

7.6 Unit 7 Pre-Test Review – Graphing with solutions

This document was composed and formatted by Annie Bernatchez.

The course material originated from www.jensenmath.ca

Section 1: Rational Functions and Combinations of Functions

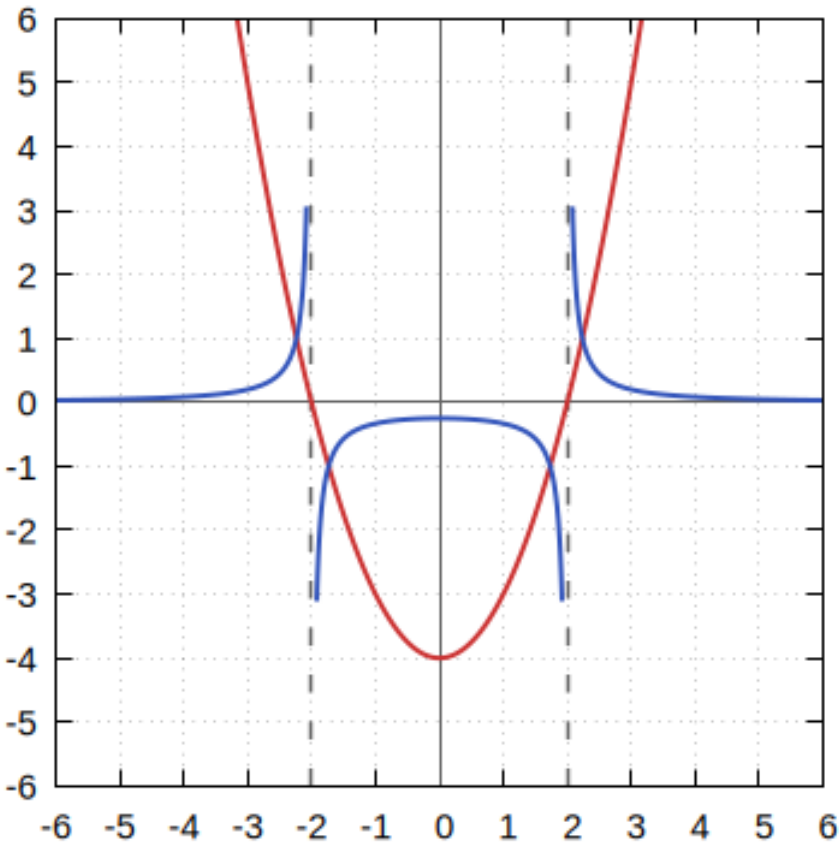
1) Graph each of the following reciprocal functions. Start by graphing the function in the denominator. Show as much characteristic information about the graph as you can (e.g. intercepts, asymptotes with equations, other defining points, etc).

a) $f(x) = \frac{1}{x^2 - 4}$

$y = x^2 - 4 = (x - 2)(x + 2)$, x-int : $x = -2, 2$

VA : $x = 2$ and $x = -2$, HA : $y = 0$

x	$y = x^2 - 4$	$y = \frac{1}{x^2 - 4}$
-3	5	0.2
-2	0	undefined
-1	-3	-0.33
0	-4	-0.25
1	-3	-0.33
2	0	undefined
3	5	0.2

