Grade 12 Advanced Functions Full Course Practice Final Exam with solutions

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Advanced Functions (MHF4U) Final Exam—Solutions

Section 1: Multiple Choice

1) What is the end behavior of the function $P(x) = -5x^4 + x^3 + 7x$

Degree 4 (even), leading coefficient -5 (negative) ⇒ both ends down.

A) Q2 → Q1	B) Q3 → Q1	C) Q3 → Q4	D) Q2 → Q4

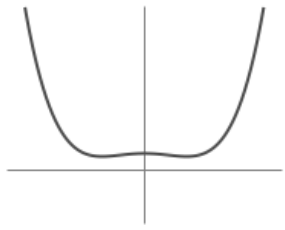
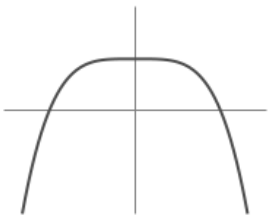
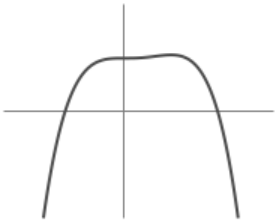
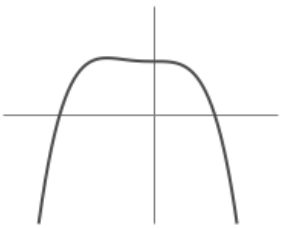
2) Which of the following is an odd function

Check $f(-x) = -f(x)$. Only B satisfies this: $f(-x) = -2(-x)^5 - 2(-x) = 2x^5 + 2x = -(-2x^5 - 2x) = -f(x)$.

A) $y = 2x^3 - 3x^2$	B) $y = -2x^5 - 2x$	C) $y = 2x^3 - x + 1$	D) All of these

3) Which of the following graphs represents the function $y = -3x^4 + 2x^3 + 1$

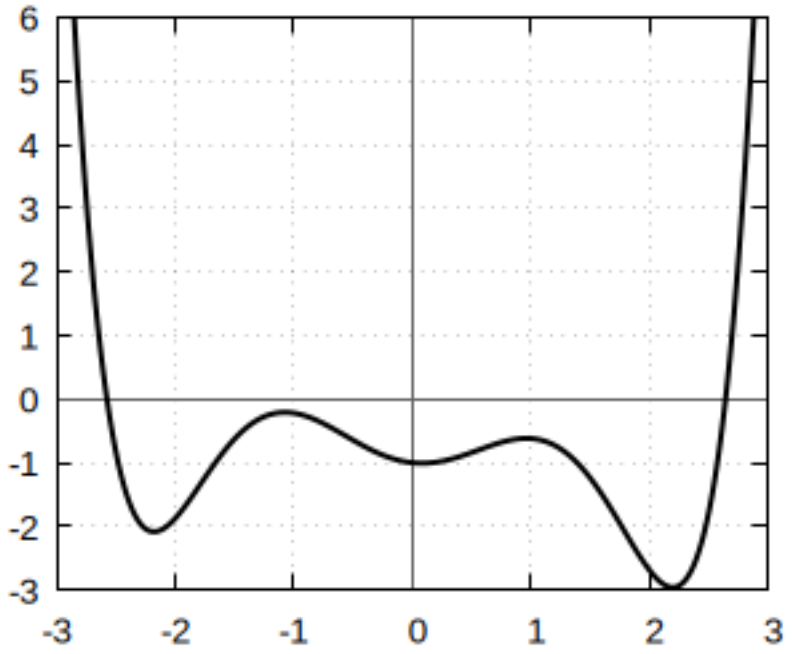
$f'(x) = -12x^3 + 6x^2 = 6x^2(1 - 2x)$; critical points at $x = 0$ (inflection, not an extremum, since $x^2 \geq 0$ doesn't change sign) and $x = 0.5$ (local max, since $f''(0.5) < 0$). A single hump, both ends down, peak just right of the y-axis at $x = 0.5$.

A)	B)	C)	D)
			

4) What is the least possible degree of the function represented in the graph to right:

Both ends up (even degree) with 5 visible turning points; a degree-4 polynomial has at most 3 turning points, so the least possible even degree that can produce 5 turning points is 6.

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A) 2	B) 4	C) 5	D) 6	

5) What is the remainder when the polynomial $y = x^3 - 5x + 2$ is divided by $x + 1$

By the remainder theorem: $f(-1) = (-1)^3 - 5(-1) + 2 = -1 + 5 + 2 = 6$

A) 8	B) −2	C) 6	D) 0

6) The polynomial $f(x) = 8x^3 - 27$ will factor into…

Difference of cubes: $(2x)^3 - 3^3 = (2x - 3)[(2x)^2 + (2x)(3) + 3^2] = (2x - 3)(4x^2 + 6x + 9)$

A) $(2x - 3)(2x + 3)^2$	B) $(2x - 3)(4x^2 + 6x + 9)$
C) $(2x + 3)(4x^2 - 6x + 9)$	D) $(2x + 3)(2x - 3)^2$

7) Which of the following divisors is a factor the polynomial $3x^4 + x^3 - 14x^2 - 4x + 8$

Test $x = -1$ (root of $x + 1$): $3(1) + (-1) - 14(1) - 4(-1) + 8 = 3 - 1 - 14 + 4 + 8 = 0$. So $x + 1$ is a factor.

A) $x - 1$	B) $4x + 3$	C) x+1	D) $3x + 4$

8) Which logarithm statement below is equivalent to the exponential statement $3^4 = 81$?

$b^y = x \Leftrightarrow \log_b(x) = y$, so $3^4 = 81 \Leftrightarrow \log_3(81) = 4$

A) $\log_3 4 = 81$	B) $\log_4 81 = 3$	C) $\log_4 3 = 81$	D) $\log_3 81 = 4$

1) C 2) B 3) C 4) D 5) C 6) B 7) C 8) D

9) Evaluate $\log_3 9$

$\log_3(9) = \log_3(3^2) = 2$

A) 3	B) 9	C) 0.5	D) 2

10) The pH of a solution with a hydronium ion concentration of 0.04 mol/L is

$pH = -\log(0.04) \approx 1.40$

A) 1.40	B) 1.09	C) 1.04	D) 2.62

11) Convert $300°$ to radian measure.

