2.2 Synthetic Division Lesson with solutions

This document was composed and formatted by Annie Bernatchez.

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In this section you will learn how to use synthetic division as an alternate method to dividing a polynomial by a binomial. Synthetic division is an efficient way to divide a polynomial by a binomial of the form $x - b$.

IMPORTANT: When using Polynomial OR Synthetic division…

- Terms must be arranged in descending order of degree, in both the divisor and the dividend.
- Zero must be used as the coefficient of any missing powers of the variable in both the divisor and the dividend.

Part 1: Synthetic Division When the Binomial Is of the Form $x - b$

Divide $3x^3 - 5x^2 - 7x - 1$ by $x - 3$. In this question, $b = 3$.

- List the coefficients of the dividend in the first row. To the left, write the b value (the zero of the divisor).
- Bring the first term down – this is the coefficient of the first term of the quotient. Multiply it by the b value and write this product below the second term of the dividend.
- Now add the terms together.
- Multiply this sum by the b value and write the product below the third term of the dividend. Repeat this process until the chart is complete.
- The last number below the chart is the remainder. The numbers before it are the coefficients of the quotient, starting with the degree that is one less than the dividend.

$$(3x^3 - 5x^2 - 7x - 1) \div (x - 3)$$

$$\begin{array}{r|rrrr} 3 & 3 & -5 & -7 & -1 \\ & & 9 & 12 & 15 \\ \hline & 3 & 4 & 5 & 14 \\ & x^2 & x & \# & R \end{array}$$

Don't forget that the answer can be written in two ways…

$$\frac{3x^3 - 5x^2 - 7x - 1}{x - 3} = 3x^2 + 4x + 5 + \frac{14}{x - 3}$$

OR

$$3x^3 - 5x^2 - 7x - 1 = (x - 3)(3x^2 + 4x + 5) + 14$$

Example 1: Use synthetic division to divide. Then write the multiplication statement that could be used to check the division.

a) $(x^4 - 2x^3 + 13x - 6) \div (x + 2)$

Rewrite the dividend with the missing power: $x^4 - 2x^3 + 0x^2 + 13x - 6$

$$(x^4 - 2x^3 + 0x^2 + 13x - 6) \div (x + 2)$$

$$\begin{array}{r|rrrrr} -2 & 1 & -2 & 0 & 13 & -6 \\ & & -2 & 8 & -16 & 6 \\ \hline & 1 & -4 & 8 & -3 & 0 \\ & x^3 & x^2 & x & \# & R \end{array}$$

$$x^4 - 2x^3 + 13x - 6 = (x + 2)(x^3 - 4x^2 + 8x - 3)$$

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Note: since the remainder is zero, both the quotient and divisor are factors of the dividend.

b) $(2x^3 - 5x^2 + 8x + 4) \div (x - 3)$

$$(2x^3 - 5x^2 + 8x + 4) \div (x - 3)$$

$$\begin{array}{r|rrrr} 3 & 2 & -5 & 8 & 4 \\ & & 6 & 3 & 33 \\ \hline & 2 & 1 & 11 & 37 \\ & x^2 & x & \# & R \end{array}$$

$$2x^3 - 5x^2 + 8x + 4 = (x - 3)(2x^2 + x + 11) + 37$$

Part 2: Synthetic Division When the Binomial Is of the Form $ax - b$

Divide $6x^3 + 5x^2 - 16x - 15$ by $2x + 3$

$2x + 3 = 2\left(x + \frac{3}{2}\right) \rightarrow b = -\frac{3}{2}$

- To use synthetic division, the divisor must be in the form $x - b$. Re-write the divisor by factoring out the coefficient of the x.
- We can now divide $6x^3 + 5x^2 - 16x - 15$ by $\left(x + \frac{3}{2}\right)$ using synthetic division as long as you remember to divide the quotient by 2 after.

