2.1 Long Division of Polynomials and the Remainder Theorem Worksheet with solutions

This document was composed and formatted by Annie Bernatchez.

The course material originated from www.jensenmath.ca

1) Use the remainder theorem to determine the remainder when $2x^3 + 7x^2 - 8x + 3$ is divided by each binomial.

a) $x + 1$

$$\begin{aligned} P(-1) &= 2(-1)^3 + 7(-1)^2 - 8(-1) + 3 \\ &= 16 \end{aligned}$$

b) $x - 2$

$$\begin{aligned} P(2) &= 2(2)^3 + 7(2)^2 - 8(2) + 3 \\ &= 31 \end{aligned}$$

c) $x + 3$

$$\begin{aligned} P(-3) &= 2(-3)^3 + 7(-3)^2 - 8(-3) + 3 \\ &= 36 \end{aligned}$$

2) a) Divide $x^3 + 3x^2 - 2x + 5$ by $x + 1$. Express the result in quotient form.

$$\begin{array}{r} x^2 +2x -4 \\ \hline x+1 \;)\; x^3 +3x^2 -2x +5 \\ \underline{x^3 +x^2} \\ 2x^2 -2x \\ \underline{2x^2 +2x} \\ -4x +5 \\ \underline{-4x -4} \\ R = 9 \end{array}$$

The result in quotient form is:

$$\frac{x^3 + 3x^2 - 2x + 5}{x + 1} = x^2 + 2x - 4 + \frac{9}{x + 1}$$

b) Write the corresponding statement that can be used to check the division.

$$x^3 + 3x^2 - 2x + 5 = (x + 1)(x^2 + 2x - 4) + 9$$

3) Divide $3x^4 - 4x^3 - 6x^2 + 17x - 8$ by $3x - 4$. Express the result in quotient form.

Note: the dividend has no x^2 term at the correct spot for this step, so a placeholder $0x^2$ is used to keep the columns lined up.

$$\begin{array}{r} x^3 +0x^2 -2x +3 \\ \hline 3x-4 \;)\; 3x^4 -4x^3 -6x^2 +17x -8 \\ \underline{3x^4 -4x^3} \\ 0x^3 -6x^2 \\ \underline{0x^3 +0x^2} \\ -6x^2 +17x \\ \underline{-6x^2 +8x} \\ 9x -8 \\ \underline{9x -12} \\ R = 4 \end{array}$$

The result in quotient form is:

$$\frac{3x^4 - 4x^3 - 6x^2 + 17x - 8}{3x - 4} = x^3 - 2x + 3 + \frac{4}{3x - 4}$$

b) Write the corresponding statement that can be used to check the division.

$$3x^4 - 4x^3 - 6x^2 + 17x - 8 = (3x - 4)(x^3 - 2x + 3) + 4$$

4) Perform each division. Express the result in quotient form.

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4) Perform each division. Express the result in quotient form.

a) $x^3 + 7x^2 - 3x + 4$ divided by $x + 2$

$$\begin{array}{r} x^2 \ +5x \ -13 \\ \hline x+2 \ ) \ x^3 \ +7x^2 \ -3x \ +4 \\ \underline{x^3 \ +2x^2} \\ 5x^2 \ -3x \\ \underline{5x^2 \ +10x} \\ -13x \ +4 \\ \underline{-13x \ -26} \\ R = 30 \end{array}$$

$$\frac{x^3 + 7x^2 - 3x + 4}{x + 2} = x^2 + 5x - 13 + \frac{30}{x + 2}$$

b) $6x^3 + x^2 - 14x - 6$ divided by $3x + 2$

$$\begin{array}{r} 2x^2 \ -x \ -4 \\ \hline 3x+2 \ ) \ 6x^3 \ +x^2 \ -14x \ -6 \\ \underline{6x^3 \ +4x^2} \\ -3x^2 \ -14x \\ \underline{-3x^2 \ -2x} \\ -12x \ -6 \\ \underline{-12x \ -8} \\ R = 2 \end{array}$$

$$\frac{6x^3 + x^2 - 14x - 6}{3x + 2} = 2x^2 - x - 4 + \frac{2}{3x + 2}$$

c) $10x^3 + 11 - 9x^2 - 8x$ divided by $5x - 2$

Note: the dividend is reordered to standard descending-degree form first: $10x^3 - 9x^2 - 8x + 11$.

