

Combinations of Functions and Inverse Functions Worksheet with solutions

This document was composed and formatted by Annie Bernatchez.

The course material originated from www.jensenmath.ca

1) Let $f(x) = 3x - 5$ and $g(x) = 2x + 3$.

a) Write the equation for $h(x) = f(x) + g(x)$ and determine the value of $h(2)$.

$$h(x) = (3x - 5) + (2x + 3) = 3x - 5 + 2x + 3 = 5x - 2$$

$$h(2) = 5(2) - 2 = 8$$

b) Write the equation for $k(x) = f(x) - g(x)$ and determine the value of $k(2)$.

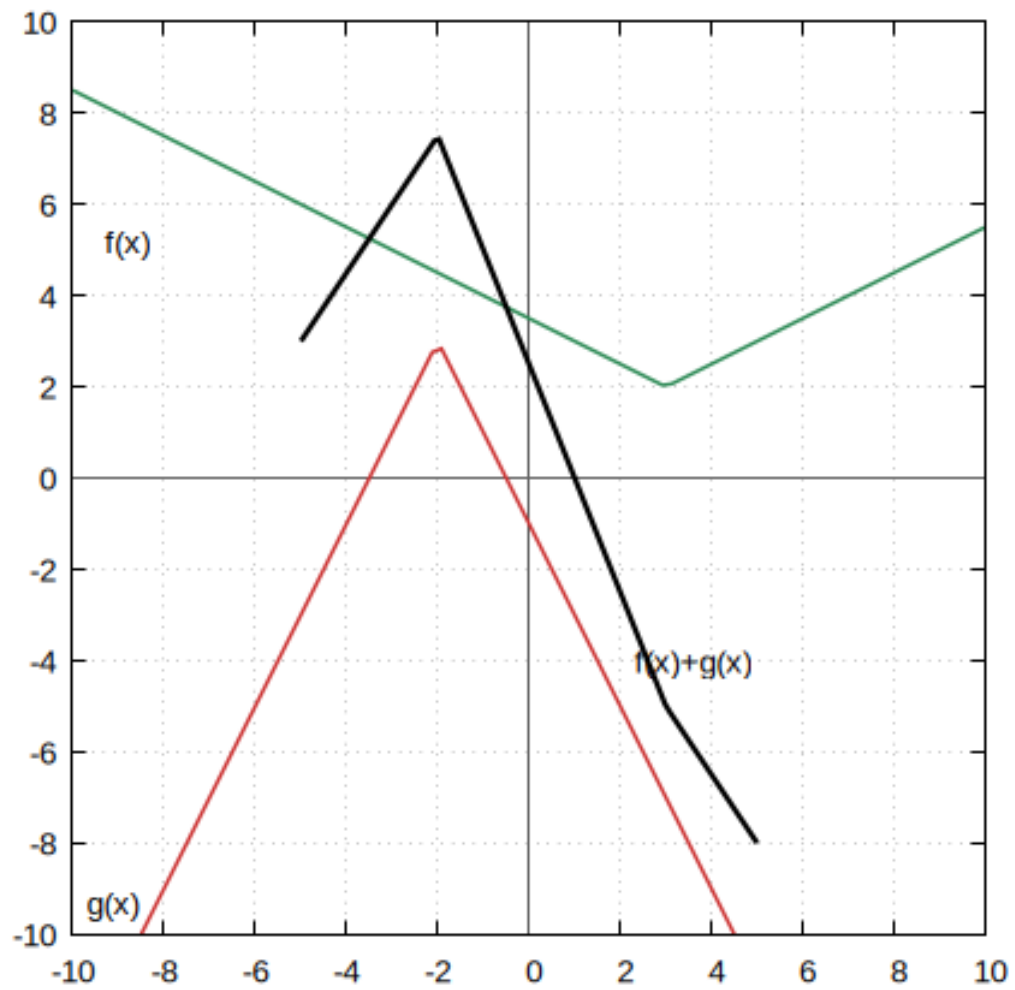
$$k(x) = (3x - 5) - (2x + 3) = 3x - 5 - 2x - 3 = x - 8$$