$$(6x^3 + 5x^2 - 16x - 15) \div (x + \tfrac{3}{2})$$

$$\begin{array}{r|rrrr} -\frac{3}{2} & 6 & 5 & -16 & -15 \\ & & -9 & 6 & 15 \\ \hline & 6 & -4 & -10 & 0 \\ & x^2 & x & \# & R \end{array}$$

$$\frac{6x^2 - 4x - 10}{2} = 3x^2 - 2x - 5$$

$$6x^3 + 5x^2 - 16x - 15 = (2x + 3)(3x^2 - 2x - 5)$$

Check answer using long division

$$\begin{array}{r} 3x^2 \quad -2x \quad -5 \\ 2x + 3 \;)\; 6x^3 + 5x^2 - 16x - 15 \\ \underline{6x^3 + 9x^2} \\ -4x^2 - 16x \\ \underline{-4x^2 - 6x} \\ -10x - 15 \\ \underline{-10x - 15} \\ R = 0 \end{array}$$

Note: Synthetic division can only be used with a linear divisor. It is most useful with a divisor of the form $x - b$. If the divisor is $ax - b$, it can be used but long division may be easier.

Example 2: Find each quotient by choosing an appropriate strategy.

a) Divide $x^3 - 4x^2 + 2x + 3$ by $x - 3$

Use synthetic division because we have a linear divisor of the form $x - b$.

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$$(x^3 - 4x^2 + 2x + 3) \div (x - 3)$$

$$\begin{array}{r|rrrr} 3 & 1 & -4 & 2 & 3 \\ & & 3 & -3 & -3 \\ \hline & 1 & -1 & -1 & 0 \\ & x^2 & x & \# & R \end{array}$$

$$x^3 - 4x^2 + 2x + 3 = (x - 3)(x^2 - x - 1)$$

b) Divide $12x^4 - 56x^3 + 59x^2 + 9x - 18$ by $2x + 1$

$$\begin{array}{r} 6x^3 - 31x^2 + 45x - 18 \\ 2x + 1 \;\overline{)\; 12x^4 - 56x^3 + 59x^2 + 9x - 18} \\ \underline{12x^4 + 6x^3} \\ -62x^3 + 59x^2 \\ \underline{-62x^3 - 31x^2} \\ 90x^2 + 9x \\ \underline{90x^2 + 45x} \\ -36x - 18 \\ \underline{-36x - 18} \\ R = 0 \end{array}$$

$$12x^4 - 56x^3 + 59x^2 + 9x - 18 = (2x + 1)(6x^3 - 31x^2 + 45x - 18)$$

c) Divide $x^4 - 2x^3 + 5x + 3$ by $x^2 + 2x + 1$

Use long division since it is a non-linear divisor.

$$\begin{array}{r} x^2 - 4x + 7 \\ x^2 + 2x + 1 \;\overline{)\; x^4 - 2x^3 + 0x^2 + 5x + 3} \\ \underline{x^4 + 2x^3 + x^2} \\ -4x^3 - x^2 + 5x \\ \underline{-4x^3 - 8x^2 - 4x} \\ 7x^2 + 9x + 3 \\ \underline{7x^2 + 14x + 7} \\ R = -5x - 4 \end{array}$$

$$x^4 - 2x^3 + 5x + 3 = (x^2 + 2x + 1)(x^2 - 4x + 7) - 5x - 4$$

d) Divide $x^4 - x^3 - x^2 + 2x + 1$ by $x^2 + 2$

Use long division since it is a non-linear divisor.

$$\begin{array}{r} x^2 - x - 3 \\ x^2 + 0x + 2 \;\overline{)\; x^4 - x^3 - x^2 + 2x + 1} \\ \underline{x^4 + 0x^3 + 2x^2} \\ -x^3 - 3x^2 + 2x \\ \underline{-x^3 + 0x^2 - 2x} \\ -3x^2 + 4x + 1 \\ \underline{-3x^2 + 0x - 6} \\ R = 4x + 7 \end{array}$$

$$x^4 - x^3 - x^2 + 2x + 1 = (x^2 + 2)(x^2 - x - 3) + 4x + 7$$

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