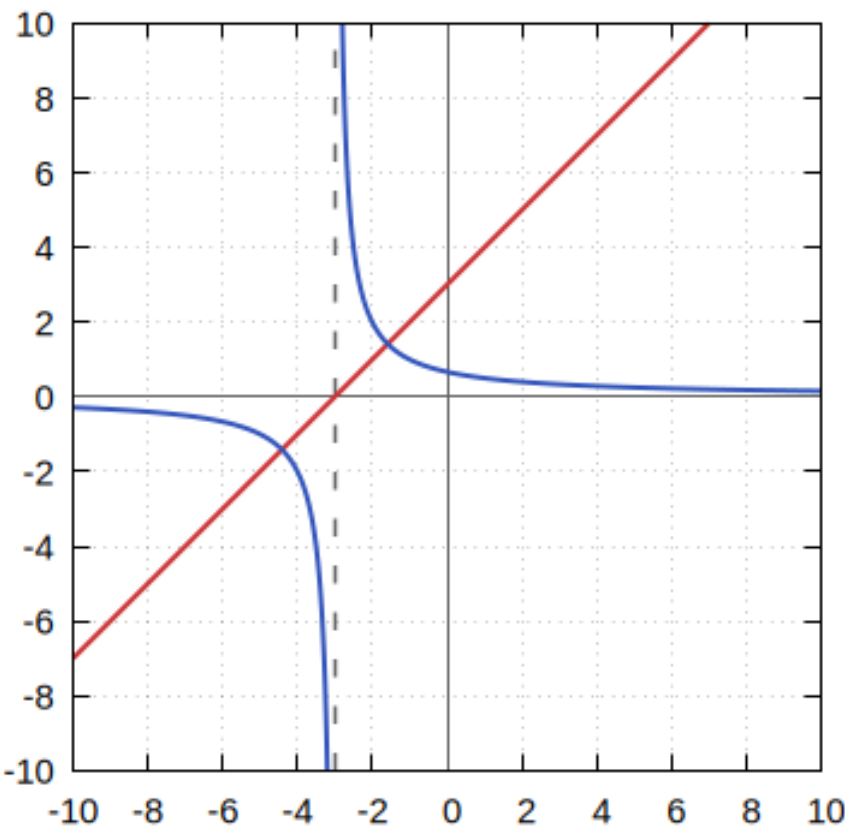


b) $g(x) = \frac{2}{x + 3}$

$y = x + 3$, x-int : $x = -3$

$VA : x = -3, HA : y = 0$

x	$y = x + 3$	$y = \frac{2}{x+3}$
-5	-2	-1
-4	-1	-2
-3	0	undefined
-2	1	2
-1	2	1



2) Graph each of the following functions. Make sure to identify key characteristics of the functions including intercepts and asymptotes.