$$k(2) = 2 - 8 = -6$$

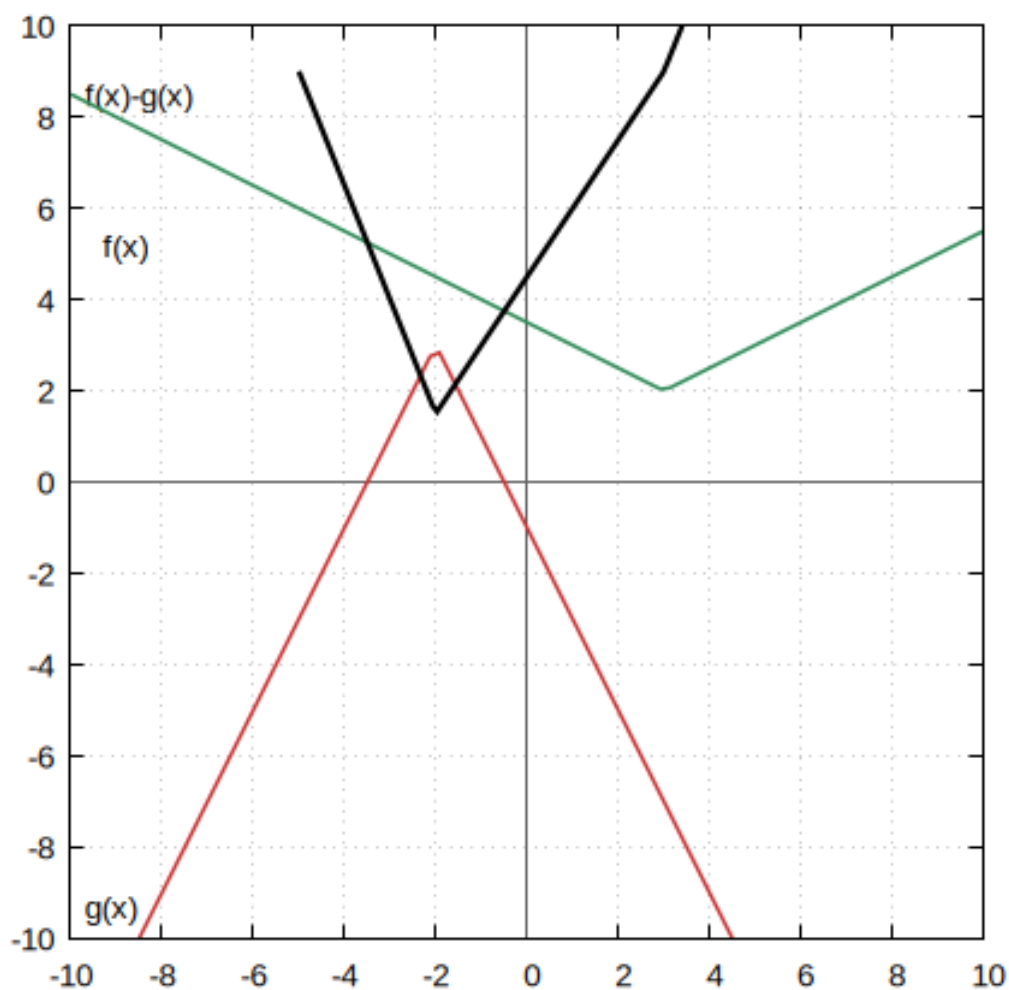
2) Use the functions $f(x)$ and $g(x)$ as shown. Apply the superposition principle to graph

a) $y = f(x) + g(x)$

x	$f(x)$	$g(x)$	$f(x) + g(x)$	$f(x) - g(x)$
-5	6	-3	3	9
-4	5.5	-1	4.5	6.5
-3	5	1	6	4
-2	4.5	3	7.5	1.5
-1	4	1	5	3
0	3.5	-1	2.5	4.5
1	3	-3	0	6
2	2.5	-5	-2.5	7.5
3	2	-7	-5	9
4	2.5	-9	-6.5	11.5
5	3	-11	-8	14