$300 \times \frac{\pi}{180} = \frac{5\pi}{3}$

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A) $\frac{5\pi}{3}$ radians	B) $\frac{4\pi}{3}$ radians	C) $\frac{5\pi}{6}$ radians	D) $\frac{7\pi}{6}$ radians

12) Convert $\frac{5\pi}{4}$ radians to degree measure.

$\frac{5\pi}{4} \times \frac{180}{\pi} = 225°$

A) 135°	B) 225°	C) 45°	D) 315°

13) Use special triangles and the CAST rule to determine the exact value of $\csc \frac{5\pi}{3}$

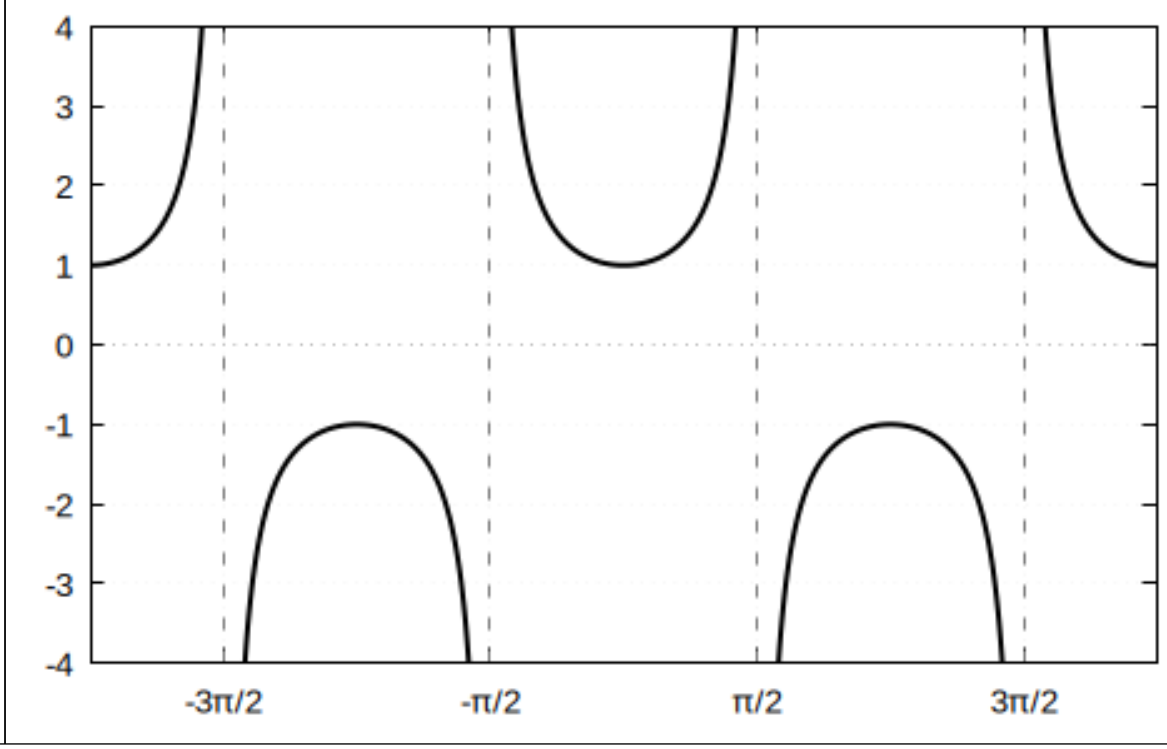
$\frac{5\pi}{3}$ is in QIV, reference angle $\frac{\pi}{3}$: $\sin\left(\frac{5\pi}{3}\right) = -\sin\left(\frac{\pi}{3}\right) = -\frac{\sqrt{3}}{2}$; $\csc = \frac{1}{\sin} = -\frac{2}{\sqrt{3}}$

A) $-\frac{\sqrt{3}}{2}$	B) $-\frac{2}{\sqrt{3}}$	C) -2	D) $\sqrt{3}$

14) The graph shown is the of the function…

Repeating up/down U-shapes with vertical asymptotes at odd multiples of $\pi/2$, minimum value 1 on the upper branches, maximum value -1 on the lower branches: this is $y = \sec x$.

A) $y = \csc x$
B) $y = \sec x$
C) $y = \cot x$
D) $y = \tan x$



15) Which statement below is TRUE about the rational function $y = \frac{3x-2}{x+1}$?

VA: $x = -1$. HA: $y = 3$ (ratio of leading coefficients). y-int: $f(0) = \frac{-2}{1} = -2$. x-int: $3x - 2 = 0 \Rightarrow x = \frac{2}{3}$.

A) VA $x = 3$ and y-int at $(0, -2)$	B) VA $x = -1$ and y-int at $(0, -2)$
C) HA $y = 3$ and x-int at $(2, 0)$	D) HA $y = -1$ and x-int at $\left(\frac{3}{2}, 0\right)$

16) Using a surrounding interval, a good estimate for the instantaneous rate of change of the temperature at 10 minutes is…

Using the surrounding interval $t = 8$ to $t = 13$: $m = \frac{290-205}{13-8} = \frac{85}{5} = 17$

Time in Minutes	**0**	**5**	**8**	**10**	**13**	**15**	**19**	**21**	**25**
Temp. (in degree Celsius)	25	120	205	250	290	280	290	285	285

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A) 250 degrees/min	B) 15 degrees/min	C) 10.4 degrees/min	D) 17 degrees/min

