2.3 Factor Theorem Worksheet with solutions

This document was composed and formatted by Annie Bernatchez.

The course material originated from www.jensenmath.ca

1) Determine if $x+3$ is a factor of each polynomial:

a) x^3+x^2-x+6

$$\begin{aligned} f(-3) &= (-3)^3+(-3)^2-(-3)+6 \\ &= -27+9+3+6 \\ &= -9 \end{aligned}$$

Not a factor.

b) $2x^3+9x^2+10x+3$

$$\begin{aligned} f(-3) &= 2(-3)^3+9(-3)^2+10(-3)+3 \\ &= -54+81-30+3 \\ &= 0 \end{aligned}$$

A factor.

c) x^3+27

$$\begin{aligned} f(-3) &= (-3)^3+27 \\ &= 0 \end{aligned}$$

A factor.

2) Find possible factors of the following polynomials using the integral zero theorem. Then, factor the polynomial.

a) x^3+3x^2-6x-8

Possible factors: $\pm 1,\ \pm 2,\ \pm 4,\ \pm 8$

$f(-1)=0;\ \therefore x+1$ is a factor

-1	1	3	-6	-8
		-1	-2	8
	1	2	-8	0
	x^2	x	#	R

$x^3+3x^2-6x-8=(x+1)(x^2+2x-8)=(x+1)(x+4)(x-2)$

b) $x^3+4x^2-15x-18$

Possible factors: $\pm 1,\ \pm 2,\ \pm 3,\ \pm 6,\ \pm 9,\ \pm 18$

$f(-1)=0;\ \therefore x+1$ is a factor

-1	1	4	-15	-18
		-1	-3	18
	1	3	-18	0
	x^2	x	#	R

$x^3+4x^2-15x-18=(x+1)(x^2+3x-18)=(x+1)(x+6)(x-3)$

c) $x^3-3x^2-10x+24$

Possible factors: $\pm 1,\ \pm 2,\ \pm 3,\ \pm 4,\ \pm 6,\ \pm 8,\ \pm 12,\ \pm 24$

$f(2)=0;\ \therefore x-2$ is a factor

2	1	-3	-10	24
		2	-2	-24
	1	-1	-12	0
	x^2	x	#	R

$x^3-3x^2-10x+24=(x-2)(x^2-x-12)=(x-2)(x-4)(x+3)$

Copyright © 2026 Annie Bernatchez—All rights reserved.

3) Factor by grouping:

a) $x^3 + x^2 - 9x - 9$

$= (x^3 + x^2) + (-9x - 9)$

$= x^2(x + 1) - 9(x + 1)$

$= (x + 1)(x^2 - 9)$

$= (x + 1)(x - 3)(x + 3)$

b) $2x^3 - x^2 - 72x + 36$

$= (2x^3 - x^2) + (-72x + 36)$

$= x^2(2x - 1) - 36(2x - 1)$

$= (2x - 1)(x^2 - 36)$

$= (2x - 1)(x - 6)(x + 6)$

4) Determine a value of k so that $x + 2$ is a factor of $x^3 - 2kx^2 + 6x - 4$.

$$\begin{aligned} f(-2) &= (-2)^3 - 2k(-2)^2 + 6(-2) - 4 \\ 0 &= -8 - 8k - 12 - 4 \\ 0 &= -24 - 8k \\ 24 &= -8k \\ k &= -3 \end{aligned}$$

5) Find possible factors of the following polynomials using the integral zero theorem. Then, factor the polynomial.

a) $3x^3 + x^2 - 22x - 24$

Possible factors: $\pm 1,\ \pm\frac{1}{3},\ \pm 2,\ \pm\frac{2}{3},\ \pm 3,\ \pm 4,\ \pm\frac{4}{3},\ \pm 6,\ \pm 8,\ \pm\frac{8}{3},\ \pm 12,\ \pm 24$

$f(-2) = 0$; $\therefore x + 2$ is a factor

$$\begin{array}{r|rrrr} -2 & 3 & 1 & -22 & -24 \\ & & -6 & 10 & 24 \\ \hline & 3 & -5 & -12 & 0 \\ & x^2 & x & \# & R \end{array}$$

