2.7 Unit 2 Pre-Test Review – Factor Theorem and Inequalities with solutions

This document was composed and formatted by Annie Bernatchez.

The course material originated from www.jensenmath.ca

This review covers all of Unit 2: Factor Theorem and Inequalities, and is meant to be used to prepare for the unit test.

Section 1: 2.1 - Long and Synthetic Division / Remainder Theorem

1) What is the remainder when $x^4 - 4x^2 - 2x + 3$ is divided by $x + 1$? Do not divide. Support your answer with an explanation.

$$\begin{aligned} P(-1) &= (-1)^4 - 4(-1)^2 - 2(-1) + 3 \\ &= 1 - 4 + 2 + 3 \\ &= 2 \end{aligned}$$

Based on the remainder theorem, the remainder will be 2.

2) Is $x - 3$ a factor of the polynomial $3x^2 - 8x - 3$? Do not divide. Support your answer with an explanation.

$$\begin{aligned} P(3) &= 3(3)^2 - 8(3) - 3 \\ &= 27 - 24 - 3 \\ &= 0 \end{aligned}$$

Yes, because the remainder is 0.

3) Divide $\frac{f(x)}{g(x)}$ and state the answer in quotient form. Use synthetic division where possible.

a) $f(x) = x^4 - 4x^2 - 2x + 3$, $g(x) = x - 2$

$$\begin{array}{r|rrrrr} 2 & 1 & 0 & -4 & -2 & 3 \\ & & 2 & 4 & 0 & -4 \\ \hline & 1 & 2 & 0 & -2 & -1 \\ & x^3 & x^2 & x & \# & R \end{array}$$

The result in quotient form is:

$$\frac{x^4 - 4x^2 - 2x + 3}{x - 2} = x^3 + 2x^2 - 2 + \frac{-1}{x - 2}$$

The expression that can be used to check the division is:

$$x^4 - 4x^2 - 2x + 3 = (x - 2)(x^3 + 2x^2 - 2) - 1$$

b) $f(x) = x^5 - x^4 + 2x^3 + 3x - 2$, $g(x) = x^2 + 2$

$$\begin{array}{r} x^3 - x^2 + 0x + 2 \\ x^2 + 0x + 2 \;\overline{)}\; x^5 - x^4 + 2x^3 + 0x^2 + 3x - 2 \\ \underline{x^5 + 0x^4 + 2x^3} \\ -x^4 + 0x^3 + 0x^2 \\ \underline{-x^4 + 0x^3 - 2x^2} \\ 0x^3 + 2x^2 + 3x \\ \underline{0x^3 + 0x^2 + 0x} \\ 2x^2 + 3x - 2 \\ \underline{2x^2 + 0x + 4} \\ R = 3x - 6 \end{array}$$

The result in quotient form is:

$$\frac{x^5 - x^4 + 2x^3 + 3x - 2}{x^2 + 2} = x^3 - x^2 + 2 + \frac{3x - 6}{x^2 + 2}$$

The expression that can be used to check the division is:

$$x^5 - x^4 + 2x^3 + 3x - 2 = (x^2 + 2)(x^3 - x^2 + 2) + (3x - 6)$$

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4) Perform each division. Express the answer in quotient form and write the statement that could be used to check the division.

a) $x^3 + 9x^2 - 5x + 3$ divided by $x - 2$

$$\begin{array}{r|rrrr} 2 & 1 & 9 & -5 & 3 \\ & & 2 & 22 & 34 \\ \hline & 1 & 11 & 17 & 37 \\ & x^2 & x & \# & R \end{array}$$

$$\frac{x^3 + 9x^2 - 5x + 3}{x - 2} = x^2 + 11x + 17 + \frac{37}{x - 2}$$

$$x^3 + 9x^2 - 5x + 3 = (x - 2)(x^2 + 11x + 17) + 37$$

b) $12x^3 - 2x^2 + x - 11$ divided by $3x + 1$

$$\begin{array}{r} 4x^2 \ -2x \ +1 \\ \hline 3x + 1 \ ) \ 12x^3 \ -2x^2 \ +x \ -11 \\ \underline{12x^3 \ +4x^2} \\ -6x^2 \ +x \\ \underline{-6x^2 \ -2x} \\ 3x \ -11 \\ \underline{3x \ +1} \\ R = \ -12 \end{array}$$