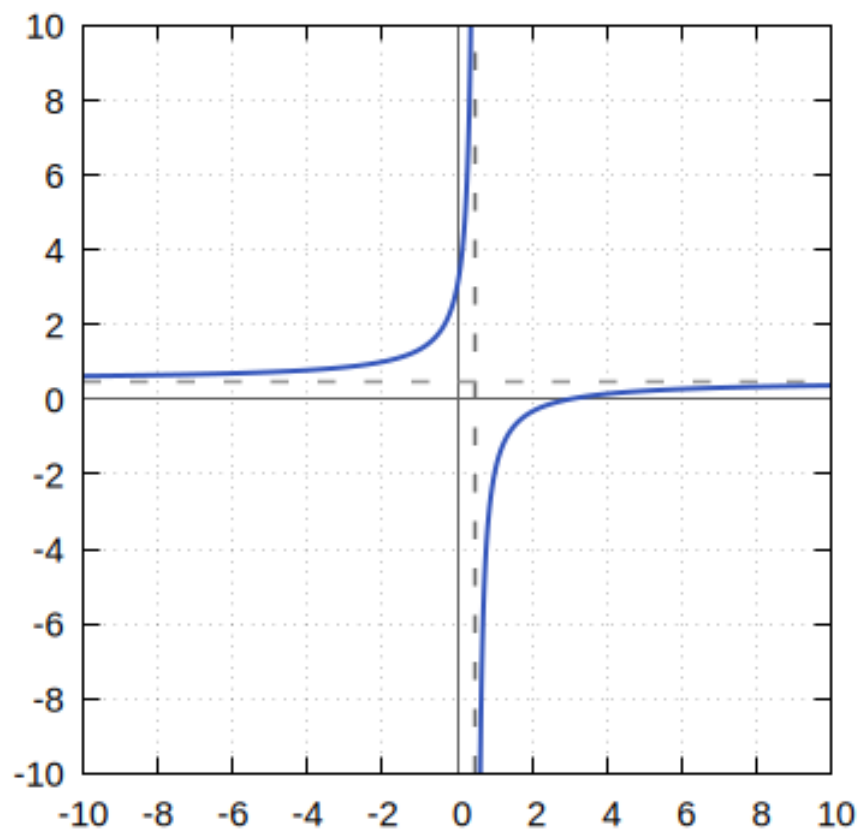
a) $f(x) = \frac{x-3}{2x-1}$

$HA : y = \frac{1}{2}, VA : x = \frac{1}{2}$

$x\text{-int} : 0 = x - 3 \Rightarrow x = 3, \text{ i.e. } (3, 0)$

$y\text{-int} : f(0) = \frac{0-3}{2(0)-1} = 3, \text{ i.e. } (0, 3)$

other point: $f(2) = \frac{2-3}{2(2)-1} = -\frac{1}{3}, \text{ i.e. } (2, -0.33)$



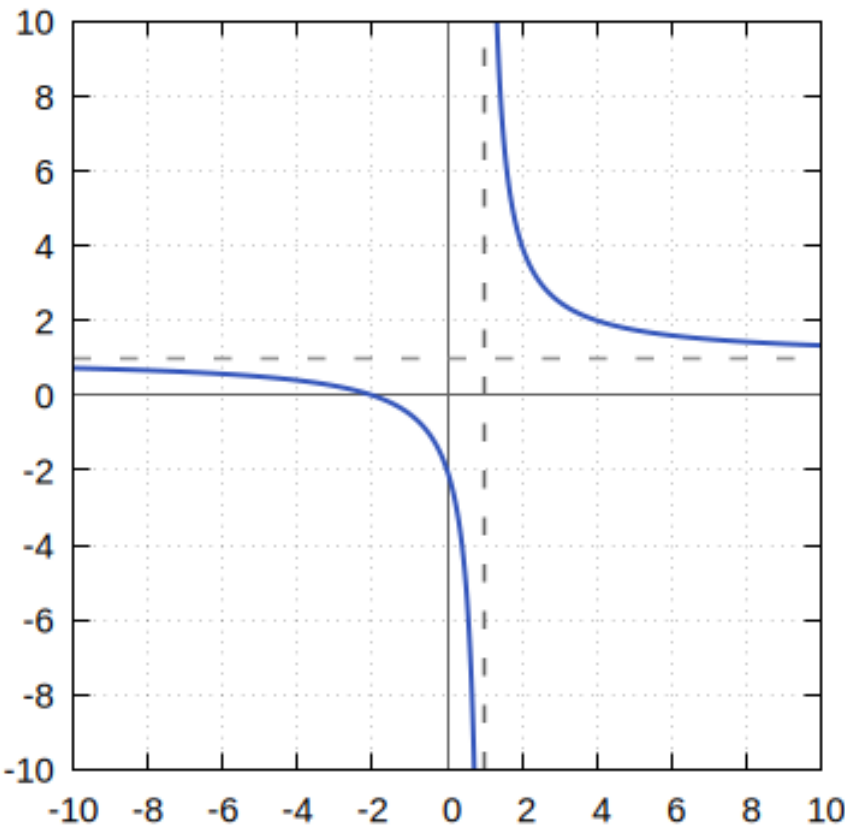
b) $g(x) = \frac{x+2}{x-1}$

$HA : y = 1, VA : x = 1$

$x\text{-int} : 0 = x + 2 \Rightarrow x = -2, \text{ i.e. } (-2, 0)$

$y\text{-int} : g(0) = \frac{0+2}{0-1} = -2, \text{ i.e. } (0, -2)$

other point: $g(-1) = \frac{-1+2}{-1-1} = -\frac{1}{2}, \text{ i.e. } (-1, -0.5)$



