

3.3 Quotient of Linear Functions Worksheet with solutions

This document was composed and formatted by Annie Bernatchez.

The course material originated from www.jensenmath.ca

1) State the equation of the vertical and horizontal asymptotes for each function.

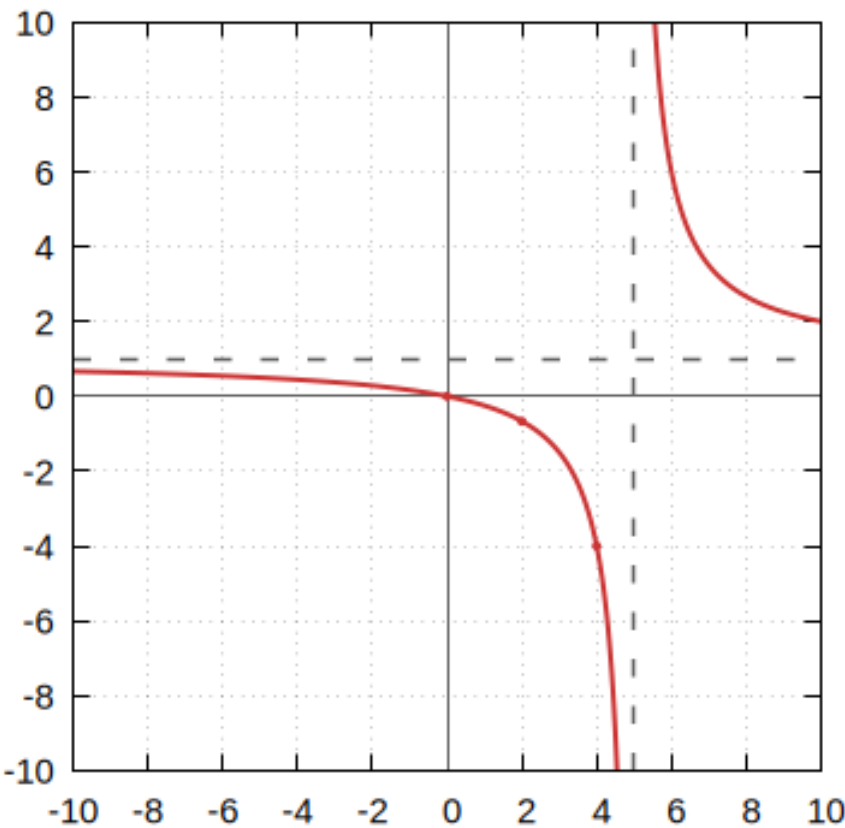
a) $p(x) = \frac{x}{x-6}$ $VA : x = 6$ $HA : y = 1$	b) $q(x) = \frac{3x}{x+4}$ $VA : x = -4$ $HA : y = 3$
c) $r(x) = \frac{x-1}{x+1}$ $VA : x = -1$ $HA : y = 1$	d) $\frac{5x-2}{2x+3}$ $VA : x = -1.5$ $HA : y = 2.5$

2) Graph each of the following functions. Make sure to identify key characteristics of the functions including intercepts and asymptotes.

a) $f(x) = \frac{x}{x-5}$

$VA : x = 5, HA : y = 1. \quad x\text{-int} : 0 = \frac{x}{x-5} \Rightarrow x = 0, \text{ SO } (0, 0); \quad y\text{-int} : f(0) = \frac{0}{-5} = 0, \text{ so also } (0, 0)$

