

3.4 Solve Rational Equations and Inequalities Lesson

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Part 1: Rational Expressions

Rational Expression: the quotient of two polynomials, $\frac{p(x)}{q(x)}$, where $q(x) \neq 0$

Example 1: Simplify and state the restrictions of each rational expression

a) $\frac{2x^2 - 8}{x^2 + 3x + 2}$	b) $\frac{x^3 - x^2 - x + 1}{3x^3 - 3}$
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Part 2: Solve Rational Equations

Steps for solving rational equations:

1. Fully factor both sides of the equation
2. Multiply both sides by a common denominator (cross multiply if appropriate)
3. Continue to solve as you would a normal polynomial equation
4. State restrictions throughout (values of x that would make denominator equal zero)

Example 2: Solve each equation

a) $\frac{4}{3x - 5} = 4$	b) $\frac{6}{x - 2} = x - 1$
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c) $\frac{x - 5}{x^2 - 3x - 4} = \frac{3x + 2}{x^2 - 1}$

Part 3: Solve Rational Inequalities

REMEMBER: Solving _____ is the same as solving _____. However, when both sides of an inequality are multiplied or divided by a _____ number, the inequality sign must be _____.

3.4 Solve Rational Equations and Inequalities Lesson

Steps for solving rational inequalities algebraically:

1. Use inverse operations to move all terms to one side of the inequality
2. Combine the expressions by using a common denominator
3. Factor the polynomial
4. Find the interval(s) where the function is positive or negative by making a factor table

To make a factor table:

- Use x -intercepts and vertical asymptotes to divide into intervals
- Use a test point within each interval to find the sign of each factor
- Determine the overall sign of the product by multiplying signs of each factor within each interval.

Example 3: Solve each inequality algebraically

a) $\frac{x^2 + 6x + 5}{2x^2 - 7x + 3} < 0$

b) $x - 2 < \frac{8}{x}$

c) $\frac{x + 3}{x + 1} \geq \frac{x - 2}{x - 3}$

d) $\frac{x^3 + 6x^2 - 2x}{x^2 + 4} \geq 2$