b) $y = f(x) - g(x)$



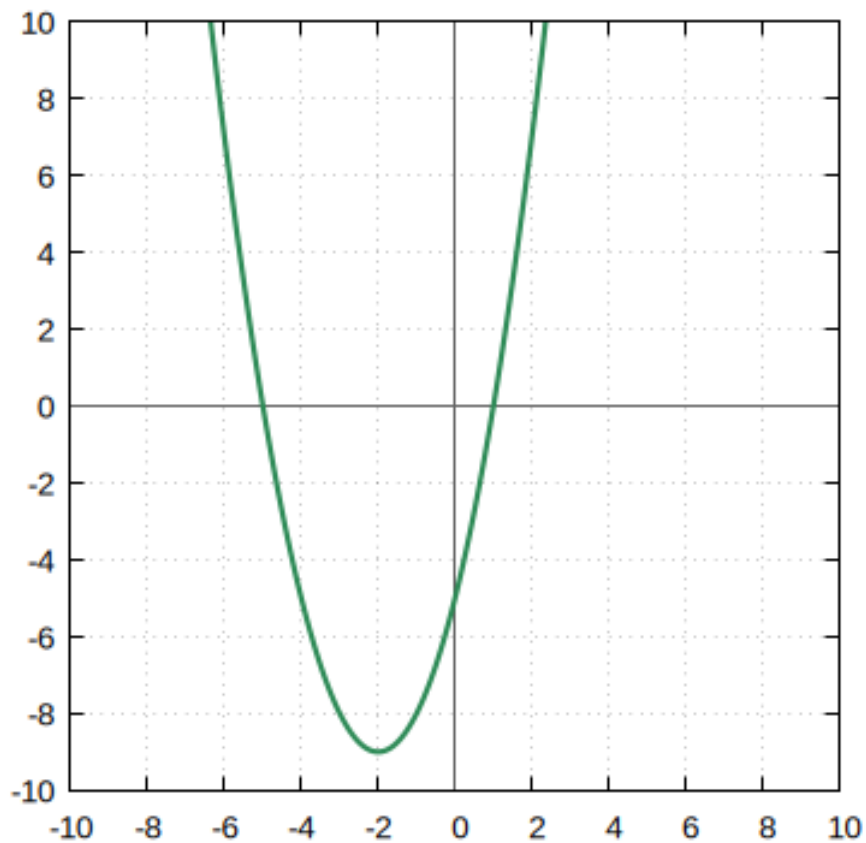
3) Let $f(x) = x - 2$ and $g(x) = x^2 + 3x - 3$. Determine an algebraic and graphical model for $h(x) = f(x) + g(x)$.

$$h(x) = (x - 2) + (x^2 + 3x - 3) = x^2 + 4x - 5 = (x + 5)(x - 1)$$

$$x\text{-int} : x = -5 \text{ and } x = 1$$

$$x\text{-vertex} = \frac{-5 + 1}{2} = -2; h(-2) = (-2)^2 + 4(-2) - 5 = -9, \text{ so vertex } (-2, -9)$$

