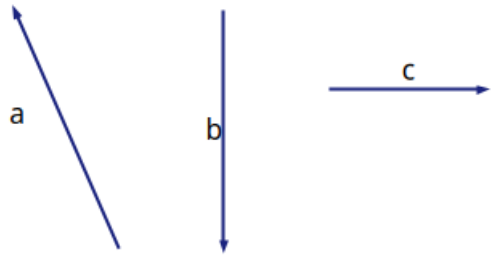
8) Graph the function



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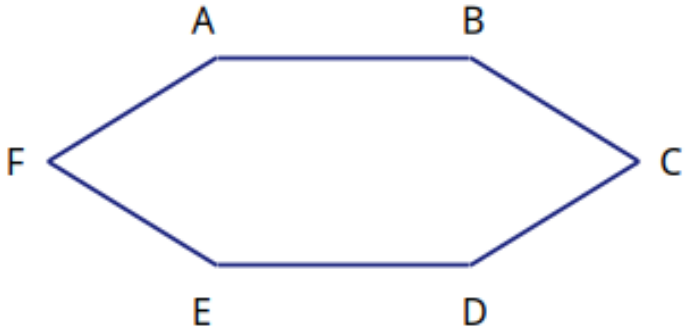
Part 3: Geometric and Algebraic Vectors [30]

11) Given vectors, sketch the following **[3]**



a) $\vec{a} + \vec{c}$	**b)** $\vec{b} - \vec{c}$	**c)** $-0.5\vec{c} + \vec{b}$

12) In hexagon ABCDEF, opposite sides are parallel and equal, and $\overrightarrow{FC} = 2\overrightarrow{AB}$. Let $\overrightarrow{AB} = \vec{u}$ and $\overrightarrow{FA} = \vec{v}$. Express each vector in terms of $\vec{u}$ and $\vec{v}$. **[4]**



a) $\overrightarrow{CF}$	**b)** $\overrightarrow{FB}$
c) $\overrightarrow{BC}$	**d)** $\overrightarrow{DA}$

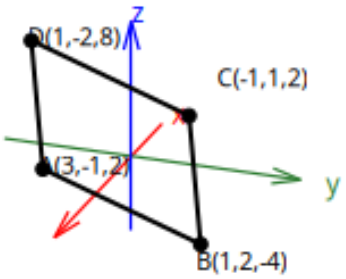
13) Given vectors $\vec{a} = [4, -2, 1]$, $\vec{b} = [1, 5, 4]$, and $\vec{c} = [-1, -3, 0]$, calculate the following if possible. If not possible explain why. **[12]**

a) $\vec{b} - 2\vec{c}$	**b)** $\lvert \text{proj}_{\vec{c}}\vec{b} \rvert$
c) $\vec{a} \cdot \vec{c}$	**d)** $(\vec{a})^2 + 4\vec{b}$

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e) $\vec{a} \times \vec{c}$	**f)** Based on your previous answers, do $\vec{a}$ and $\vec{c}$ form a right angle? How do you know?

14) Given the following parallelogram $A(3, -1, 2)$, $B(1, 2, -4)$, $C(-1, 1, 2)$, $D(1, -2, 8)$. **[6]**



a) calculate $|\overrightarrow{AB}|$

b) Find the area of the parallelogram.

c) Calculate the measure of angle A in degrees.

15) A pilot flies on a heading of $N10°W$ with an airspeed of 260 km/h. Her actual ground velocity is 310 km/h at a bearing of $N20°E$. Find the magnitude and direction of the wind. **[3]**

16) Given that $|\vec{u}| = 10$ and $|\vec{v}| = 6$ and the angle between $\vec{u}$ and $\vec{v}$ is $40°$ determine an expression for a unit vector in the direction of $\vec{u} + \vec{v}$ **[2]**

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Part 4: Lines and Planes [20]

17) Find the vector equation of each of the following lines **[4]**

a) the line passing through the points $P(2,\ 5,\ 4)$ and $Q(-3,\ 1,\ 5)$

b) With the same x-intercept as $[x,\ y,\ z] = [3,\ 0,\ 0] + t[4,\ -4,\ 1]$ and the same z-intercept as $[x,\ y,\ z] = [6,\ -2,\ -3] + t[3,\ -1,\ 8]$

18) Find the vector and scalar equation of each of the following planes **[4]**

a) the plane passing through the points $P(4,\ -2,\ 2)$, $Q(-1,\ 6,\ 8)$, and $R(-2,\ -2,\ 14)$.

b) the plane parallel to the xy-plane and through the point $P(4,\ -2,\ 7)$.