$$\frac{12x^3 - 2x^2 + x - 11}{3x + 1} = 4x^2 - 2x + 1 + \frac{-12}{3x + 1}$$

$$12x^3 - 2x^2 + x - 11 = (3x + 1)(4x^2 - 2x + 1) - 12$$

c) $-8x^4 - 4x + 10x^3 - x^2 + 15$ divided by $2x - 1$

$$\begin{array}{r} -4x^3 \ +3x^2 \ +x \ -\frac{3}{2} \\ \hline 2x - 1 \ ) \ -8x^4 \ +10x^3 \ -x^2 \ -4x \ +15 \\ \underline{-8x^4 \ +4x^3} \\ 6x^3 \ -x^2 \\ \underline{6x^3 \ -3x^2} \\ 2x^2 \ -4x \\ \underline{2x^2 \ -x} \\ -3x \ +15 \\ \underline{-3x \ +\frac{3}{2}} \\ R = \frac{27}{2} \end{array}$$

$$\frac{-8x^4 + 10x^3 - x^2 - 4x + 15}{2x - 1} = -4x^3 + 3x^2 + x - \frac{3}{2} + \frac{27}{2(2x - 1)}$$

$$-8x^4 + 10x^3 - x^2 - 4x + 15 = (2x - 1)\left(-4x^3 + 3x^2 + x - \frac{3}{2}\right) + \frac{27}{2}$$

d) $x^3 + 4x^2 - 3$ divided by $x - 2$

$$\begin{array}{r|rrrr} 2 & 1 & 4 & 0 & -3 \\ & & 2 & 12 & 24 \\ \hline & 1 & 6 & 12 & 21 \\ & x^2 & x & \# & R \end{array}$$

$$\frac{x^3 + 4x^2 - 3}{x - 2} = x^2 + 6x + 12 + \frac{21}{x - 2}$$

$$x^3 + 4x^2 - 3 = (x - 2)(x^2 + 6x + 12) + 21$$

5) Determine the value of k such that when $f(x) = x^4 + kx^3 - 3x - 5$ is divided by $x - 3$, the remainder is -10.

$$f(3) = (3)^4 + k(3)^3 - 3(3) - 5$$
$$-10 = 81 + 27k - 9 - 5$$
$$-10 = 27k + 67$$
$$-77 = 27k$$

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$k = -\frac{77}{27}$

Section 2: 2.2 - Factor Theorem

6) Suppose the cubic polynomial $8x^3 + mx^2 + nx - 6$ has both $2x + 3$ and $x - 1$ as factors. Find m and n. Do not divide.

Since $2x + 3$ is a factor, $P\left(-\frac{3}{2}\right) = 0$:

$$0 = 8\left(-\frac{3}{2}\right)^3 + m\left(-\frac{3}{2}\right)^2 + n\left(-\frac{3}{2}\right) - 6$$
$$0 = -27 + \frac{9}{4}m - \frac{3}{2}n - 6$$
$$33 = \frac{9}{4}m - \frac{3}{2}n \quad (1)$$

Since $x - 1$ is a factor, $P(1) = 0$:

$$0 = 8(1)^3 + m(1)^2 + n(1) - 6$$
$$0 = 8 + m + n - 6$$
$$-2 = m + n \quad (2)$$