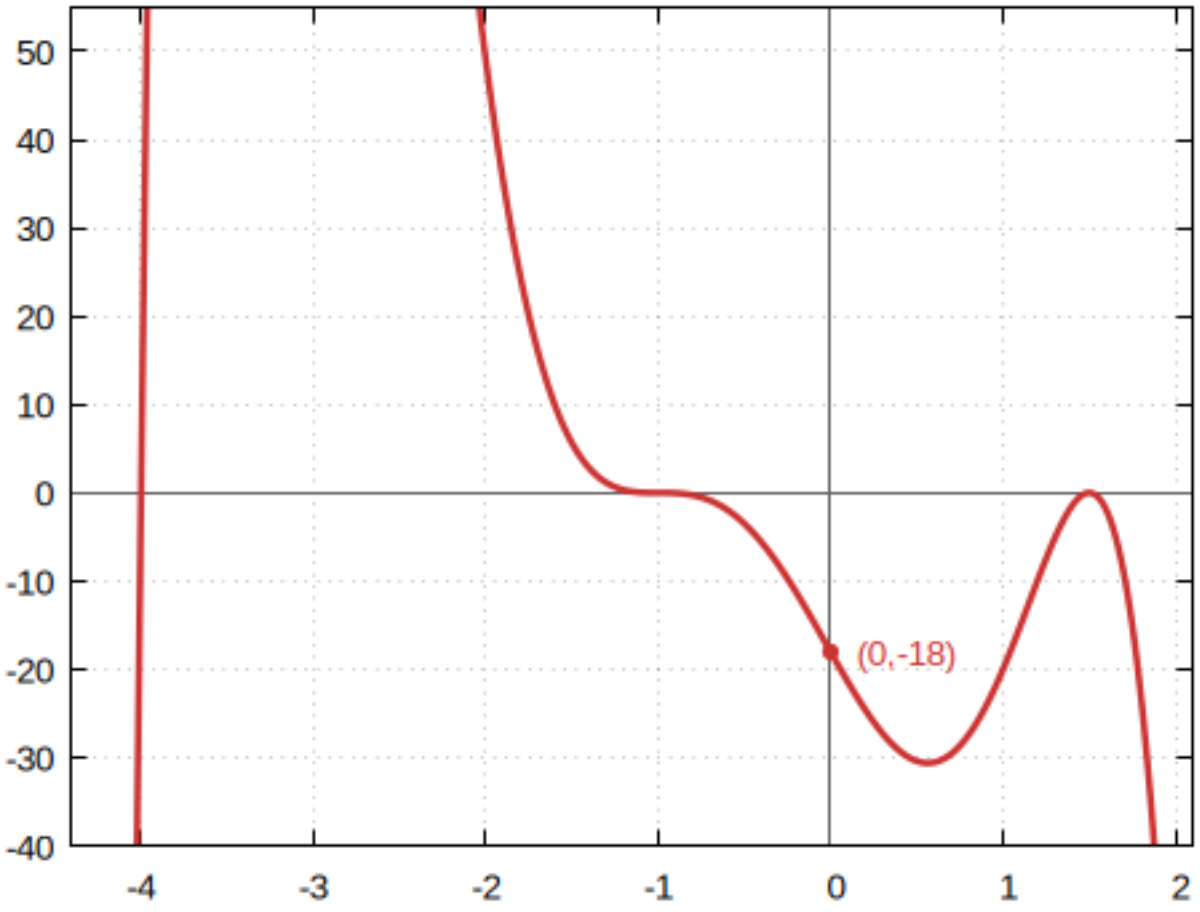
9) D 10) A 11) A 12) B 13) B 14) B 15) B 16) D

Section 2: Polynomial Functions

17) Complete the chart and sketch a possible graph of the function labelling the x and y intercepts. [6]

$f(x) = -\frac{1}{2}(x+4)(2x-3)^2(x+1)^3$

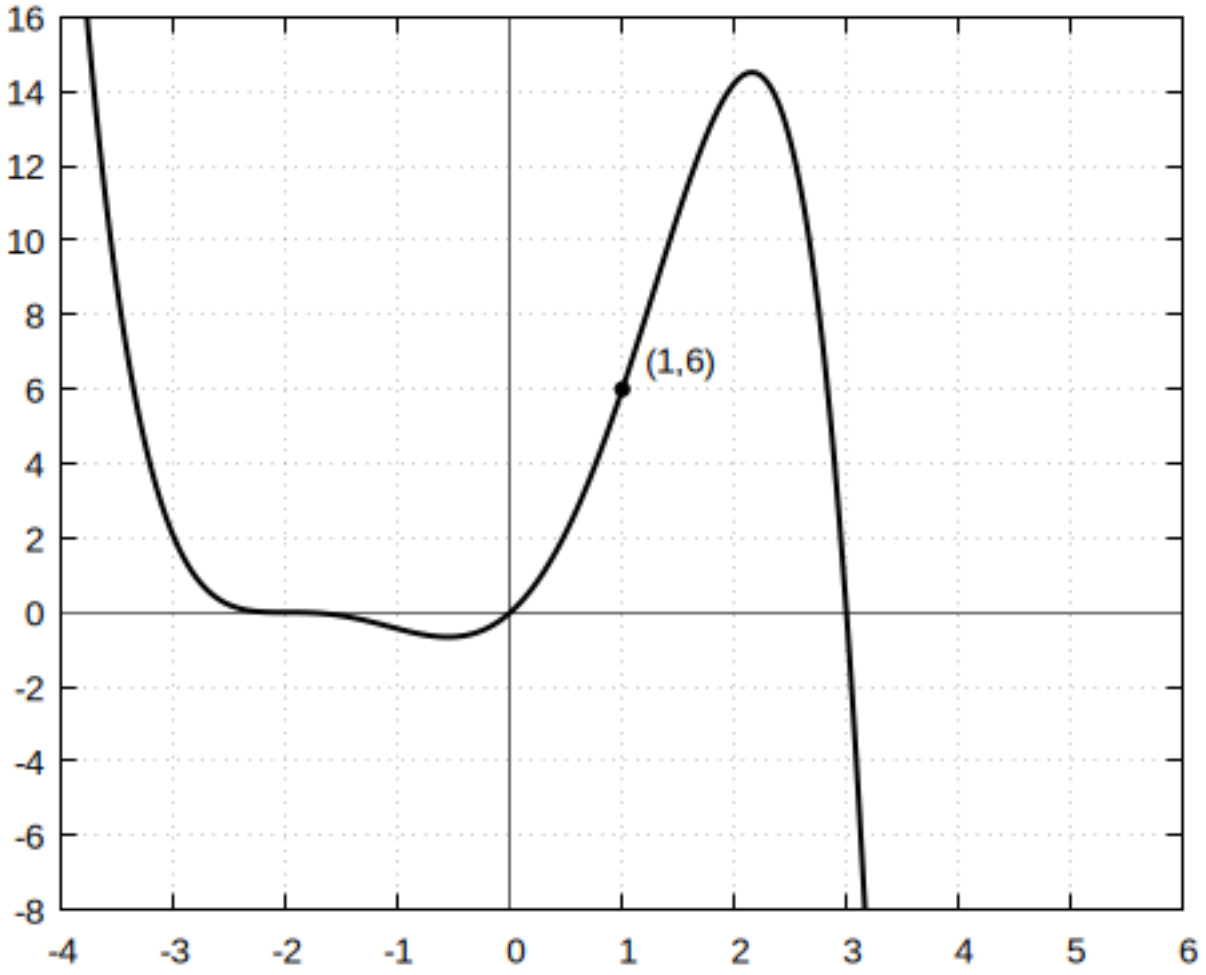
Degree	Leading Coefficient	End Behaviour	x-intercepts with orders	y-intercept
$1+2+3=6$	$-\frac{1}{2}(1)(2)^2(1) = -2$	Q3 → Q4	$(-4, 0)$ order 1 $(1.5, 0)$ order 2 $(-1, 0)$ order 3	$f(0) = -\frac{1}{2}(4)(-3)^2(1)^3 = -18$