3) Use a calculator to approximate each to the nearest thousandth

a) $\ln 7$

$\ln 7 \approx 1.946$

b) $\ln e$

$\ln e = 1.000$

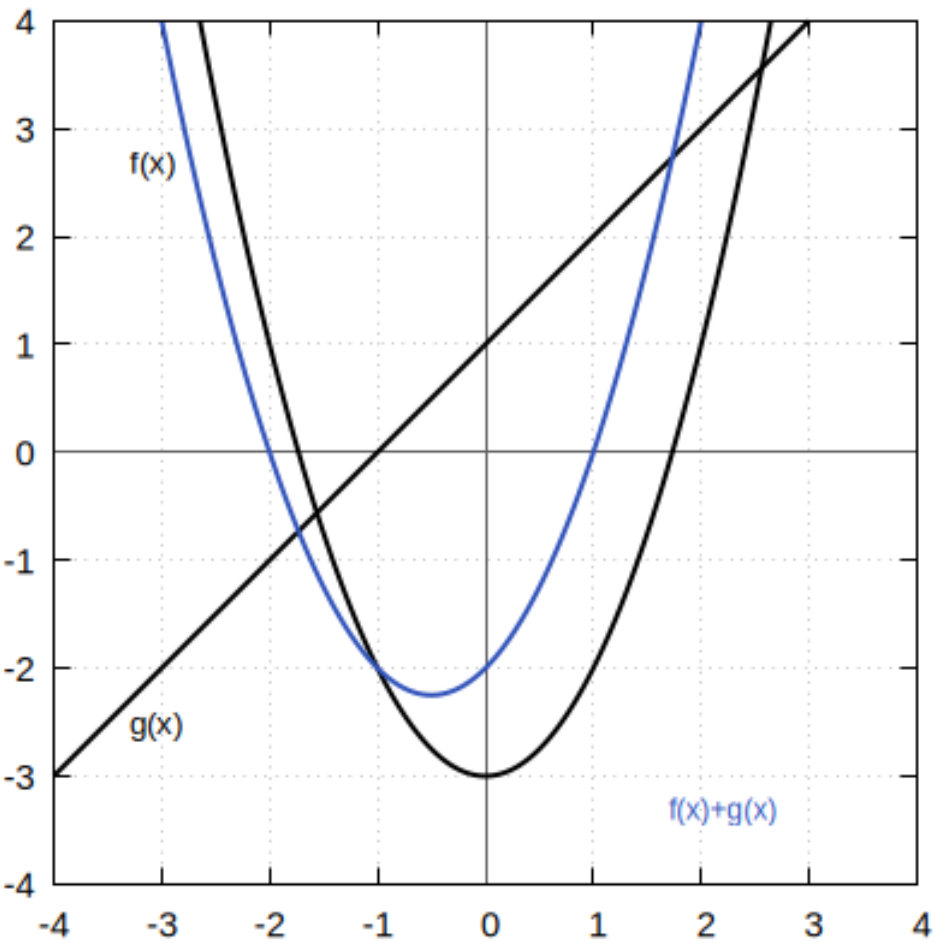
c) e^5

$e^5 \approx 148.413$

4) Use the superposition principle to draw a graph of each.

