

## 8.1/8.2 Sum/Difference and Product/Quotient of Functions Lesson with solutions

This document was composed and formatted by Annie Bernatchez.

The course material originated from [www.jensenmath.ca](http://www.jensenmath.ca)

### Part 1: Sum and Difference of Functions

When two functions  $f(x)$  and  $g(x)$  are combined to form the function  $(f + g)(x)$  or  $(f - g)(x)$ , the new function is called the sum or difference of  $f$  and  $g$ .

The graph of  $f + g$  or  $f - g$  can be obtained by adding or subtracting corresponding  $y$ -coordinates. This is called the **superposition principle**.

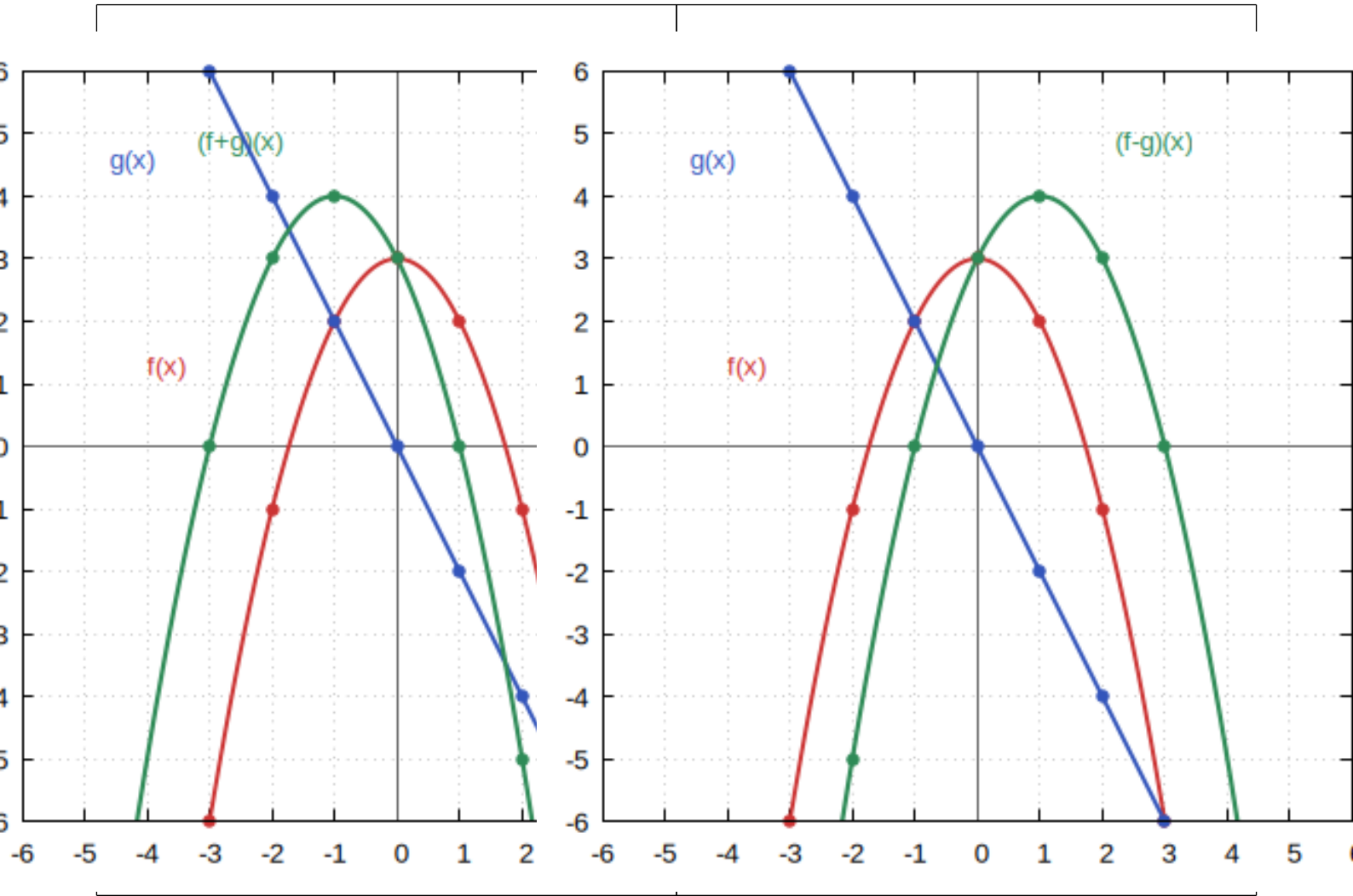
$$(f + g)(x) = f(x) + g(x)$$

$$(f - g)(x) = f(x) - g(x)$$

**Example 1:** Given  $f(x) = -x^2 + 3$  and  $g(x) = -2x$  determine the graphs of  $(f + g)(x)$  and  $(f - g)(x)$ .

**Method 1: Graphically**

| $x$ | $f(x)$ | $g(x)$ | $f(x) + g(x)$ | $f(x) - g(x)$ |