(Sketch: passes through $(-4, 0)$ heading up from below on the far left since Q3 → Q4 means both ends point down — the left arm comes up from $-\infty$, crosses at $x = -4$, wiggles through the order-3 touch/inflection at $x = -1$, dips to the y-intercept $(0, -18)$, rises to touch the x-axis at the order-2 zero $x = 1.5$, then turns back down to $-\infty$ on the right.)

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18) Find an equation in factored form for the quintic polynomial whose graph is shown below. State the final equation in factored form. [3]



Zeros: $x = -2$ (order 3, the graph flattens/wiggles through this crossing rather than a clean line), $x = 0$ (order 1), $x = 3$ (order 1). Degree $3 + 1 + 1 = 5$ ✓ quintic.

$f(x) = k(x+2)^3 x(x-3)$

Using the marked point $(1, 6)$: $f(1) = k(3)^3(1)(-2) = -54k = 6 \Rightarrow k = -\frac{1}{9}$

Equation: $f(x) = -\frac{1}{9}(x+2)^3 x(x-3)$

19) Use long division to divide $6x^3 + x^2 - 13x + 5$ by $2x + 1$. Express your answer in quotient form: [3]

$$
\begin{array}{r}
3x^2 \quad -x \quad -6 \\
2x+1\ \overline{)\ 6x^3 \quad +x^2 \quad -13x \quad +5} \\
\underline{6x^3 \quad +3x^2} \\
\quad -2x^2 \quad -13x \\
\underline{-2x^2 \quad -x} \\
\quad\quad -12x \quad +5 \\
\quad\quad \underline{-12x \quad -6} \\
\quad\quad R = 11
\end{array}
$$

Final Answer in Quotient Form: $\frac{6x^3 + x^2 - 13x + 5}{2x+1} = 3x^2 - x - 6 + \frac{11}{2x+1}$

20) Use synthetic division to divide $2x^3 + 5x^2 - 5$ by $x + 2$. Express your answer using the multiplication statement

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that can be used to check the division. [3]

$$(2x^3 + 5x^2 + 0x - 5) \div (x + 2)$$

$$\begin{array}{r|cccc} -2 & 2 & 5 & 0 & -5 \\ & & -4 & -2 & 4 \\ \hline & 2 & 1 & -2 & -1 \\ & x^2 & x & \# & R \end{array}$$

Multiplication Statement to Check Division: $2x^3 + 5x^2 - 5 = (x + 2)(2x^2 + x - 2) - 1$

21) Fully factor the polynomial $P(x) = x^3 + x^2 - 10x + 8$. I am looking to see a FULL list of possible zeros, your test of the zero, and your polynomial division. [5]

Possible Zeros:

$x = \pm 1, \pm 2, \pm 4, \pm 8$

Test(s):

$f(1) = 1 + 1 - 10 + 8 = 0$

$\therefore x - 1$ is a factor

$$(x^3 + x^2 - 10x + 8) \div (x - 1)$$

$$\begin{array}{r|cccc} 1 & 1 & 1 & -10 & 8 \\ & & 1 & 2 & -8 \\ \hline & 1 & 2 & -8 & 0 \\ & x^2 & x & \# & R \end{array}$$

$x^2 + 2x - 8 = (x + 4)(x - 2)$

Factored Form: $P(x) = (x - 1)(x + 4)(x - 2)$

22) Solve the following equation using any algebraic method. [3]

$2x^3 + 5x^2 - 14x - 8 = 0$

Test $x = 2$: $2(8) + 5(4) - 14(2) - 8 = 16 + 20 - 28 - 8 = 0$ $\therefore x - 2$ is a factor

$$(2x^3 + 5x^2 - 14x - 8) \div (x - 2)$$

$$\begin{array}{r|cccc} 2 & 2 & 5 & -14 & -8 \\ & & 4 & 18 & 8 \\ \hline & 2 & 9 & 4 & 0 \\ & x^2 & x & \# & R \end{array}$$

$2x^2 + 9x + 4 = (2x + 1)(x + 4)$

Solution(s): $x = 2, \quad x = -4, \quad x = -\frac{1}{2}$

23) Solve the following inequality using any method. Show your work. [3]

$2x^3 - 6x^2 \geq 18x - 54$

$2x^3 - 6x^2 - 18x + 54 \geq 0$

$x^3 - 3x^2 - 9x + 27 \geq 0$

$x^2(x - 3) - 9(x - 3) \geq 0$

$(x - 3)(x^2 - 9) \geq 0$

$(x - 3)^2(x + 3) \geq 0$

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Since $(x-3)^2 \geq 0$ always, the sign is controlled by $(x+3)$: the product is ≥ 0 whenever $x \geq -3$ (and also exactly 0 at $x=3$, already included).