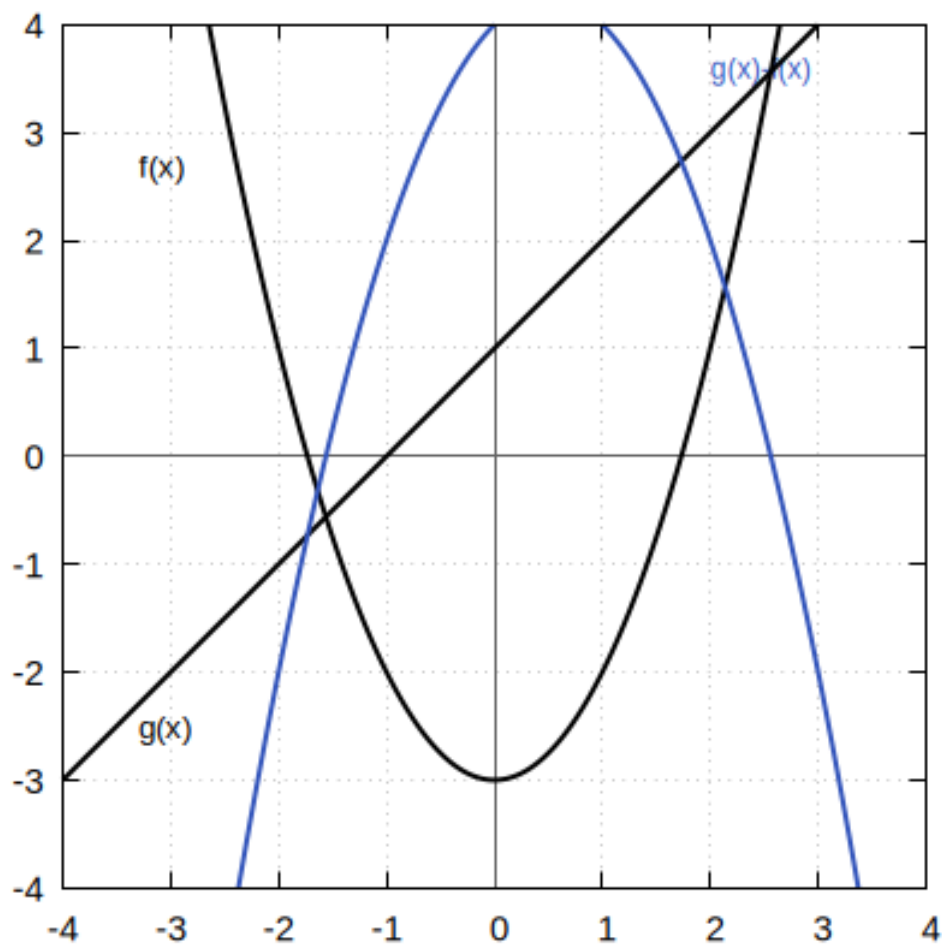
a) $y = f(x) + g(x)$

x	$f(x)$	$g(x)$	$f(x) + g(x)$	
-2	1	-1	0	
-1	-2	0	-2	
0	-3	1	-2	
1	-2	2	0	
2	1	3	4	



b) $y = g(x) - f(x)$

x	$g(x)$	$f(x)$	$g(x) - f(x)$	
-2	-1	1	-2	
-1	0	-2	2	
0	1	-3	4	
1	2	-2	4	
2	3	1	2	



5) Given that $f(x) = 2x + 3$ and $g(x) = x^2 + x$, and $h(x) = x^2 - 3$, develop an algebraic model for...

a) $y = f(x) - g(x)$

$$y = (2x + 3) - (x^2 + x)$$

$$y = -x^2 + x + 3$$

b) $y = f(x) - g(x) + h(x)$

$$y = (-x^2 + x + 3) + (x^2 - 3)$$

$$y = x$$

6) If $f(x) = 2x + 3$ and $g(x) = x^2 + x$, and $h(x) = x^2 - 3$, develop algebraic models for each.

a) $y = f(h(x))$

$$y = 2(x^2 - 3) + 3$$

$$y = 2x^2 - 6 + 3$$

$$y = 2x^2 - 3$$

b) $y = g(f(x))$

$$y = (2x + 3)^2 + (2x + 3)$$

$$y = 4x^2 + 12x + 9 + 2x + 3$$

$$y = 4x^2 + 14x + 12$$

c) $y = f(f^{-1}(x))$

$$f^{-1}(x): \text{let } y = 2x + 3, \text{ swap } x \text{ and } y: x = 2y + 3$$