19) Solve the following systems of equations. If it is an inconsistent system, give proof and explain how you know.

a) $\ell_1 : [x,\ y,\ z] = [-2,\ 0,\ -3] + s[5,\ 1,\ 3]$, $\ell_2 : [x,\ y,\ z] = [5,\ 8,\ -6] + t[-1,\ 2,\ -3]$ **[4]**

b) $\pi_1 : x - 5y + 2z - 10 = 0$, $\pi_2 : x + 7y - 2z + 6 = 0$, $\pi_3 : 8x + 5y + z - 20 = 0$ **[4]**

20) Find the distance between the point $P(7,\ -3,\ 2)$, and the plane $2x + 4y - 3z - 9 = 0$. **[4]**

Answer Key

1)a) $12x^2 + \frac{12}{x^2}$ **b)** $\frac{-3(2x^2+1)}{(2x^2+4x-1)^2}$ **c)** $(x-2)^2(x^2+9)^3(11x^2-16x+27)$ **d)** $\frac{3}{2\sqrt{3x+4}}$ **e)** $2(6^x)\ln 6$ **f)** $3\sec^2 x$ **g)** $e^{3x}[2\cos(2x) + 3\sin(2x)]$ **h)** $-4\sin(8x)$ **2)** $y = -\frac{4}{3}x + \frac{4}{3}$ **3)** parabola through $(-3,\ 0)$ and $(-1,\ 0)$ **4)** cubic-shaped curve with local extrema at $x = \pm 2$ **5)a)** $k \approx -0.01675$ **b)** ≈ -1.446 kg/week **6)a)** $b = 3,\ c = 2$ **b)** local max $(-2,\ 6)$, local min $(0,\ 2)$ **7)** $x = 0$ neither; $x \approx -4.328$ local min **8)** local max $(-2,\ 8)$, inflection $(0,\ 4)$, local min $(2,\ 0)$ **9)** $x = 6$ m, $y = 4$ m **10)** local max $(-3,\ 0)$, POI $(-2,\ -2)$, local min $(-1,\ -4)$, intercepts $(0,\ 0)$, $(-3,\ 0)$ **11)a)** $[1,\ 3]$ **b)** $[-3,\ -3]$ **c)** $[-1.5,\ -3]$ **12)a)** $-2\vec{u}$ **b)** $\vec{u} + \vec{v}$ **c)** $\vec{u} - \vec{v}$ **d)** $2\vec{v} - 2\vec{u}$ **13)a)** $[3,\ 11,\ 4]$ **b)** ≈ 5.060 **c)** 2 **d)** not possible (scalar + vector) **e)** $[3,\ -1,\ -14]$ **f)** no **14)a)** 7 **b)** 28 square units **c)** $\approx 141.343°$ **15)** ≈ 155.233 km/h at $N76.9°E$ **16)** $\approx [0.967,\ 0.256]$ **17)a)** $[x,\ y,\ z] = [2,\ 5,\ 4] + t[-5,\ -4,\ 1]$ **b)** $[x,\ y,\ z] = [3,\ 0,\ 0] + t[-3,\ 0,\ -19]$ **18)a)** $4x + y + 2z - 18 = 0$ **b)** $z - 7 = 0$ **19)a)** intersect at $(8,\ 2,\ 3)$ **b)** line of intersection $[x,\ y,\ z] = [2,\ 0,\ 4] + t[-1,\ 1,\ 3]$ **20)** $\frac{13}{\sqrt{29}} \approx 2.414$ units

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