Solution: $x \geq -3$, i.e., $x \in [-3, \infty)$

Section 3: Exponential and Logarithmic Functions

24) Rewrite each of the following as a single logarithm and then evaluate. Round to the nearest hundredth if necessary. [6]

a) $\log_5 7 + \log_5 4$	b) $2\log 8 + \log 2 - \log 4$	c) $\frac{\log_8 80}{\log_8 4}$
$= \log_5(28)$ ≈ 2.07	$= \log(64) + \log(2) - \log(4)$ $= \log(32)$ ≈ 1.51	$= \log_4(80)$ (change of base) ≈ 3.16

25) Solve for x in each of the following equation. Round to 3 decimal places where necessary. Show all of your work. Check for extraneous roots where necessary. [12]

a) $\log(x+2) + \log(x-1) = 1$	b) $\ln(2x-10) = 6$
$\log[(x+2)(x-1)] = 1$ $(x+2)(x-1) = 10$ $x^2 + x - 2 = 10$ $x^2 + x - 12 = 0$ $(x+4)(x-3) = 0$ $x = -4$ (extraneous, needs $x > 1$) or $x = 3$ $x = 3$	$2x - 10 = e^6$ $x = \frac{10 + e^6}{2}$ $x \approx 206.714$
c) $4^{2x-5} = 8^x$	d) $3^{x+5} = 5^{2x-1}$
$(2^2)^{2x-5} = (2^3)^x$ $2^{4x-10} = 2^{3x}$ $4x - 10 = 3x$ $x = 10$	$(x+5)\ln 3 = (2x-1)\ln 5$ $x\ln 3 + 5\ln 3 = 2x\ln 5 - \ln 5$ $x(\ln 3 - 2\ln 5) = -\ln 5 - 5\ln 3$ $x = \frac{\ln 5 + 5\ln 3}{2\ln 5 - \ln 3}$ $x \approx 3.350$

26) The intensities of sound pollution were measured at a small airport runway and a local highway. The airport was 6420.4 times as intense as the highway. If the sound level on the local highway is 91 dB, determine the sound level, in dB, on the runway. [2]

$\beta_2 - \beta_1 = 10\log\left(\frac{I_2}{I_1}\right)$

$\beta_2 - 91 = 10\log(6420.4)$

$\beta_2 = 91 + 10\log(6420.4)$

$\beta_2 \approx 129.076$ dB

Final Answer: $\beta_2 \approx 129.076$ dB

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Section 4: Trigonometry

27) Suppose $\sin\theta = \frac{21}{29}$ and $\frac{\pi}{2} < \theta < \pi$. State an exact value for $\sin(2\theta)$. (Hint: use an identity, DO NOT SOLVE FOR θ) [3]

QII, so $\cos\theta < 0$. Using $21^2 + 20^2 = 29^2$ (a 20-21-29 Pythagorean triple): $\cos\theta = -\frac{20}{29}$

$\sin(2\theta) = 2\sin\theta\cos\theta = 2\left(\frac{21}{29}\right)\left(-\frac{20}{29}\right) = -\frac{840}{841}$

$\sin(2\theta) = -\frac{840}{841}$

28) Use a compound angle formula to find an exact value for $\cos\left(\frac{7\pi}{12}\right)$. Show all of your work. [4]

$\frac{7\pi}{12} = \frac{4\pi}{12} + \frac{3\pi}{12} = \frac{\pi}{3} + \frac{\pi}{4}$

$\cos\left(\frac{\pi}{3} + \frac{\pi}{4}\right) = \cos\frac{\pi}{3}\cos\frac{\pi}{4} - \sin\frac{\pi}{3}\sin\frac{\pi}{4}$

$= \left(\frac{1}{2}\right)\left(\frac{\sqrt{2}}{2}\right) - \left(\frac{\sqrt{3}}{2}\right)\left(\frac{\sqrt{2}}{2}\right)$

$= \frac{\sqrt{2}}{4} - \frac{\sqrt{6}}{4} = \frac{\sqrt{2}-\sqrt{6}}{4}$

$\cos\left(\frac{7\pi}{12}\right) = \frac{\sqrt{2}-\sqrt{6}}{4}$

29) Determine solutions for each equation in the interval $0 \le x \le 2\pi$, to the nearest hundredth of a radian. Give exact answers where possible. [6]

a) $2\sin x + \sqrt{3} = 0$

$\sin x = -\frac{\sqrt{3}}{2}$; reference angle $\frac{\pi}{3}$, place in QIII and QIV

$x = \frac{4\pi}{3}, \frac{5\pi}{3}$

Solution(s): $x = \frac{4\pi}{3}, \frac{5\pi}{3}$

b) $2\cos^2 x - \cos x = 0$

$\cos x(2\cos x - 1) = 0$

$\cos x = 0 \Rightarrow x = \frac{\pi}{2}, \frac{3\pi}{2}$

$\cos x = \frac{1}{2} \Rightarrow$ reference angle $\frac{\pi}{3}$, QI and QIV: $x = \frac{\pi}{3}, \frac{5\pi}{3}$

Solution(s): $x = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{\pi}{3}, \frac{5\pi}{3}$