$$\frac{x - 3}{2} = y \Rightarrow f^{-1}(x) = \frac{x - 3}{2}$$

$$f(f^{-1}(x)) = 2\left(\frac{x-3}{2}\right) + 3$$

$$= x - 3 + 3 = x$$

7) Solve the following equations

a) $\frac{10}{x+1} = x - 2$ $10 = (x+1)(x-2)$ $0 = x^2 - x - 12$ $0 = (x-4)(x+3)$ $x = 4 \text{ or } x = -3$	b) $\frac{x+3}{x-1} = 2x + 1$ $x+3 = (x-1)(2x+1)$ $0 = 2x^2 - 2x - 4$ $0 = (x-2)(x+1)$ $x = 2 \text{ or } x = -1$
c) $\frac{3}{3x+2} = \frac{6}{5x}$ $3(5x) = 6(3x+2)$ $15x = 18x + 12$ $-3x = 12$ $x = -4$	d) $\frac{x-2}{x+3} = \frac{x-4}{x+5}$ $(x+5)(x-2) = (x-4)(x+3)$ $x^2 + 3x - 10 = x^2 - x - 12$ $4x = -2$ $x = -\frac{1}{2}$
e) $\frac{2}{x} + \frac{5}{3} = \frac{7}{x}$ $3(2) + 5(x) = 3(7)$ $6 + 5x = 21$ $x = 3$	f) $\frac{10}{x+3} + \frac{10}{3} = 6$ $3(10) + 10(x+3) = 18(x+3)$ $30 + 10x + 30 = 18x + 54$ $6 = 8x$ $x = \frac{3}{4}$
g) $\frac{3}{x} + \frac{4}{x+1} = 2$ $3(x+1) + 4x = 2x(x+1)$ $3x + 3 + 4x = 2x^2 + 2x$ $0 = 2x^2 - 5x - 3$ $0 = (x-3)(2x+1)$ $x = 3 \text{ or } x = -\frac{1}{2}$	h) $3 = \frac{6}{2x^2 - x - 4}$ $2x^2 - x - 4 = 2$ $2x^2 - x - 6 = 0$ $(x-2)(2x+3) = 0$ $x = 2 \text{ or } x = -\frac{3}{2}$

Section 2: Rational Inequalities

8) Solve the following rational inequalities using a factor table

a) $\frac{6x}{x+1} > 4$

$$\frac{6x}{x+1} - 4 > 0$$

$$\frac{6x - 4(x+1)}{x+1} > 0$$

$$\frac{2x-4}{x+1} > 0$$

x-intercept: $x = 2$; restriction: $x \neq -1$

Interval	$x < -1$	$-1 < x < 2$	$x > 2$
$x - 2$	-	-	+
$x + 1$	-	+	+
overall	+	-	+

Solution: $x < -1$ or $x > 2$, i.e. $x \in (-\infty, -1) \cup (2, \infty)$

$$b) \frac{1}{2x+10} \geq \frac{1}{x+3}$$

$$\frac{1}{2x+10} - \frac{1}{x+3} \geq 0$$

$$\frac{(x+3) - (2x+10)}{(2x+10)(x+3)} \geq 0$$

$$\frac{-x-7}{(2x+10)(x+3)} \geq 0$$

x-intercept: $x = -7$; restrictions: $x \neq -5, -3$

Interval	$x < -7$	$-7 < x < -5$	$-5 < x < -3$	$x > -3$
$-x-7$	+	-	-	-
$2x+10$	-	-	+	+
$x+3$	-	-	-	+
overall	+	-	+	-

Solution: $x \leq -7$ or $-5 < x < -3$, i.e. $x \in (-\infty, -7] \cup (-5, -3)$

$$c) \frac{2x-3}{x+5} \geq \frac{2x+7}{x-3}$$

$$\frac{(2x-3)(x-3) - (2x+7)(x+5)}{(x+5)(x-3)} \geq 0$$

$$\frac{-26x-26}{(x+5)(x-3)} \geq 0$$

$$\frac{-26(x+1)}{(x+5)(x-3)} \geq 0$$

x-intercept: $x = -1$; restrictions: $x \neq -5, 3$

Interval	$x < -5$	$-5 < x < -1$	$-1 < x < 3$	$x > 3$
$x+1$	-	-	+	+
$x+5$	-	+	+	+
$x-3$	-	-	-	+
overall	+	-	+	-