$$\begin{array}{r} 2x^2 \ -x \ -2 \\ \hline 5x-2 \ ) \ 10x^3 \ -9x^2 \ -8x \ +11 \\ \underline{10x^3 \ -4x^2} \\ -5x^2 \ -8x \\ \underline{-5x^2 \ +2x} \\ -10x \ +11 \\ \underline{-10x \ +4} \\ R = 7 \end{array}$$

$$\frac{10x^3 - 9x^2 - 8x + 11}{5x - 2} = 2x^2 - x - 2 + \frac{7}{5x - 2}$$

d) $11x - 4x^4 - 7$ divided by $x - 3$

Note: the dividend is reordered to standard descending-degree form first: $-4x^4 + 0x^3 + 0x^2 + 11x - 7$.

$$\begin{array}{r} -4x^3 \ -12x^2 \ -36x \ -97 \\ \hline x-3 \ ) \ -4x^4 \ +0x^3 \ +0x^2 \ +11x \ -7 \\ \underline{-4x^4 \ +12x^3} \\ -12x^3 \ +0x^2 \\ \underline{-12x^3 \ +36x^2} \\ -36x^2 \ +11x \\ \underline{-36x^2 \ +108x} \\ -97x \ -7 \\ \underline{-97x \ +291} \\ R = -298 \end{array}$$

$$\frac{-4x^4 + 11x - 7}{x - 3} = -4x^3 - 12x^2 - 36x - 97 - \frac{298}{x - 3}$$

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e) $6x^3 + x^2 + 7x + 3$ divided by $3x + 2$

$$\begin{array}{r} 2x^2 \ -x \ +3 \\ 3x+2 \ \overline{)\ 6x^3 \ +x^2 \ +7x \ +3} \\ \underline{6x^3 \ +4x^2} \\ -3x^2 \ +7x \\ \underline{-3x^2 \ -2x} \\ 9x \ +3 \\ \underline{9x \ +6} \\ R = \ -3 \end{array}$$

$$\frac{6x^3 + x^2 + 7x + 3}{3x + 2} = 2x^2 - x + 3 - \frac{3}{3x + 2}$$

f) $8x^3 + 4x^2 - 31$ divided by $2x - 3$

Note: the dividend has no x term, so a placeholder $0x$ is used to keep the columns lined up.

$$\begin{array}{r} 4x^2 \ +8x \ +12 \\ 2x-3 \ \overline{)\ 8x^3 \ +4x^2 \ +0x \ -31} \\ \underline{8x^3 \ -12x^2} \\ 16x^2 \ +0x \\ \underline{16x^2 \ -24x} \\ 24x \ -31 \\ \underline{24x \ -36} \\ R = 5 \end{array}$$

$$\frac{8x^3 + 4x^2 - 31}{2x - 3} = 4x^2 + 8x + 12 + \frac{5}{2x - 3}$$

g) $6x^2 - 6 + 8x^3$ divided by $4x - 3$

Note: the dividend is reordered to standard descending-degree form first: $8x^3 + 6x^2 + 0x - 6$.

$$\begin{array}{r} 2x^2 + 3x + \frac{9}{4} \\ 4x-3 \ \overline{)\ 8x^3 + 6x^2 + 0x - 6} \\ \underline{8x^3 - 6x^2} \\ 12x^2 + 0x \\ \underline{12x^2 - 9x} \\ 9x - 6 \\ \frac{3}{4} \end{array}$$

$$\frac{6x^2 - 6 + 8x^3}{4x - 3} = 2x^2 + 3x + \frac{9}{4} + \frac{3}{4(4x - 3)}$$

5) The volume, in cubic cm, of a rectangular box can be modelled by the polynomial expression $2x^3 + 17x^2 + 38x + 15$. Determine possible dimensions of the box if the height, in cm, is given by $x + 5$.

$$\begin{array}{r} 2x^2 \ +7x \ +3 \\ x+5 \ \overline{)\ 2x^3 \ +17x^2 \ +38x \ +15} \\ \underline{2x^3 \ +10x^2} \\ 7x^2 \ +38x \\ \underline{7x^2 \ +35x} \\ 3x \ +15 \\ \underline{3x \ +15} \\ R = 0 \end{array}$$

$$2x^3 + 17x^2 + 38x + 15 = (x + 5)(2x^2 + 7x + 3) = (x + 5)(x + 3)(2x + 1)$$

Possible dimensions of the box are $x + 5$ (height), $x + 3$ (length), and $2x + 1$ (width).

6) Determine the value of k such that when $P(x) = kx^3 + 5x^2 - 2x + 3$ is divided by $x + 1$, the remainder is 7.

The remainder is $P(-1)$. Solve for k when $P(-1)$ is set to equal 7.

$$\begin{aligned} P(-1) &= k(-1)^3 + 5(-1)^2 - 2(-1) + 3 \\ 7 &= -k + 5 + 2 + 3 \\ 7 &= -k + 10 \\ -3 &= -k \\ 3 &= k \end{aligned}$$

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Therefore, the value of k is 3.

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