$$y\text{-int} : h(0) = 0^2 + 4(0) - 5 = -5, \text{ so } (0, -5)$$



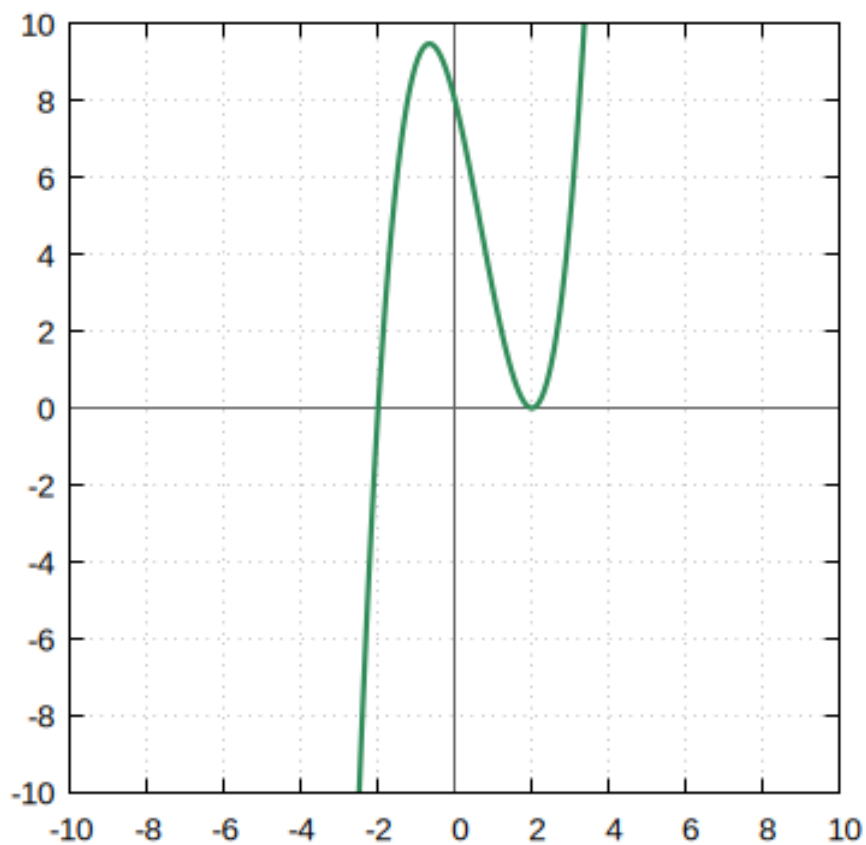
4) Let $f(x) = x - 2$ and $g(x) = x^2 - 4$. Develop an algebraic and graphical model for each of the following:

a) $y = f(x)g(x)$

$$y = (x - 2)(x^2 - 4) = (x - 2)(x - 2)(x + 2) = (x - 2)^2(x + 2)$$

x-int : $(2, 0)$ order 2, $(-2, 0)$ order 1

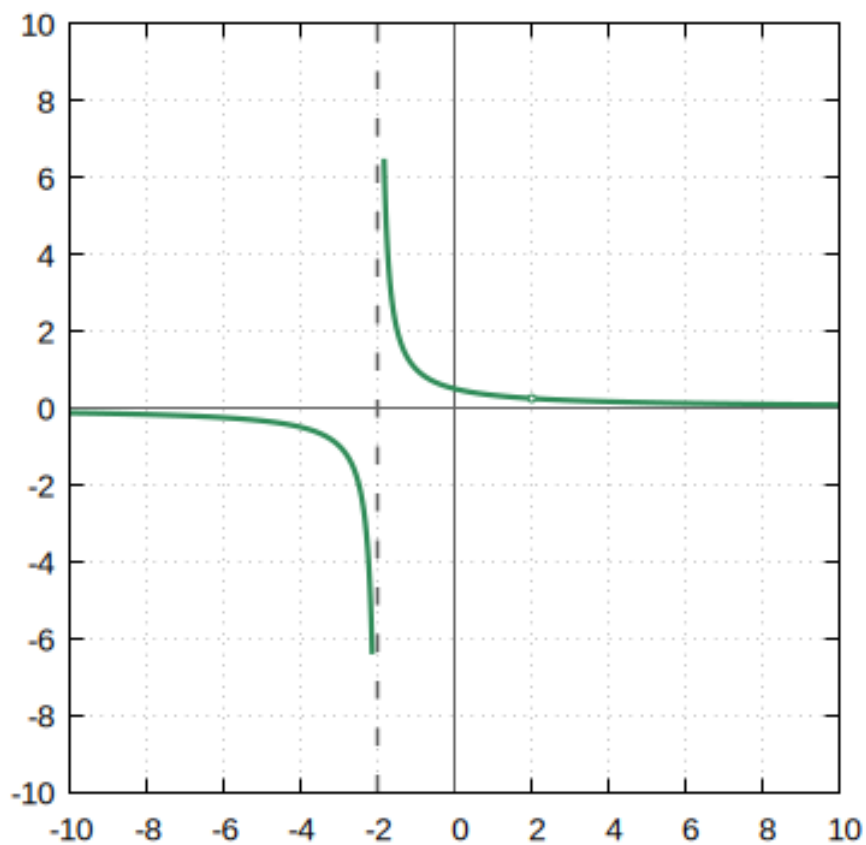
y-int : $y = (0 - 2)^2(0 + 2) = 8$, so $(0, 8)$