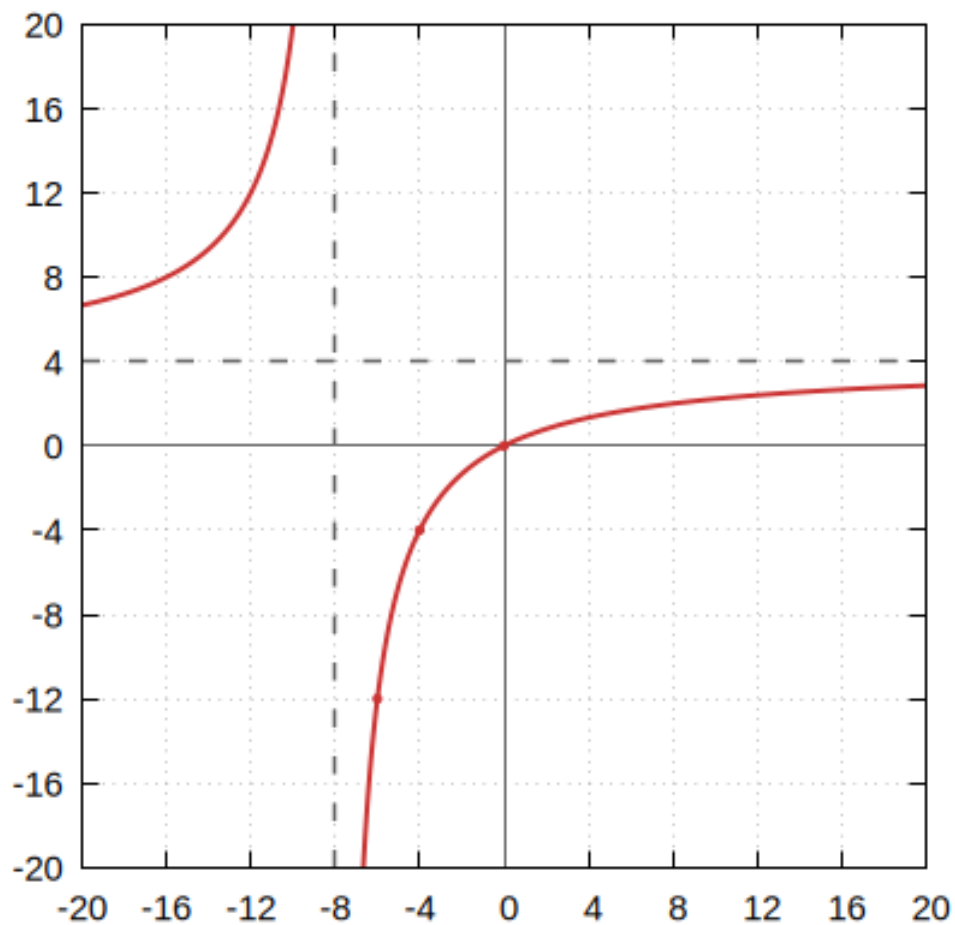
Other points: $f(2) = \frac{2}{-3} = -0.67, \text{ SO } (2, -0.67); \quad f(4) = \frac{4}{-1} = -4, \text{ SO } (4, -4)$



b) $c(x) = \frac{4x}{x+8}$

$VA : x = -8, HA : y = 4. \quad x\text{-int} : 0 = \frac{4x}{x+8} \Rightarrow x = 0, \text{ SO } (0, 0), \text{ which is also the } y\text{-int}$

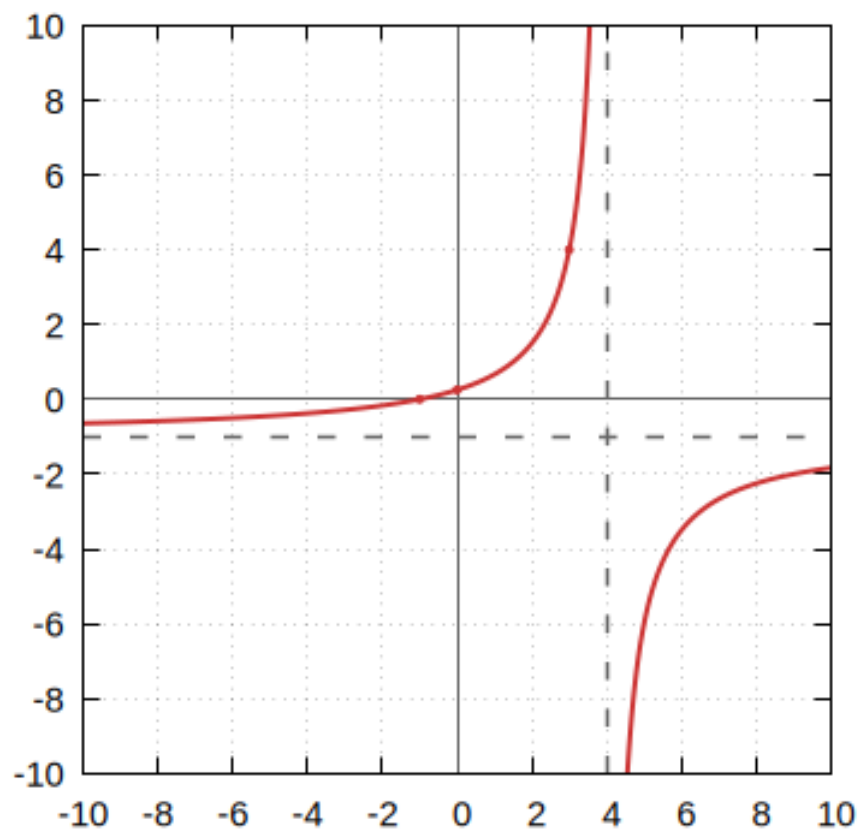
Other points: $c(-6) = \frac{4(-6)}{-6+8} = \frac{-24}{2} = -12$, SO $(-6, -12)$; $c(-4) = \frac{4(-4)}{-4+8} = \frac{-16}{4} = -4$, SO $(-4, -4)$