|-----|--------|--------|---------------|---------------|
| -3  | -6     | 6      | 0             | -12           |
| -2  | -1     | 4      | 3             | -5            |
| -1  | 2      | 2      | 4             | 0             |
| 0   | 3      | 0      | 3             | 3             |
| 1   | 2      | -2     | 0             | 4             |
| 2   | -1     | -4     | -5            | 3             |
| 3   | -6     | -6     | -12           | 0             |



Method 2: Algebraically

$(f + g)(x) = f(x) + g(x) = (-x^2 + 3) + (-2x) = -x^2 - 2x + 3$

Complete the square to find the vertex:

$(f + g)(x) = -(x^2 + 2x) + 3 = -(x^2 + 2x + 1 - 1) + 3 = -(x^2 + 2x + 1) + 1 + 3 = -(x + 1)^2 + 4$

vertex is (-1, 4)

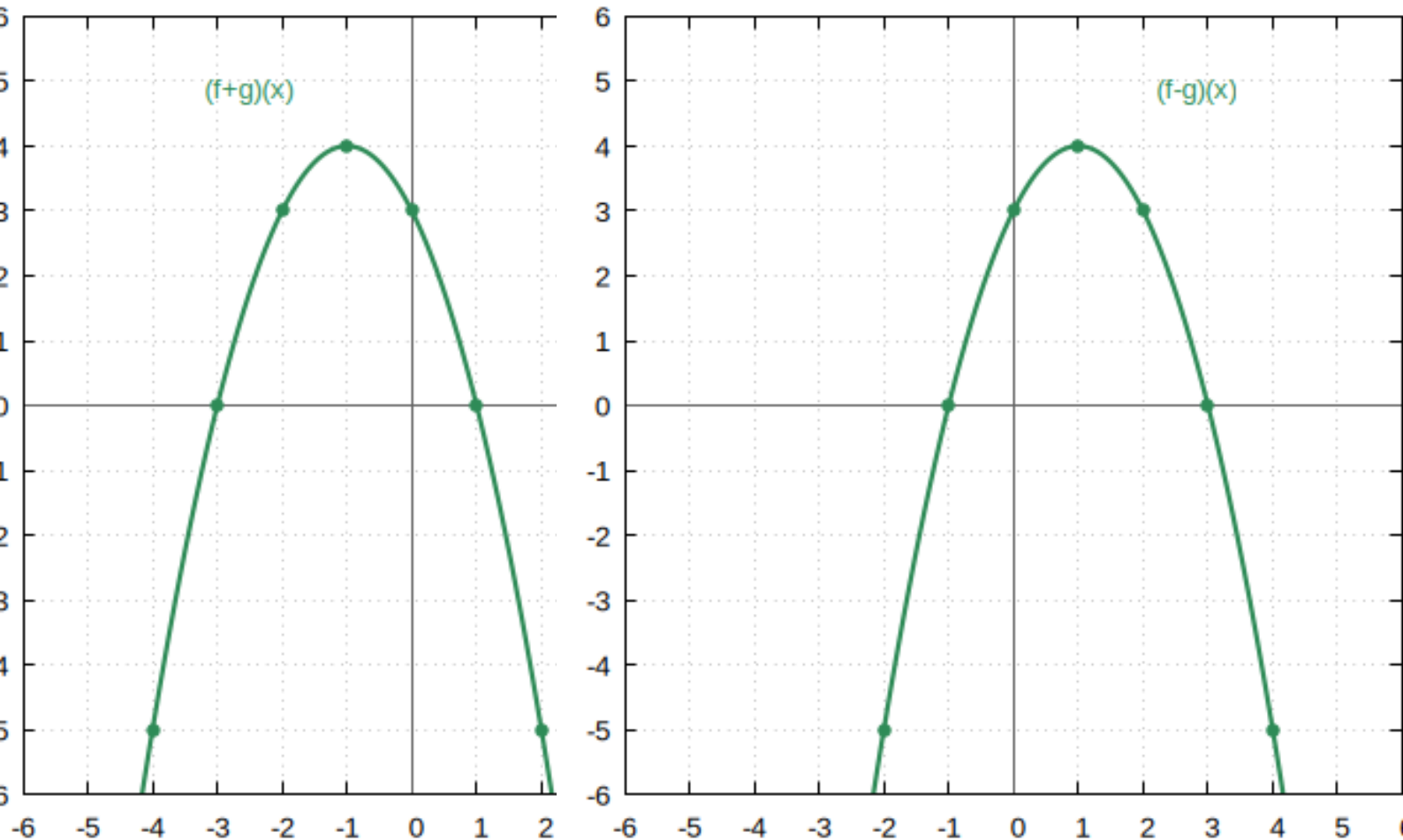
$(f - g)(x) = f(x) - g(x) = (-x^2 + 3) - (-2x) = -x^2 + 2x + 3$