30) Prove the following trig identity. Show ALL of your work. [3]

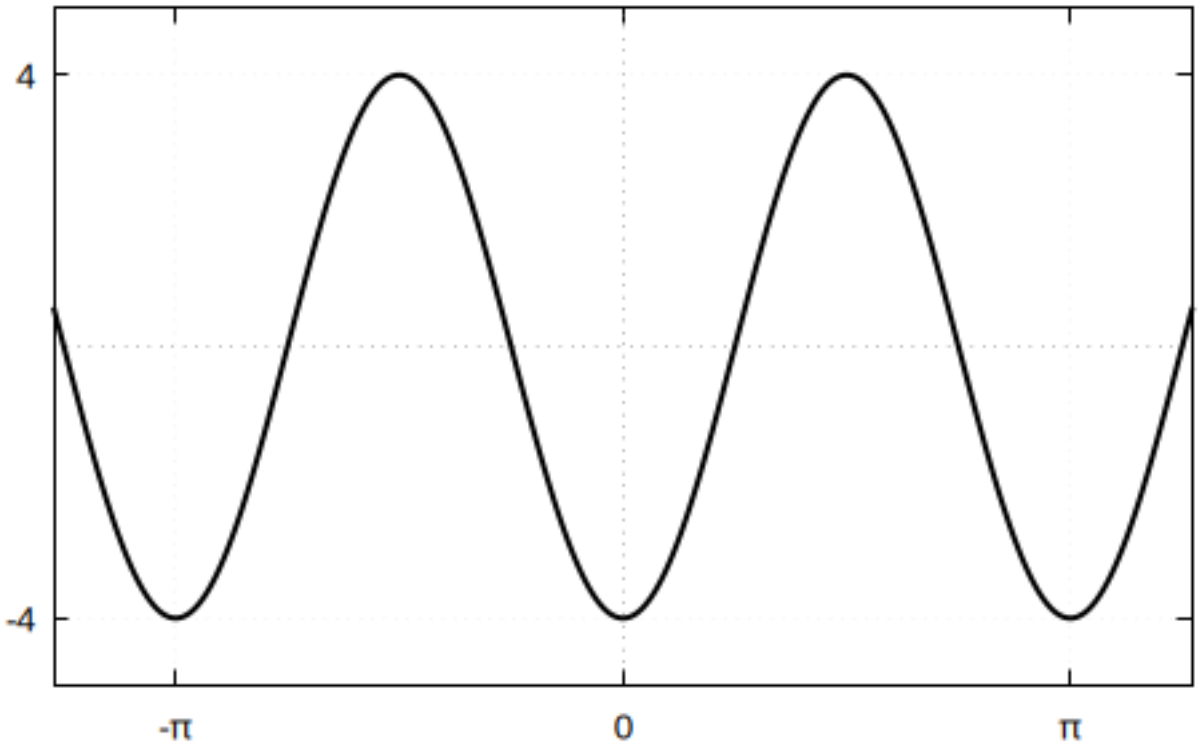
$\frac{1-\cos(2x)}{\sin(2x)} = \tan x$

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LS	RS
$= \frac{1-\cos(2x)}{\sin(2x)}$ $= \frac{1-(1-2\sin^2 x)}{2\sin x\cos x}$ $= \frac{2\sin^2 x}{2\sin x\cos x}$ $= \frac{\sin x}{\cos x}$ $= \tan x$	$= \tan x$

LS = RS

31) Find two equations (one sine and one cosine) to represent the function on the graph below. Show your calculations for full marks. [5]



$a = \frac{\max - \min}{2} = \frac{4-(-4)}{2} = 4$

$k = \frac{2\pi}{\text{period}} = \frac{2\pi}{\pi} = 2$

$c = 0$ (midline $y = 0$)

Minimum at $x = 0$, so this is directly a cosine function with a negative amplitude: $y = -4\cos(2x)$

For the sine form, minimum occurs at $2(x-d) = -\frac{\pi}{2}$ when $x = 0$: $-2d = -\frac{\pi}{2} \Rightarrow d = \frac{\pi}{4}$

$y = 4\sin\left[2\left(x-\frac{\pi}{4}\right)\right]$

Equation 1: $y = -4\cos(2x)$	**Equation 2:** $y = 4\sin\left[2\left(x-\frac{\pi}{4}\right)\right]$

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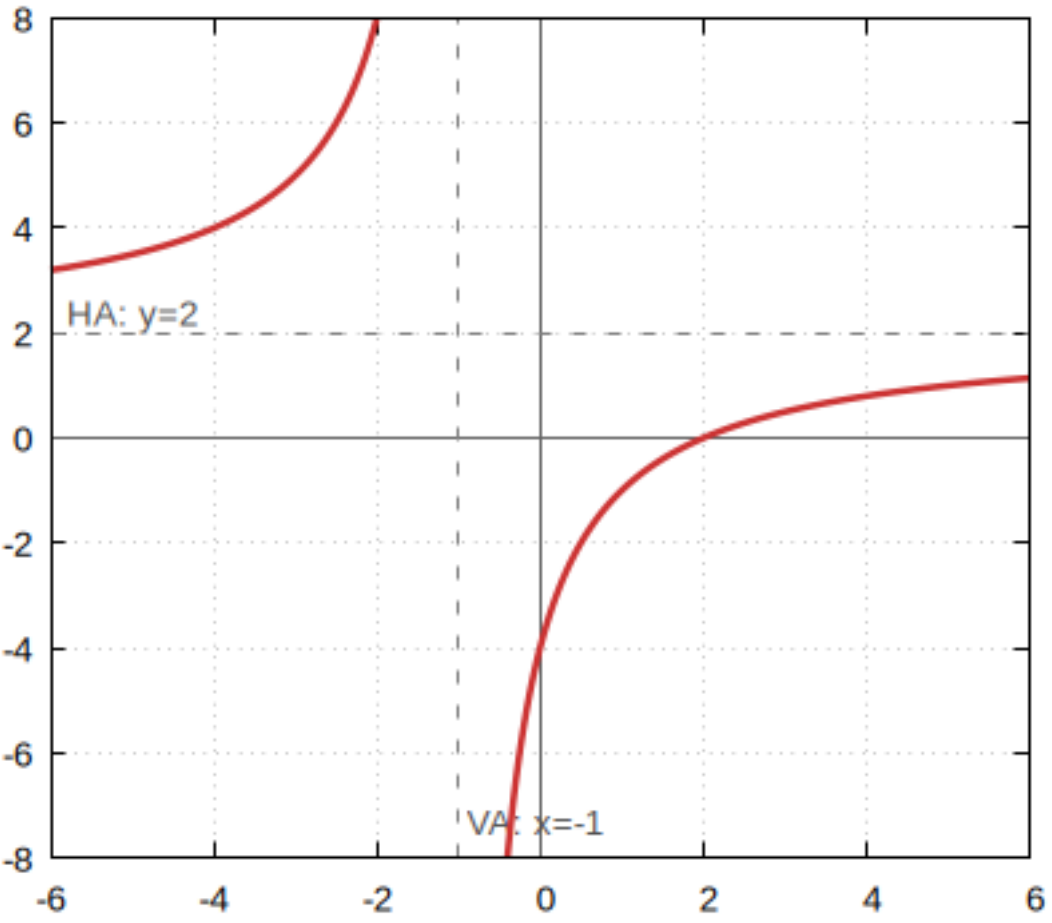
Section 5: Rational Functions

