

8.3 Composite Functions Lesson with solutions

This document was composed and formatted by Annie Bernatchez.

The course material originated from www.jensenmath.ca

Two functions, f and g can be combined using a process called composition, which can be represented by:

$f(g(x))$ OR $(f \circ g)(x)$

This is read as “ f composite g ”

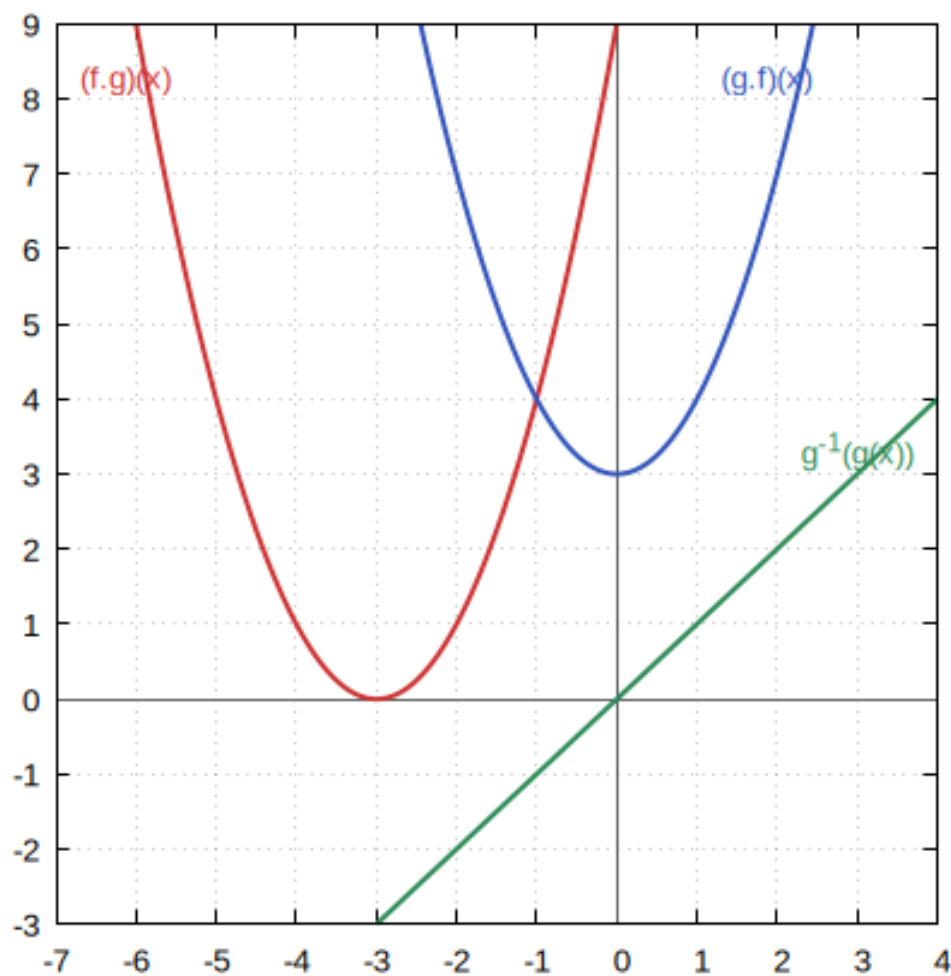
Part 1: Determine the Composition of Two Functions

To determine an equation for a composite function, substitute the second function into the first.

To determine $f(g(x))$, substitute $g(x)$ in for x into $f(x)$

Example 1: If $f(x) = x^2$ and $g(x) = x + 3$, determine an equation for each composite function and then graph the function.

a) $(f \circ g)(x)$ $= f(g(x)) = f(x + 3) = (x + 3)^2$		b) $(g \circ f)(x)$ $= g(f(x)) = g(x^2) = x^2 + 3$		c) $g^{-1}(g(x))$ Start by finding $g^{-1}(x)$: $y = x + 3$, $x = y + 3$, $x - 3 = y$, SO $g^{-1}(x) = x - 3$ Now find $g^{-1}(g(x)) = g^{-1}(x + 3) = (x + 3) - 3 = x$																																					
<table><tr><th>x</th><th>$(f \circ g)(x)$</th></tr><tr><td>-5</td><td>4</td></tr><tr><td>-4</td><td>1</td></tr><tr><td>-3</td><td>0</td></tr><tr><td>-2</td><td>1</td></tr><tr><td>-1</td><td>4</td></tr></table>		x	$(f \circ g)(x)$	-5	4	-4	1	-3	0	-2	1	-1	4	<table><tr><th>x</th><th>$(g \circ f)(x)$</th></tr><tr><td>-2</td><td>7</td></tr><tr><td>-1</td><td>4</td></tr><tr><td>0</td><td>3</td></tr><tr><td>1</td><td>4</td></tr><tr><td>2</td><td>7</td></tr></table>		x	$(g \circ f)(x)$	-2	7	-1	4	0	3	1	4	2	7	<table><tr><th>x</th><th>$g^{-1}(g(x))$</th></tr><tr><td>-2</td><td>-2</td></tr><tr><td>-1</td><td>-1</td></tr><tr><td>0</td><td>0</td></tr><tr><td>1</td><td>1</td></tr><tr><td>2</td><td>2</td></tr></table>		x	$g^{-1}(g(x))$	-2	-2	-1	-1	0	0	1	1	2	2
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Part 2: Evaluate a Composite Function

To evaluate a composite function $f(g(x))$ at a specific value, evaluate $g(x)$ at the specific value and then substitute the result into $f(x)$.

Example 2: If $u(x) = x^2 + 3x + 2$ and $w(x) = \frac{1}{x-1}$

a) Evaluate $(u \circ w)(2)$ $w(2) = \frac{1}{2-1} = 1$ $(u \circ w)(2) = u(w(2)) = u(1) = (1)^2 + 3(1) + 2 = 6$	b) Evaluate $w(u(-3))$ $u(-3) = (-3)^2 + 3(-3) + 2 = 2$ $w(u(-3)) = w(2) = \frac{1}{2-1} = 1$
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Part 3: Application

Example 3: The number of rabbits, R , in a wildlife reserve as a function of time, t , in years can be modelled by the function $R(t) = 50\cos(t) + 100$. The number of wolves, W , in the same reserve can be modelled by the function $W(t) = 0.2[R(t-2)]$. Find the full equation for $W(t)$

$$R(t-2) = 50\cos(t-2) + 100$$

$$W(t) = 0.2[R(t-2)] = 0.2[50\cos(t-2) + 100]$$

$$W(t) = 10\cos(t-2) + 20$$