c) $k(x) = \frac{x+1}{4-x}$

VA : $x = 4$, HA : $y = -1$. x-int : $0 = x + 1 \Rightarrow x = -1$, SO $(-1, 0)$; y-int : $k(0) = \frac{1}{4} = 0.25$, SO $(0, 0.25)$

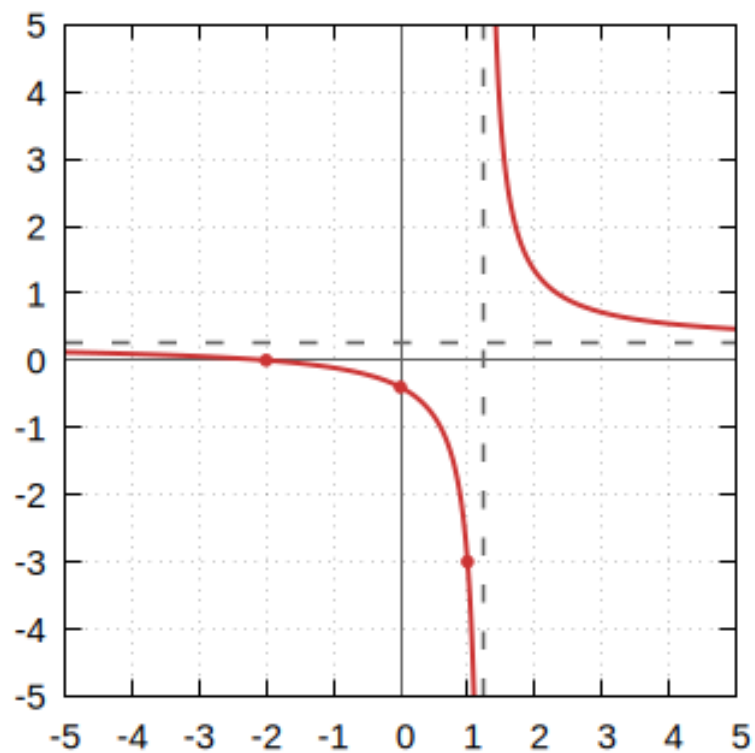
Other point: $k(3) = \frac{3+1}{4-3} = 4$, SO $(3, 4)$



d) $w(x) = \frac{x+2}{4x-5}$

VA : $x = 1.25$, HA : $y = 0.25$. x-int : $0 = x + 2 \Rightarrow x = -2$, SO $(-2, 0)$; y-int : $w(0) = \frac{2}{-5} = -0.4$, SO $(0, -0.4)$

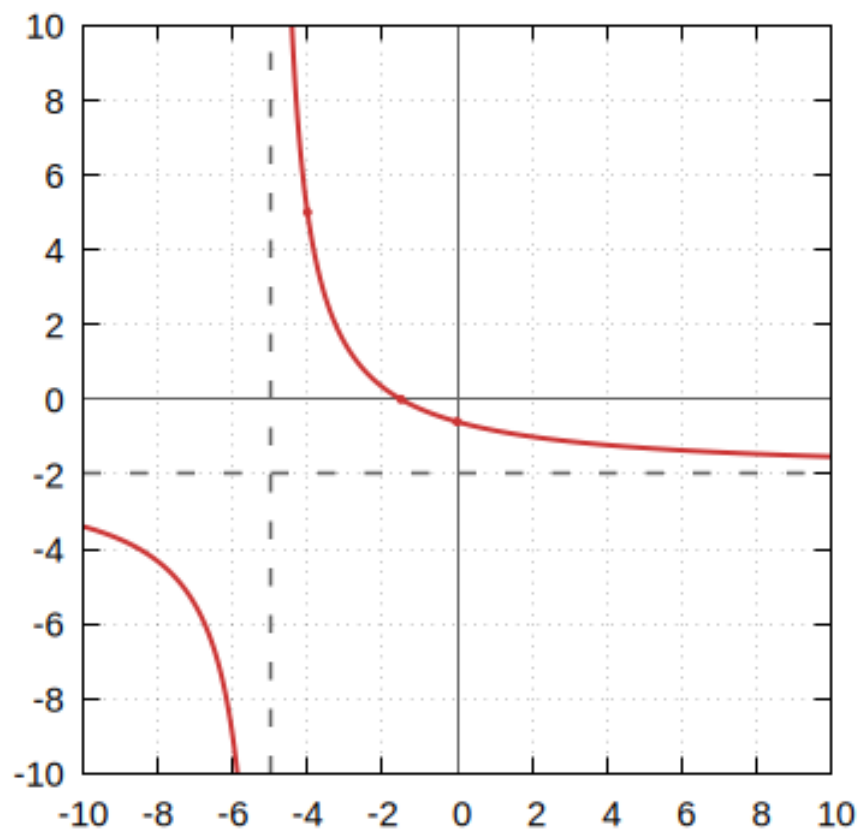
Other point: $w(1) = \frac{1+2}{4-5} = -3$, SO $(1, -3)$