32) Graph $f(x)$ OR $g(x)$, NOT BOTH. Circle your function of choice. Show your work and any key information that you used to graph your function. Label any asymptotes and label your x and y scales appropriately. [4]

Either function is an acceptable choice; both are solved below for completeness.

$f(x) = \frac{2x-4}{x+1}$

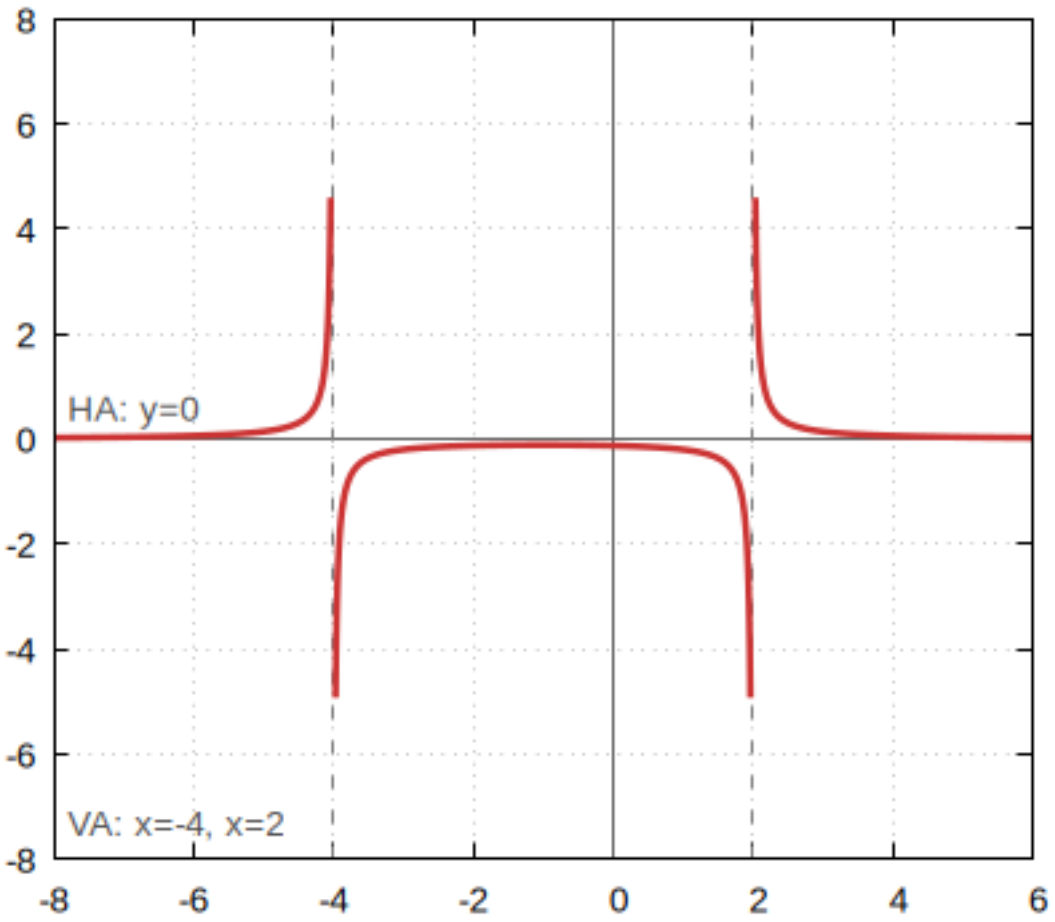
VA: $x = -1$ (denominator zero). HA: $y = 2$ (ratio of leading coefficients). x-int: $2x - 4 = 0 \Rightarrow x = 2$. y-int: $f(0) = \frac{-4}{1} = -4$.



$g(x) = \frac{1}{x^2+2x-8} = \frac{1}{(x+4)(x-2)}$

VA: $x = -4,\ x = 2$. HA: $y = 0$. No x-intercepts (numerator is never 0). y-int: $g(0) = \frac{1}{-8} = -\frac{1}{8}$.