b) $y = \frac{f(x)}{g(x)}$

$$y = \frac{x-2}{x^2-4} = \frac{x-2}{(x-2)(x+2)} = \frac{1}{x+2}; x \neq -2, 2$$

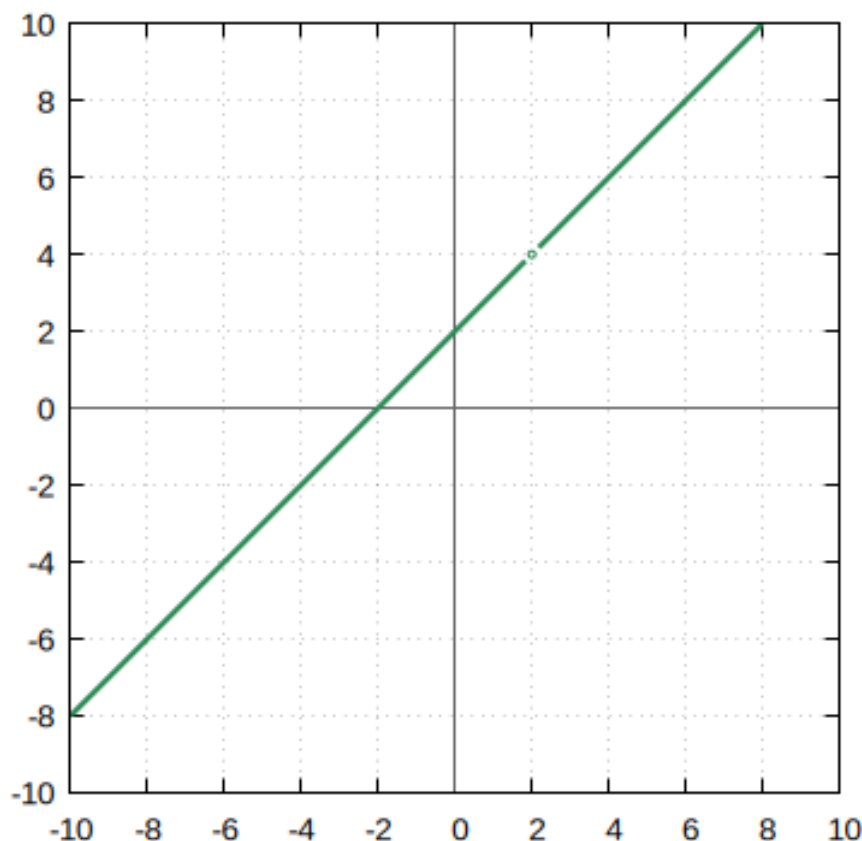
Hole at $x = 2$; VA : $x = -2$; HA : $y = 0$



c) $y = \frac{g(x)}{f(x)}$

$$y = \frac{x^2 - 4}{x - 2} = \frac{(x - 2)(x + 2)}{x - 2} = x + 2; x \neq 2$$

Hole at $x = 2$



5) Let $f(x) = x^2 + 2x - 4$ and $g(x) = \frac{1}{x+1}$.