$3x^3 + x^2 - 22x - 24 = (x + 2)(3x^2 - 5x - 12) = (x + 2)(x - 3)(3x + 4)$

b) $2x^3 - 9x^2 + 10x - 3$

Possible factors: $\pm 1,\ \pm\frac{1}{2},\ \pm 3,\ \pm\frac{3}{2}$

$f(1) = 0$; $\therefore x - 1$ is a factor

$$\begin{array}{r|rrrr} 1 & 2 & -9 & 10 & -3 \\ & & 2 & -7 & 3 \\ \hline & 2 & -7 & 3 & 0 \\ & x^2 & x & \# & R \end{array}$$

$2x^3 - 9x^2 + 10x - 3 = (x - 1)(2x^2 - 7x + 3) = (x - 1)(x - 3)(2x - 1)$

c) $6x^3 - 11x^2 - 26x + 15$

Possible factors: $\pm 1,\ \pm\frac{1}{2},\ \pm\frac{1}{3},\ \pm\frac{1}{6},\ \pm 3,\ \pm\frac{3}{2},\ \pm 5,\ \pm\frac{5}{2},\ \pm\frac{5}{3},\ \pm\frac{5}{6}$

$f(3) = 0$; $\therefore x - 3$ is a factor

$$\begin{array}{r|rrrr} 3 & 6 & -11 & -26 & 15 \\ & & 18 & 21 & -15 \\ \hline & 6 & 7 & -5 & 0 \\ & x^2 & x & \# & R \end{array}$$

$6x^3 - 11x^2 - 26x + 15 = (x - 3)(6x^2 + 7x - 5) = (x - 3)(3x + 5)(2x - 1)$

Copyright © 2026 Annie Bernatchez—All rights reserved.

d) $4x^3 + 3x^2 - 4x - 3$

Possible factors: $\pm 1,\ \pm \frac{1}{2},\ \pm \frac{1}{4},\ \pm 3,\ \pm \frac{3}{2},\ \pm \frac{3}{4}$

$f(1) = 0;\ \therefore x - 1$ is a factor

1	4	3	−4	−3
		4	7	3
	4	7	3	0
	x^2	x	#	R

$4x^3 + 3x^2 - 4x - 3 = (x - 1)(4x^2 + 7x + 3) = (x - 1)(x + 1)(4x + 3)$

6) Factor each polynomial

a) $2x^3 + 5x^2 - x - 6$

Possible factors: $\pm 1,\ \pm \frac{1}{2},\ \pm 2,\ \pm 3,\ \pm \frac{3}{2},\ \pm 6$

$f(1) = 0;\ \therefore x - 1$ is a factor

1	2	5	−1	−6
		2	7	6
	2	7	6	0
	x^2	x	#	R

$2x^3 + 5x^2 - x - 6 = (x - 1)(2x^2 + 7x + 6) = (x - 1)(x + 2)(2x + 3)$

b) $4x^3 - 7x - 3$

Possible factors: $\pm 1,\ \pm \frac{1}{2},\ \pm \frac{1}{4},\ \pm 3,\ \pm \frac{3}{2},\ \pm \frac{3}{4}$

$f(-1) = 0;\ \therefore x + 1$ is a factor

−1	4	0	−7	−3
		−4	4	3
	4	−4	−3	0
	x^2	x	#	R

$4x^3 - 7x - 3 = (x + 1)(4x^2 - 4x - 3) = (x + 1)(2x - 3)(2x + 1)$

c) $x^4 - 15x^2 - 10x + 24$

Possible factors: $\pm 1,\ \pm 2,\ \pm 4$

$f(1) = 0;\ \therefore x - 1$ is a factor

1	1	0	−15	−10	24
		1	1	−14	−24
	1	1	−14	−24	0
	x^3	x^2	x	#	R

$x^4 - 15x^2 - 10x + 24 = (x - 1)(x^3 + x^2 - 14x - 24)$

Now further factor $x^3 + x^2 - 14x - 24$:

Possible factors: $\pm 1,\ \pm 2,\ \pm 3,\ \pm 4,\ \pm 6,\ \pm 8,\ \pm 12,\ \pm 24$

$f(-2) = 0;\ \therefore x + 2$ is a factor

−2	1	1	−14	−24
		−2	2	24
	1	−1	−12	0
	x^2	x	#	R

Copyright © 2026 Annie Bernatchez—All rights reserved.

$x^4 - 15x^2 - 10x + 24 = (x - 1)(x + 2)(x^2 - x - 12) = (x - 1)(x + 2)(x - 4)(x + 3)$

Copyright © 2026 Annie Bernatchez—All rights reserved.