

3.4 Solve Rational Equations and Inequalities Worksheet with solutions

This document was composed and formatted by Annie Bernatchez.

The course material originated from www.jensenmath.ca

1) Solve each equation algebraically.

a) $\frac{6}{2x-1} = 5$

$$(2x-1)\left(\frac{6}{2x-1}\right) = 5(2x-1)$$

$$6 = 10x - 5 \Rightarrow 11 = 10x \Rightarrow x = \frac{11}{10} = 1.1$$

b) $\frac{6}{x} = x - 5$

$$x\left(\frac{6}{x}\right) = x(x-5) \Rightarrow 6 = x^2 - 5x \Rightarrow 0 = x^2 - 5x - 6 \Rightarrow 0 = (x-6)(x+1)$$

$$x-6=0 \Rightarrow x_1=6; x+1=0 \Rightarrow x_2=-1$$

c) $1 = \frac{5}{3x^2-8x+2}$

$$(3x^2-8x+2)(1) = 5 \Rightarrow 3x^2-8x-3=0$$

$$3x^2-9x+1x-3=0 \Rightarrow 3x(x-3)+1(x-3)=0 \Rightarrow (x-3)(3x+1)=0$$

$$x-3=0 \Rightarrow x_1=3; 3x+1=0 \Rightarrow x_2=-\frac{1}{3}$$

d) $\frac{x+5}{x-1} = \frac{x+1}{x-3}$

$$(x-3)(x+5) = (x+1)(x-1) \Rightarrow x^2+2x-15 = x^2-1$$

$$2x = 14 \Rightarrow x = 7$$

e) $\frac{3}{x+5} + \frac{4}{x} = 0$

$$\frac{3}{x+5} = \frac{-4}{x} \Rightarrow 3x = -4(x+5) \Rightarrow 3x = -4x-20$$

$$7x = -20 \Rightarrow x = -\frac{20}{7}$$

f) $2x = 5 - \frac{3}{x}$

$$x(2x) = x\left(5 - \frac{3}{x}\right) \Rightarrow 2x^2 = 5x - 3 \Rightarrow 2x^2 - 5x + 3 = 0$$

$$2x^2-2x-3x+3=0 \Rightarrow 2x(x-1)-3(x-1)=0 \Rightarrow (x-1)(2x-3)=0$$

$$x-1=0 \Rightarrow x_1=1; 2x-3=0 \Rightarrow x_2=\frac{3}{2}$$

g) $\frac{2}{1-x} + \frac{3}{x+1} = \frac{1}{x}$

$$\frac{2(x+1)+3(1-x)}{(1-x)(x+1)} = \frac{1}{x} \Rightarrow \frac{2x+2+3-3x}{1-x^2} = \frac{1}{x} \Rightarrow \frac{-x+5}{1-x^2} = \frac{1}{x}$$

$$x(-x+5) = 1(1-x^2) \Rightarrow -x^2+5x = 1-x^2 \Rightarrow 5x = 1 \Rightarrow x = \frac{1}{5}$$

h) $\frac{3}{x-1} + 5 + \frac{2}{x} = 0$

$$x(x-1)\left[\frac{3}{x-1} + 5 + \frac{2}{x}\right] = 0(x)(x-1) \Rightarrow 3x + 5x(x-1) + 2(x-1) = 0$$

$$3x + 5x^2 - 5x + 2x - 2 = 0 \Rightarrow 5x^2 = 2 \Rightarrow x^2 = \frac{2}{5} \Rightarrow x = \pm\sqrt{0.4}$$

2) Solve each inequality without using technology.

a) $\frac{4}{2x-3} < \frac{1}{x+4}$

$$\frac{4(x+4)-1(2x-3)}{(2x-3)(x+4)} < 0 \Rightarrow \frac{4x+16-2x+3}{(2x-3)(x+4)} < 0 \Rightarrow \frac{2x+19}{(2x-3)(x+4)} < 0; \text{zero: } x = -9.5; \text{restrictions: } x \neq -4, 1.5$$

3.4 Solve Rational Equations and Inequalities Worksheet with solutions

interval	$x < -9.5$	$-9.5 < x < -4$	$-4 < x < 1.5$	$x > 1.5$
test point	-10	-5	0	2
$2x + 19$	-	+	+	+
$2x - 3$	-	-	-	+
$x + 4$	-	-	+	+
overall	-	+	-	+

Solution: $x < -9.5$ or $-4 < x < 1.5$, i.e. $x \in (-\infty, -9.5) \cup (-4, 1.5)$

b) $\frac{2x+3}{x-3} \geq \frac{6x-5}{3x+1}$

$$\frac{(3x+1)(2x+3) - (6x-5)(x-3)}{(3x+1)(x-3)} \geq 0 \Rightarrow \frac{6x^2 + 11x + 3 - (6x^2 - 23x + 15)}{(3x+1)(x-3)} \geq 0$$

$$\frac{34x - 12}{(3x+1)(x-3)} \geq 0 \Rightarrow \frac{2(17x-6)}{(3x+1)(x-3)} \geq 0; \text{zero: } x = \frac{6}{17}; \text{restrictions: } x \neq -\frac{1}{3}, 3$$

interval	$x < -\frac{1}{3}$	$-\frac{1}{3} < x \leq \frac{6}{17}$	$\frac{6}{17} < x < 3$	$x > 3$
test point	-1	0	1	4
$17x - 6$	-	-	+	+
$3x + 1$	-	+	+	+
$x - 3$	-	-	-	+
overall	-	+	-	+