e) $d(x) = \frac{-2x-3}{x+5}$

$VA : x = -5$, $HA : y = -2$. x-int : $0 = -2x - 3 \Rightarrow x = -1.5$, SO $(-1.5, 0)$; y-int : $d(0) = \frac{-3}{5} = -0.6$, SO $(0, -0.6)$

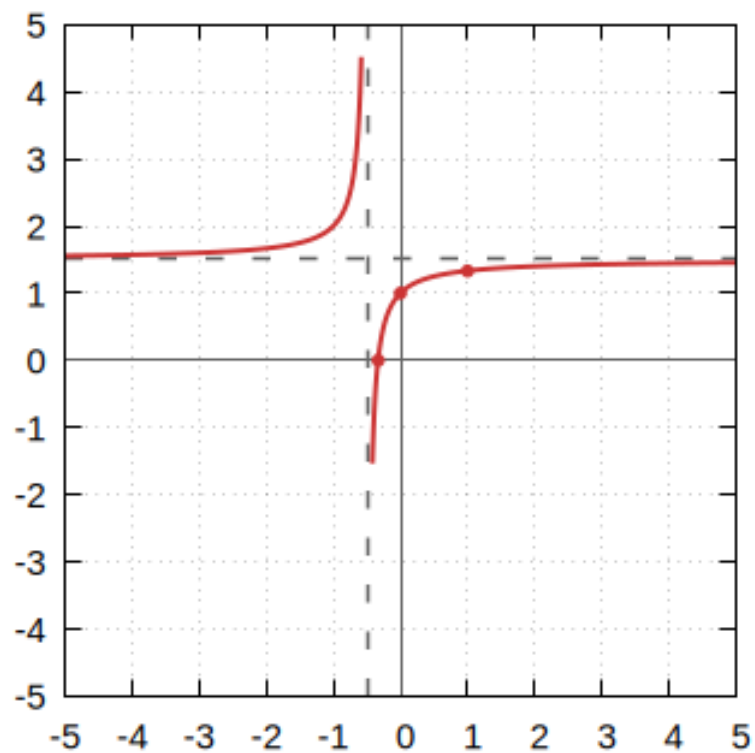
Other point: $d(-4) = \frac{-2(-4)-3}{-4+5} = \frac{5}{1} = 5$, SO $(-4, 5)$



$$f) m(x) = \frac{3x+1}{2x+1}$$

VA : $x = -0.5$, HA : $y = 1.5$. x-int : $0 = 3x + 1 \Rightarrow x = -0.33$, SO $(-0.33, 0)$; y-int : $m(0) = \frac{1}{1} = 1$, SO $(0, 1)$

Other point: $m(1) = \frac{3(1)+1}{2(1)+1} = \frac{4}{3} = 1.33$, SO $(1, 1.33)$



$$g) \ g(x) = \frac{x-2}{x^2+3x+2}$$

(use a graphing calculator/Desmos)

$$g(x) = \frac{x-2}{x^2+3x+2} = \frac{x-2}{(x+2)(x+1)}$$

HA : $y = 0$, VA : $x = -2$ and $x = -1$

x-int : $0 = x - 2 \Rightarrow x = 2$, SO $(2, 0)$; y-int : $g(0) = \frac{0-2}{(0+2)(0+1)} = -1$, SO $(0, -1)$