Multiplying (1) by 4: $132 = 9m - 6n$. From (2), $n = -2 - m$. Substituting:

$$132 = 9m - 6(-2 - m)$$
$$132 = 9m + 12 + 6m$$
$$120 = 15m$$
$$m = 8$$

Substituting $m = 8$ into (2): $n = -2 - 8 = -10$

$m = 8,\ n = -10$

7) Factor each of the following:

a) $x^3 - 4x^2 + x + 6$

Possible factors: $\pm 1,\ \pm 2,\ \pm 3,\ \pm 6$

$f(2) = 0$, so $x - 2$ is a factor

$$\begin{array}{c|cccc} 2 & 1 & -4 & 1 & 6 \\ & & 2 & -4 & -6 \\ \hline & 1 & -2 & -3 & 0 \\ & x^2 & x & \# & R \end{array}$$

$$x^3 - 4x^2 + x + 6 = (x - 2)(x^2 - 2x - 3) = (x - 2)(x - 3)(x + 1)$$

b) $3x^3 - 5x^2 - 26x - 8$

Possible factors: $\pm 1,\ \pm \frac{1}{3},\ \pm 2,\ \pm \frac{2}{3},\ \pm 4,\ \pm \frac{4}{3},\ \pm 8,\ \pm \frac{8}{3}$

$f(-2) = 0$, so $x + 2$ is a factor

$$\begin{array}{c|cccc} -2 & 3 & -5 & -26 & -8 \\ & & -6 & 22 & 8 \\ \hline & 3 & -11 & -4 & 0 \\ & x^2 & x & \# & R \end{array}$$

$$3x^3 - 5x^2 - 26x - 8 = (x + 2)(3x^2 - 11x - 4) = (x + 2)(x - 4)(3x + 1)$$

c) $-4x^3 - 4x^2 + 16x + 16$

$$\begin{aligned} -4x^3 - 4x^2 + 16x + 16 &= -4(x^3 + x^2 - 4x - 4) \\ &= -4\left[x^2(x + 1) - 4(x + 1)\right] \\ &= -4(x + 1)(x^2 - 4) \\ &= -4(x + 1)(x - 2)(x + 2) \end{aligned}$$

d) $x^3 - 64$

Difference of cubes: $a^3 - b^3 = (a - b)(a^2 + ab + b^2)$

$$x^3 - 64 = x^3 - 4^3 = (x - 4)(x^2 + 4x + 16)$$

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Section 3: 2.3 & 2.6 - Factoring to Solve Equations and Inequalities

8) Determine the real roots of each equation.

a) $(5x^2 + 20)(3x^2 - 48) = 0$

$5x^2 + 20 = 0 \Rightarrow x^2 = -4$, no real solution

$3x^2 - 48 = 0 \Rightarrow x^2 = 16 \Rightarrow x = \pm 4$

b) $(2x^2 - x - 13)(x^2 + 1) = 0$

$2x^2 - x - 13$ is not factorable, so use the quadratic formula:

$$x = \frac{1 \pm \sqrt{(-1)^2 - 4(2)(-13)}}{2(2)} = \frac{1 \pm \sqrt{105}}{4}$$

$x^2 + 1 = 0 \Rightarrow x = \pm\sqrt{-1}$, no real solution

9) Solve the following polynomial equations.

a) $2x^3 + 1 = x^2 + 2x$

$$\begin{aligned} 2x^3 - x^2 - 2x + 1 &= 0 \\ x^2(2x - 1) - 1(2x - 1) &= 0 \\ (2x - 1)(x^2 - 1) &= 0 \\ (2x - 1)(x - 1)(x + 1) &= 0 \end{aligned}$$

$x = \frac{1}{2}, 1, -1$

b) $x^3 + 6x^2 + 11x + 6 = 0$

Possible factors: $\pm 1, \pm 2, \pm 3, \pm 6$

$f(-1) = 0$, so $x + 1$ is a factor

$$\begin{array}{r|cccc} -1 & 1 & 6 & 11 & 6 \\ & & -1 & -5 & -6 \\ \hline & 1 & 5 & 6 & 0 \\ & x^2 & x & \# & R \end{array}$$

$$\begin{aligned} (x + 1)(x^2 + 5x + 6) &= 0 \\ (x + 1)(x + 2)(x + 3) &= 0 \end{aligned}$$

$x = -1, -2, -3$

c) $x^5 - 4x^3 - x^2 + 4 = 0$

$$\begin{aligned} x^3(x^2 - 4) - 1(x^2 - 4) &= 0 \\ (x^2 - 4)(x^3 - 1) &= 0 \\ (x - 2)(x + 2)(x - 1)(x^2 + x + 1) &= 0 \end{aligned}$$