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33) Solve the inequality $\frac{2x-5}{x-1} \geq 1$. Show your work including a factor table. [4]

$\frac{2x-5}{x-1} - 1 \geq 0$

$\frac{(2x-5)-(x-1)}{x-1} \geq 0$

$\frac{x-4}{x-1} \geq 0$

x-intercept: $x = 4$; restriction: $x \neq 1$

Interval	$x < 1$	$1 < x < 4$
$x-4$	-	-
$x-1$	-	+
overall	+	-

Interval	$x > 4$
$x-4$	+
$x-1$	+
overall	+

Solution: $x < 1$ or $x \geq 4$, i.e., $x \in (-\infty, 1) \cup [4, \infty)$

Section 6: Problem Solving

34) Choose 3 of the following 4 questions to complete. It must be clear which 3 questions you chose. [12]

All 4 choices are solved below for completeness.

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Choice 1: Determine ALL solutions for the equation $\cos(2x) = \frac{\sqrt{3}}{2}$ in the interval $0 \le x \le 2\pi$. Give exact answers where possible.

Let $\theta = 2x$. $\cos\theta = \frac{\sqrt{3}}{2}$; reference angle $\frac{\pi}{6}$, place in QI and QIV. Since θ ranges over $[0, 4\pi]$ as x ranges over $[0, 2\pi]$, all coterminal solutions in that range are needed.

$\theta = \frac{\pi}{6}, \frac{11\pi}{6}, \frac{\pi}{6} + 2\pi = \frac{13\pi}{6}, \frac{11\pi}{6} + 2\pi = \frac{23\pi}{6}$

$x = \frac{\theta}{2} = \frac{\pi}{12}, \frac{11\pi}{12}, \frac{13\pi}{12}, \frac{23\pi}{12}$

Choice 2: Solve the equation $4^{2x} - 2(4^x) - 15 = 0$. Round to 3 decimal places where necessary. Make sure to check for extraneous roots where necessary.

Let $k = 4^x$: $k^2 - 2k - 15 = 0$

$(k - 5)(k + 3) = 0 \Rightarrow k = 5$ or $k = -3$

$k = -3$ is rejected ($4^x > 0$ always).

$4^x = 5 \Rightarrow x = \log_4(5) \approx 1.161$

Choice 3: Given the equation $f(x) = x^4 - 2x^3 + kx - 5$, solve for k given that $f(x)$ divided by $x + 1$ has a remainder of -6.

By the remainder theorem, $f(-1) = -6$.

$f(-1) = (-1)^4 - 2(-1)^3 + k(-1) - 5 = 1 + 2 - k - 5 = -2 - k$

$-2 - k = -6 \Rightarrow k = 4$

Choice 4: Use the given graph to find the following limits:

a) $\lim_{x\to\infty} f(x)$

$= 0$

b) $\lim_{x\to -2^+} f(x)$

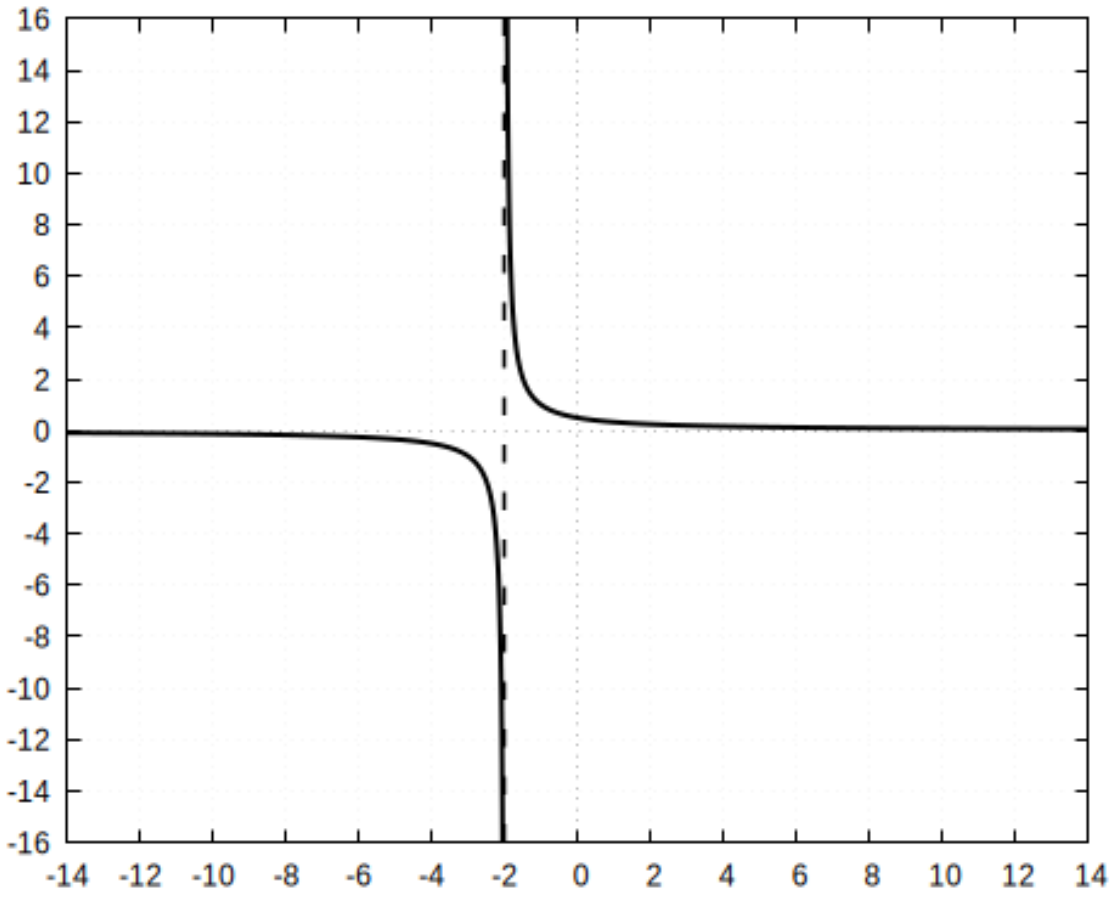
$= \infty$

c) $\lim_{x\to -2^-} f(x)$

$= -\infty$

d) $\lim_{x\to -2} f(x)$

Does Not Exist



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