a) Evaluate $g(f(0))$

$$f(0) = 0^2 + 2(0) - 4 = -4$$

$$g(f(0)) = g(-4) = \frac{1}{-4+1} = -\frac{1}{3}$$

b) Evaluate $f(g(-2))$

$$g(-2) = \frac{1}{-2+1} = -1$$

$$f(g(-2)) = f(-1) = (-1)^2 + 2(-1) - 4 = -5$$

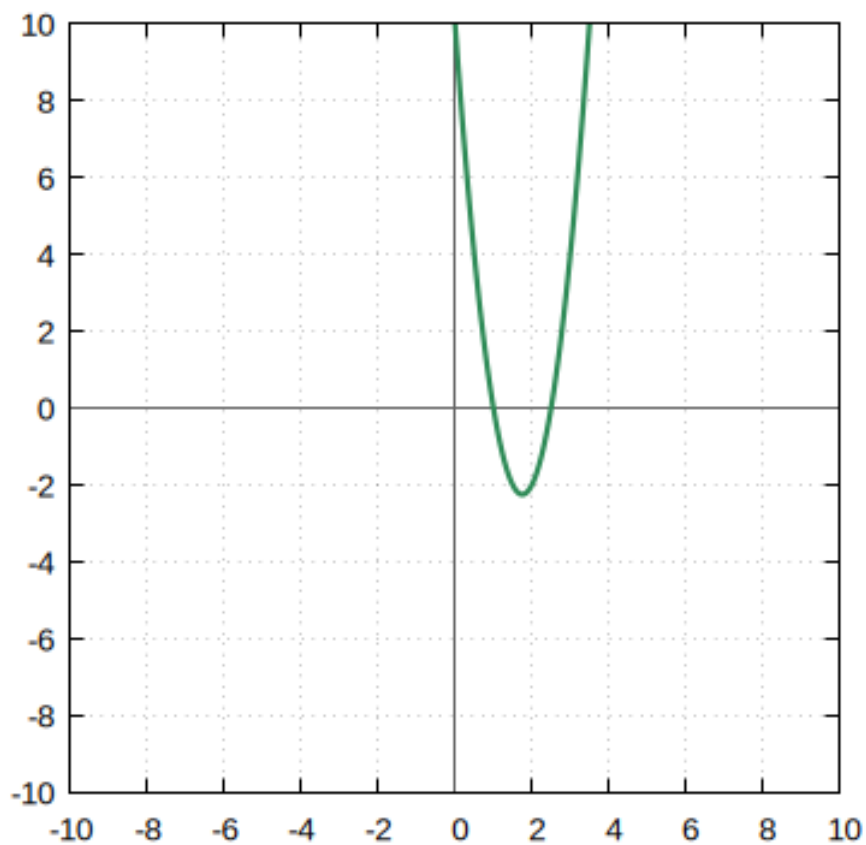
6) Let $f(x) = x^2 + 3x$ and $g(x) = 2x - 5$. Determine an equation for each composite function and graph it.

a) $y = f(g(x))$

$$f(g(x)) = (2x - 5)^2 + 3(2x - 5) = 4x^2 - 20x + 25 + 6x - 15 = 4x^2 - 14x + 10 = 2(2x^2 - 7x + 5) = 2[x(2x - 5) - 1(2x - 5)] = 2(2x - 5)(x - 1)$$

x-int : $x = 1$ and $x = 2.5$; y-int : $(0, 10)$

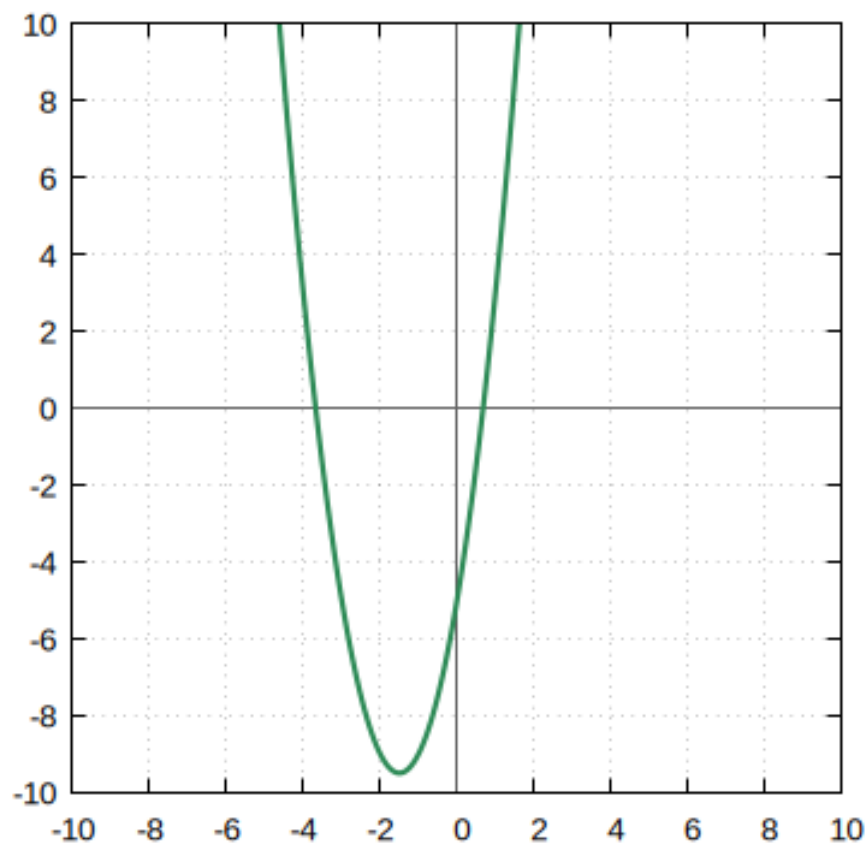
$$x\text{-vertex} = \frac{1 + 2.5}{2} = 1.75; y\text{-vertex} = -2.25$$



