2.4 Solving Polynomial Equations Worksheet with solutions

This document was composed and formatted by Annie Bernatchez.

The course material originated from www.jensenmath.ca

1) Determine the solutions of the following polynomials.

a) $(3x+2)(x+9)(x-2)=0$

$3x+2=0 \rightarrow x_1=-\frac{2}{3}$

$x+9=0 \rightarrow x_2=-9$

$x-2=0 \rightarrow x_3=2$

Solutions: $\left(-\frac{2}{3},0\right)$, $(-9,0)$, and $(2,0)$

b) $(x^2+1)(x-4)=0$

$x^2+1=0 \rightarrow x^2=-1$—no real solutions

$x-4=0 \rightarrow x=4$

Solution: $(4,0)$

2) Determine the solutions of the following polynomials by factoring. Use the tools you have learned this unit to help you. (remainder theorem, integral zero theorem, division etc.)

a) $x^3-4x^2-3x+18=0$

Possible zeros: $\pm 1,\ \pm 2,\ \pm 3,\ \pm 6,\ \pm 9,\ \pm 18$

$f(-2)=0;\ \therefore x+2$ is a factor

-2	1	-4	-3	18
		-2	12	-18
	1	-6	9	0
	x^2	x	#	R

$(x+2)(x^2-6x+9)=0$

$(x+2)(x-3)^2=0$

$x_1=-2,\ x_2=3$

Solutions: $(-2,0)$ and $(3,0)$

b) $x^3-3x^2-4x+12=0$

Possible zeros: $\pm 1,\ \pm 2,\ \pm 3,\ \pm 4,\ \pm 6,\ \pm 12$

$f(2)=0;\ \therefore x-2$ is a factor

2	1	-3	-4	12
		2	-2	-12
	1	-1	-6	0
	x^2	x	#	R

$(x-2)(x^2-x-6)=0$

$(x-2)(x-3)(x+2)=0$

$x_1=2,\ x_2=3,\ x_3=-2$

Solutions: $(2,0)$, $(3,0)$, and $(-2,0)$

c) $x^4-x^3-11x^2+9x+18=0$

Possible zeros: $\pm 1,\ \pm 2,\ \pm 3,\ \pm 6,\ \pm 9,\ \pm 18$

$f(-1)=0;\ \therefore x+1$ is a factor

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$$\begin{array}{r|rrrrr} -1 & 1 & -1 & -11 & 9 & 18 \\ & & -1 & 2 & 9 & -18 \\ \hline & 1 & -2 & -9 & 18 & 0 \\ & x^3 & x^2 & x & \# & R \end{array}$$

$(x+1)(x^3-2x^2-9x+18)=0$

$(x+1)[x^2(x-2)-9(x-2)]=0$

$(x+1)(x-2)(x^2-9)=0$

$(x+1)(x-2)(x-3)(x+3)=0$

$x_1=-1,\ x_2=2,\ x_3=3,\ x_4=-3$

Solutions: $(-1,0)$, $(2,0)$, $(3,0)$, and $(-3,0)$

d) $x^3-64=0$

$a^3+b^3=(a+b)(a^2-ab+b^2),\ a^3-b^3=(a-b)(a^2+ab+b^2)$

$(x-4)(x^2+4x+16)=0$

$x=4$

Check $b^2-4ac=(4)^2-4(1)(16)=-48$—no real solutions

Solution: $(4,0)$

e) $2x^3-7x^2+10x-5=0$

Possible zeros: $\pm 1,\ \pm\frac{1}{2},\ \pm 5,\ \pm\frac{5}{2}$

$f(1)=0;\ \therefore x-1$ is a factor

$$\begin{array}{r|rrrr} 1 & 2 & -7 & 10 & -5 \\ & & 2 & -5 & 5 \\ \hline & 2 & -5 & 5 & 0 \\ & x^2 & x & \# & R \end{array}$$

$(x-1)(2x^2-5x+5)=0$

$x-1=0\rightarrow x=1$

Check $b^2-4ac=(-5)^2-4(2)(5)=-15$—no real solutions

Solution: $(1,0)$

3) Solve each equation by first factoring the sum or difference of cubes.

a) $x^3-8=0$

$(x-2)(x^2+2x+4)=0$

$x=2$

Check $b^2-4ac=(2)^2-4(1)(4)=-12$—no real solution

Solution: $(2,0)$

b) $x^3+27=0$

$(x+3)(x^2-3x+9)=0$

$x=-3$

Check $b^2-4ac=(-3)^2-4(1)(9)=-27$—no real solution

Solution: $(-3,0)$

4) Solve by factoring

a) $x^3-4x^2-7x+10=0$

Possible zeros: $\pm 1,\ \pm 2,\ \pm 5,\ \pm 10$

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$f(1) = 0$; $\therefore x - 1$ is a factor

$$\begin{array}{r|rrrr} 1 & 1 & -4 & -7 & 10 \\ & & 1 & -3 & -10 \\ \hline & 1 & -3 & -10 & 0 \\ & x^2 & x & \# & R \end{array}$$

$(x - 1)(x^2 - 3x - 10) = 0$

$(x - 1)(x - 5)(x + 2) = 0$

$x_1 = 1,\ x_2 = 5,\ x_3 = -2$

Solutions: $(1,\ 0)$, $(5,\ 0)$, and $(-2,\ 0)$

b) $2x^3 - 11x^2 + 12x + 9 = 0$

Possible zeros: $\pm 1,\ \pm \frac{1}{2},\ \pm 3,\ \pm \frac{3}{2},\ \pm 9,\ \pm \frac{9}{2}$

$f(3) = 0$; $\therefore x - 3$ is a factor

$$\begin{array}{r|rrrr} 3 & 2 & -11 & 12 & 9 \\ & & 6 & -15 & -9 \\ \hline & 2 & -5 & -3 & 0 \\ & x^2 & x & \# & R \end{array}$$

$(x - 3)(2x^2 - 5x - 3) = 0$

$(x - 3)(x - 3)(2x + 1) = 0$

$(x - 3)^2(2x + 1) = 0$

$x_1 = 3,\ x_2 = -\frac{1}{2}$

Solutions: $(3,\ 0)$ and $\left(-\frac{1}{2},\ 0\right)$

c) $x^4 - x^3 - 2x - 4 = 0$

Possible zeros: $\pm 1,\ \pm 2,\ \pm 4$

$f(-1) = 0$; $\therefore x + 1$ is a factor

$$\begin{array}{r|rrrrr} -1 & 1 & -1 & 0 & -2 & -4 \\ & & -1 & 2 & -2 & 4 \\ \hline & 1 & -2 & 2 & -4 & 0 \\ & x^3 & x^2 & x & \# & R \end{array}$$

$(x + 1)(x^3 - 2x^2 + 2x - 4) = 0$

$(x + 1)[x^2(x - 2) + 2(x - 2)] = 0$

$(x + 1)(x - 2)(x^2 + 2) = 0$

$x_1 = -1,\ x_2 = 2$; $x^2 + 2 = 0$ has no real solution

Solutions: $(-1,\ 0)$ and $(2,\ 0)$

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