2.4 Solving Polynomial Equations Lesson

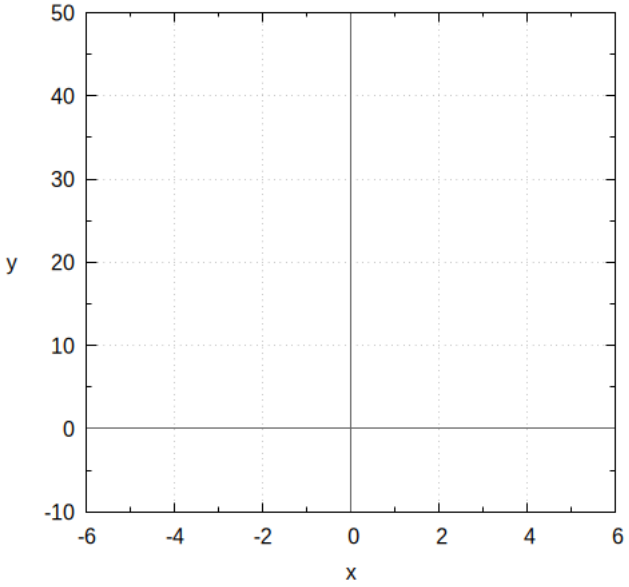
This document was composed and formatted by Annie Bernatchez.

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In this section, you will learn methods of solving polynomial equations of degree higher than two by factoring (using the factor theorem). You will also identify the connection between the roots of polynomial equations, the x-intercepts of the graph of a polynomial function, and the zeros of the function.

Part 1: Investigation

a) Use technology to graph the function $f(x) = x^4 - 13x^2 + 36$



b) Determine the x-intercepts from the graph

c) Factor $f(x)$. Then, use the factors to determine the zeros (roots) of $f(x)$.

Remember: The zeros of the function are the values of x that make $f(x) = 0$. If the polynomial equation is factorable, then the values of the zeros (roots) can be determined algebraically by solving each linear or quadratic factor.

d) How are the x-intercepts from the graph related to the roots (zeros) of the equation?

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Example 1: State the solutions to the following polynomials that are already in factored form

a) $x(2x + 3)(x - 5) = 0$

b) $(2x^2 - 3)(3x^2 + 1) = 0$

Example 2: Solve each polynomial equation by factoring

a) $x^3 - x^2 - 2x = 0$

b) $3x^3 + x^2 - 12x - 4 = 0$

Methods of factoring:

- Long division and synthetic division
- Factor by grouping

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- Difference of squares: $a^2 - b^2 = (a - b)(a + b)$
- Common Factoring
- Trinomial factoring (sum and product)
- Sum and difference of cubes: $a^3 + b^3 = (a + b)(a^2 - ab + b^2)$, $a^3 - b^3 = (a - b)(a^2 + ab + b^2)$

Example 3:

a) Solve $0 = 2x^3 + 3x^2 - 11x - 6$

b) What do your answers to part a) represent?

Example 4: Find the zeros of the polynomial function $f(x) = x^4 - 2x^3 - 7x^2 + 8x + 12$

Example 5:

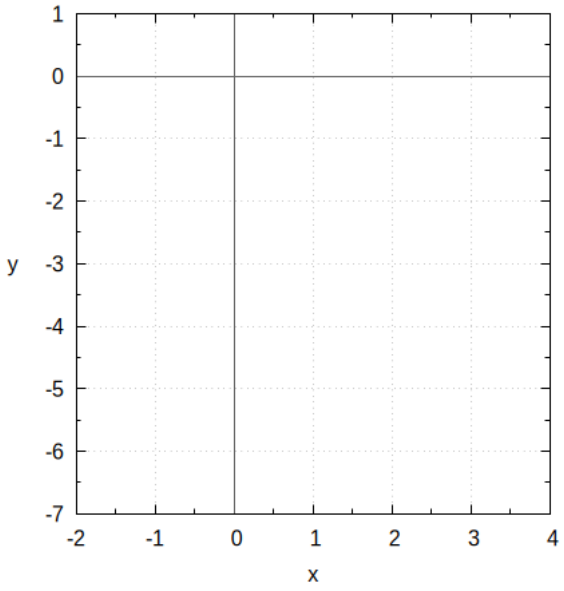
a) Find the roots of the polynomial function $f(x) = x^3 + x - 3x^2 - 3$

Start by rearranging in descending order of degree:

Note: Since the square root of a negative number is not a real number, the only REAL root is $x = 3$. $x = \pm\sqrt{-1}$ is considered a NON-REAL root.

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b) Use technology to look at the graph of the function $f(x)$. Comment on how x-intercept(s) of the graph are related to the REAL and NON-REAL roots of the equation.



Example 6: Find all real solutions for each polynomial equation

a) $0 = 2x^3 - 3x^2 - x - 2$

b) $0 = 8x^3 + 125$

Hint: This is a difference of cubes $\rightarrow a^3 + b^3 = (a + b)(a^2 - ab + b^2)$

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