Solution: $-\frac{1}{3} < x \leq \frac{6}{17}$ or $x > 3$, i.e. $x \in (-\frac{1}{3}, \frac{6}{17}] \cup (3, \infty)$

c) $\frac{(x-3)(2x-1)}{(x+4)(x-5)} > 0$

zeros: $x = \frac{1}{2}, 3$; restrictions: $x \neq -4, 5$

interval	$x < -4$	$-4 < x < 0.5$	$0.5 < x < 3$	$3 < x < 5$	$x > 5$
test point	-5	0	1	4	6
$x - 3$	-	-	-	+	+
$2x - 1$	-	-	+	+	+
$x + 4$	-	+	+	+	+
$x - 5$	-	-	-	-	+
overall	+	-	+	-	+

Solution: $x < -4$ or $0.5 < x < 3$ or $x > 5$, i.e. $x \in (-\infty, -4) \cup (0.5, 3) \cup (5, \infty)$

d) $\frac{2x^2 + 5x - 3}{x^2 + 5x + 4} \leq 0$

$$\frac{(2x-1)(x+3)}{(x+4)(x+1)} \leq 0; \text{zeros: } x = -3, \frac{1}{2}; \text{restrictions: } x \neq -4, -1$$

interval	$x < -4$	$-4 < x \leq -3$	$-3 < x < -1$	$-1 < x \leq 0.5$	$x > 0.5$
test point	-5	-3.5	-2	0	1
$2x - 1$	-	-	-	-	+
$x + 3$	-	-	+	+	+
$x + 4$	-	+	+	+	+
$x + 1$	-	-	-	+	+
overall	+	-	+	-	+

Solution: $-4 < x \leq -3$ or $-1 < x \leq 0.5$, i.e. $x \in (-4, -3] \cup (-1, 0.5]$

e) $\frac{4}{x-3} < 1$

$$\frac{4}{x-3} - \frac{x-3}{x-3} < 0 \Rightarrow \frac{4-(x-3)}{x-3} < 0 \Rightarrow \frac{7-x}{x-3} < 0; \text{zero: } x = 7; \text{restriction: } x \neq 3$$

interval	$x < 3$	$3 < x < 7$	$x > 7$
test point	0	4	8
$7 - x$	+	+	-
$x - 3$	-	+	+
overall	-	+	-

3.4 Solve Rational Equations and Inequalities Worksheet with solutions

Solution: $x < 3$ or $x > 7$, i.e. $x \in (-\infty, 3) \cup (7, \infty)$

f) $\frac{2x^2 + 5x - 3}{x^2 + 8x + 16} < 0$

$\frac{(2x-1)(x+3)}{(x+4)^2} < 0$; zeros: $x = -3, \frac{1}{2}$; restriction: $x \neq -4$

interval	$x < -4$	$-4 < x < -3$	$-3 < x < 0.5$	$x > 0.5$
test point	-5	-3.5	0	1
$2x - 1$	-	-	-	+
$x + 3$	-	-	+	+
$(x + 4)^2$	+	+	+	+
overall	+	+	-	+

Solution: $-3 < x < \frac{1}{2}$, i.e. $x \in (-3, \frac{1}{2})$

g) $\frac{x}{x+3} > \frac{x}{x-1}$

$\frac{x(x-1) - x(x+3)}{(x+3)(x-1)} > 0 \Rightarrow \frac{x^2 - x - x^2 - 3x}{(x+3)(x-1)} > 0 \Rightarrow \frac{-4x}{(x+3)(x-1)} > 0$; zero: $x = 0$; restrictions: $x \neq -3, 1$

interval	$x < -3$	$-3 < x < 0$	$0 < x < 1$	$x > 1$
test point	-4	-1	0.5	2
$-4x$	+	+	-	-
$x + 3$	-	+	+	+
$x - 1$	-	-	-	+
overall	+	-	+	-

Solution: $x < -3$ or $0 < x < 1$, i.e. $x \in (-\infty, -3) \cup (0, 1)$

h) $\frac{2x+3}{x} > \frac{x+1}{x}$

$\frac{2x+3 - (x+1)}{x} > 0 \Rightarrow \frac{x+2}{x} > 0$; zero: $x = -2$; restriction: $x \neq 0$

interval	$x < -2$	$-2 < x < 0$	$x > 0$
test point	-3	-1	1
$x + 2$	-	+	+
x	-	-	+
overall	+	-	+

Solution: $x < -2$ or $x > 0$, i.e. $x \in (-\infty, -2) \cup (0, \infty)$