$x = 2, -2, 1$. The factor $x^2 + x + 1$ has $b^2 - 4ac = 1 - 4 = -3$, so it contributes no real solutions.

d) $3x^3 + 2x^2 - 11x - 10 = 0$

Possible factors: $\pm 1, \pm \frac{1}{3}, \pm 2, \pm \frac{2}{3}, \pm 5, \pm \frac{5}{3}, \pm 10, \pm \frac{10}{3}$

$f(-1) = 0$, so $x + 1$ is a factor

$$\begin{array}{r|cccc} -1 & 3 & 2 & -11 & -10 \\ & & -3 & 1 & 10 \\ \hline & 3 & -1 & -10 & 0 \\ & x^2 & x & \# & R \end{array}$$

$$\begin{aligned} (x + 1)(3x^2 - x - 10) &= 0 \\ (x + 1)(x - 2)(3x + 5) &= 0 \end{aligned}$$

$x = -1, 2, -\frac{5}{3}$

10) Solve the following polynomial inequalities. (Refer to #9 where you factored the polynomials)

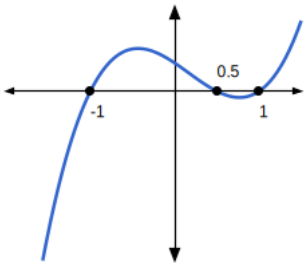
a) $2x^3 + 1 < x^2 + 2x$

$2x^3 - x^2 - 2x + 1 < 0$

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$(2x-1)(x-1)(x+1)<0$

x-intercepts at $x=-1,\ \frac{1}{2},\ 1$; positive leading coefficient, odd degree, so Q3 → Q1



Interval	$(-\infty,\ -1)$	$(-1,\ 0.5)$	$(0.5,\ 1)$	$(1,\ \infty)$
Sign	-	+	-	+

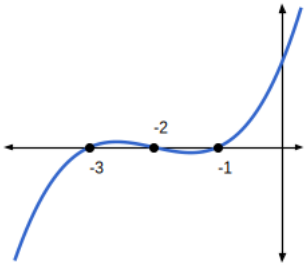
Solution: when $x<-1$ or $0.5<x<1$

$x\in(-\infty,\ -1)\cup(0.5,\ 1)$

b) $x^3+6x^2+11x+6>0$

$(x+1)(x+2)(x+3)>0$

x-intercepts at $x=-3,\ -2,\ -1$; positive leading coefficient, odd degree, so Q3 → Q1



Interval	$(-\infty,\ -3)$	$(-3,\ -2)$	$(-2,\ -1)$	$(-1,\ \infty)$
Sign	-	+	-	+

Solution: when $-3<x<-2$ or $x>-1$

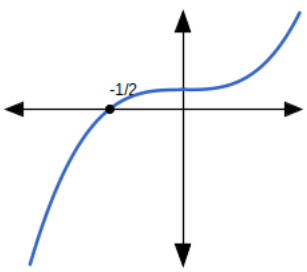
$x\in(-3,\ -2)\cup(-1,\ \infty)$

11) Where is the polynomial $y=8x^3+1$ positive? Justify your solution.

Sum of cubes:

$$8x^3+1>0$$
$$(2x+1)\left[(2x)^2-(2x)(1)+(1)^2\right]>0$$
$$(2x+1)(4x^2-2x+1)>0$$

Check the discriminant of $4x^2-2x+1$: $b^2-4ac=(-2)^2-4(4)(1)=-12$, no real roots, so this factor is always positive (positive leading coefficient). The sign is therefore determined entirely by $2x+1=0\Rightarrow x=-\frac{1}{2}$.



Solution: $8x^3+1>0$ when $x>-\frac{1}{2}$

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$x \in \left(-\frac{1}{2}, \infty\right)$

12) Solve $6x^3 + 13x^2 - 41x + 12 \leq 0$ using a sign chart.