Solution: $x < -5$ or $-1 \leq x < 3$, i.e. $x \in (-\infty, -5) \cup [-1, 3)$

$$d) \frac{7}{x-3} \geq \frac{2}{x+4}$$

$$\frac{7(x+4) - 2(x-3)}{(x-3)(x+4)} \geq 0$$

$$\frac{5x+34}{(x-3)(x+4)} \geq 0$$

x-intercept: $x = -6.8$; restrictions: $x \neq -4, 3$

Interval	$x < -6.8$	$-6.8 < x < -4$	$-4 < x < 3$	$x > 3$
$5x+34$	-	+	+	+
$x-3$	-	-	-	+
$x+4$	-	-	+	+
overall	-	+	-	+

Solution: $-6.8 \leq x < -4$ or $x > 3$, i.e. $x \in [-6.8, -4) \cup (3, \infty)$

$$e) \frac{x^2 - x - 12}{x-1} < 0$$

$$\frac{(x-4)(x+3)}{x-1} < 0$$

x-intercepts: $x = -3, 4$; restriction: $x \neq 1$

Interval	$x < -3$	$-3 < x < 1$	$1 < x < 4$	$x > 4$
$x-4$	-	-	-	+
$x+3$	-	+	+	+
$x-1$	-	-	+	+

overall	-	+	-	+
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Solution: $x < -3$ or $1 < x < 4$, i.e. $x \in (-\infty, -3) \cup (1, 4)$

$$f) \frac{6x^2 - 5x + 1}{2x + 1} < 0$$

$$\frac{(3x - 1)(2x - 1)}{2x + 1} < 0$$

x-intercepts: $x = \frac{1}{3}, \frac{1}{2}$; **restriction:** $x \neq -\frac{1}{2}$

Interval	$x < -\frac{1}{2}$	$-\frac{1}{2} < x < \frac{1}{3}$	$\frac{1}{3} < x < \frac{1}{2}$	$x > \frac{1}{2}$
$3x - 1$	-	-	+	+
$2x - 1$	-	-	-	+
$2x + 1$	-	+	+	+
overall	-	+	-	+

Solution: $x < -\frac{1}{2}$ or $\frac{1}{3} < x < \frac{1}{2}$, i.e. $x \in (-\infty, -\frac{1}{2}) \cup (\frac{1}{3}, \frac{1}{2})$

$$g) \frac{2x - 10}{x} > x - 5$$

$$\frac{2x - 10}{x} - x + 5 > 0$$

$$\frac{2x - 10 - x^2 + 5x}{x} > 0$$

$$\frac{-x^2 + 7x - 10}{x} > 0$$

$$\frac{-(x - 5)(x - 2)}{x} > 0$$

x-intercepts: $x = 2, 5$; **restriction:** $x \neq 0$

Interval	$x < 0$	$0 < x < 2$	$2 < x < 5$	$x > 5$
$-(x - 5)$	+	+	+	-
$x - 2$	-	-	+	+
x	-	+	+	+
overall	+	-	+	-

Solution: $x < 0$ or $2 < x < 5$, i.e. $x \in (-\infty, 0) \cup (2, 5)$

Answer Key

1) a) VA $x = -2, 2$, HA $y = 0$ b) VA $x = -3$, HA $y = 0$

2) a) VA $x = \frac{1}{2}$, HA $y = \frac{1}{2}$ b) VA $x = 1$, HA $y = 1$

3) a) 1.946 b) 1.000 c) 148.413

4) a) $x^2 + x - 2$ b) $-x^2 + x + 4$

5) a) $-x^2 + x + 3$ b) x

6) a) $2x^2 - 3$ b) $4x^2 + 14x + 12$ c) x

7) a) 4, -3 b) 2, -1 c) -4 d) $-\frac{1}{2}$ e) 3 f) $\frac{3}{4}$ g) 3, $-\frac{1}{2}$ h) 2, $-\frac{3}{2}$

8) a) $x < -1$ or $x > 2$ b) $x \leq -7$ or $-5 < x < -3$ c) $x < -5$ or $-1 \leq x < 3$

d) $-6.8 \leq x < -4$ or $x > 3$ e) $x < -3$ or $1 < x < 4$ f) $x < -\frac{1}{2}$ or $\frac{1}{3} < x < \frac{1}{2}$ g) $x < 0$ or $2 < x < 5$