b) $y = g(f(x))$

$$g(f(x)) = 2(x^2 + 3x) - 5 = 2x^2 + 6x - 5$$

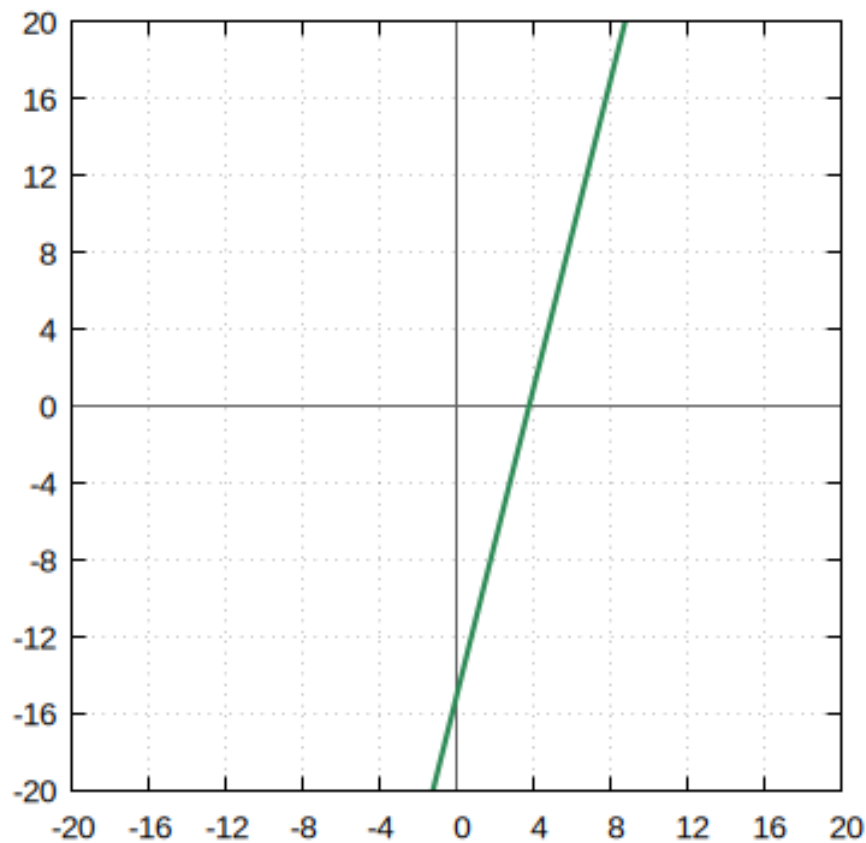
$$x\text{-vertex} = -\frac{b}{2a} = -\frac{6}{2(2)} = -1.5; \quad y\text{-vertex} = -9.5$$



c) $y = g(g(x))$

$$g(g(x)) = 2(2x - 5) - 5 = 4x - 10 - 5 = 4x - 15$$

$$m = 4, b = -15$$



d) $y = g^{-1}(g(x))$

To find $g^{-1}(x)$: let $y = 2x - 5$, swap to get $x = 2y - 5$, solve: $\frac{x+5}{2} = y$, so $g^{-1}(x) = \frac{x+5}{2}$

$$g^{-1}(g(x)) = \frac{(2x-5)+5}{2} = \frac{2x}{2} = x$$

$m = 1$, $b = 0$ — this is the identity line, since applying a function's inverse right after the function itself always returns the original input.