Possible zeros: $\pm 1, \ \pm\frac{1}{2}, \ \pm\frac{1}{3}, \ \pm\frac{1}{6}, \ \pm 2, \ \pm\frac{2}{3}, \ \pm 3, \ \pm\frac{3}{2}, \ \pm 4, \ \pm\frac{4}{3}, \ \pm 6, \ \pm 12$

$f(-4) = 0$, so $x + 4$ is a factor

$$\begin{array}{r|cccc} -4 & 6 & 13 & -41 & 12 \\ & & -24 & 44 & -12 \\ \hline & 6 & -11 & 3 & 0 \\ & x^2 & x & \# & R \end{array}$$

$$(x+4)(6x^2 - 11x + 3) \leq 0$$
$$(x+4)(2x-3)(3x-1) \leq 0$$

x-intercepts at $x = -4, \ \frac{1}{3}, \ \frac{3}{2}$

Interval	$(-\infty, -4)$	$\left(-4, \frac{1}{3}\right)$	$\left(\frac{1}{3}, \frac{3}{2}\right)$	$\left(\frac{3}{2}, \infty\right)$
Sign	-	+	-	+

Solution: when $x \leq -4$ or $\frac{1}{3} \leq x \leq \frac{3}{2}$

$x \in (-\infty, -4] \cup \left[\frac{1}{3}, \frac{3}{2}\right]$

Section 4: 2.4 - Families of Polynomials

13) Find the equation for the family of quartic polynomials that have real roots of 3 (order 2) and $2 \pm \sqrt{2}$.

Factor from $x = 3$: $(x-3)$ order 2

$$x - 2 = \pm\sqrt{2}$$
$$(x-2)^2 = 2$$
$$x^2 - 4x + 4 = 2$$
$$x^2 - 4x + 2 = 0$$

Equation for the family:

$$y = k(x-3)^2(x^2 - 4x + 2)$$

14) A family of cubic polynomials has roots of -2, -3 and -5. Find the member of this family that passes through the point $(2, -35)$. What is this polynomial's y-intercept?

$$f(x) = k(x+2)(x+3)(x+5)$$
$$f(2) = k(2+2)(2+3)(2+5)$$
$$-35 = k(4)(5)(7)$$
$$-35 = 140k$$
$$k = -\frac{1}{4}$$
$$f(0) = -\frac{1}{4}(0+2)(0+3)(0+5)$$
$$= -\frac{1}{4}(30)$$
$$= -\frac{15}{2}$$

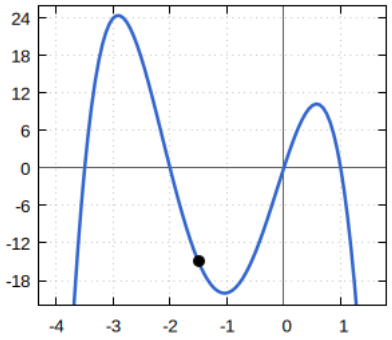
The equation is $f(x) = -\frac{1}{4}(x+2)(x+3)(x+5)$. It has a y-intercept of $\left(0, -\frac{15}{2}\right)$.

15) Find an equation for each of the following functions

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15) Find an equation for each of the following functions

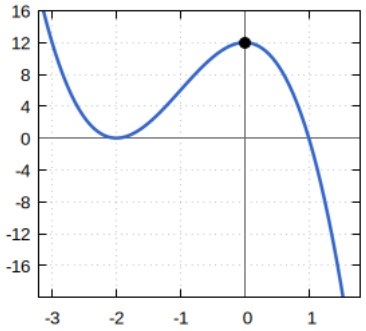
a)



$f(x) = k(2x + 7)(x + 2)(x)(x - 1)$
$-15 = k[2(-1.5) + 7](-1.5 + 2)(-1.5)(-1.5 - 1)$
$-15 = k(4)(0.5)(-1.5)(-2.5)$
$-15 = 7.5k$
$k = -2$

$f(x) = -2x(2x + 7)(x + 2)(x - 1)$

b)



$f(x) = k(x + 2)^2(x - 1)$
$12 = k(0 + 2)^2(0 - 1)$
$12 = k(4)(-1)$
$12 = -4k$
$k = -3$

$f(x) = -3(x + 2)^2(x - 1)$

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