Complete the square to find the vertex:

$(f - g)(x) = -(x^2 - 2x) + 3 = -(x^2 - 2x + 1 - 1) + 3 = -(x^2 - 2x + 1) + 1 + 3 = -(x - 1)^2 + 4$

vertex is (1, 4)

| x  | $(f + g)(x)$ | x  | $(f - g)(x)$ |
|----|--------------|----|--------------|
| -4 | -5           | -2 | -5           |
| -3 | 0            | -1 | 0            |
| -2 | 3            | 0  | 3            |
| -1 | 4            | 1  | 4            |
| 0  | 3            | 2  | 3            |
| 1  | 0            | 3  | 0            |
| 2  | -5           | 4  | -5           |



$$(f+g)(x): D: \{x \in \mathbb{R}\}, R: \{y \in \mathbb{R} | y \leq 4\}$$

$$(f-g)(x): D: \{x \in \mathbb{R}\}, R: \{y \in \mathbb{R} | y \leq 4\}$$

**Note:** The domain of the sum or difference of functions is the intersection of the domains of  $f$  and  $g$ .

## Part 2: Product and Quotient of Functions

When two functions  $f(x)$  and  $g(x)$  are combined to form the function  $(f \cdot g)(x)$  or  $(f \div g)(x)$ , the new function is called the product or quotient of  $f$  and  $g$ .

The graph of  $f \cdot g$  or  $f \div g$  can be obtained by multiplying or dividing corresponding  $y$ -coordinates.

$$(f \times g)(x) = f(x) \times g(x)$$

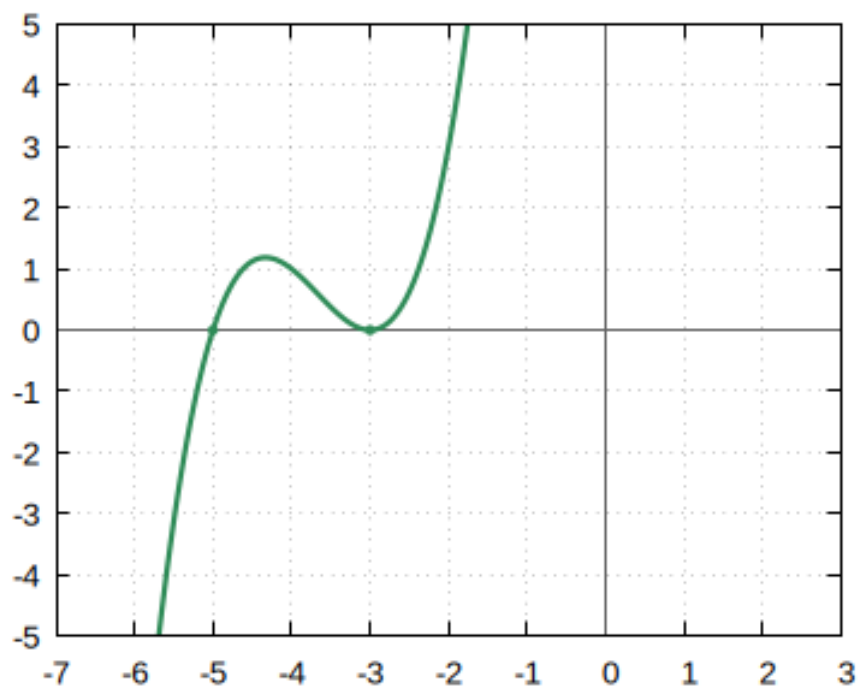
$$(f \div g)(x) = f(x) \div g(x)$$

**Example 2:** Let  $f(x) = x + 3$  and  $g(x) = x^2 + 8x + 15$ . Determine an equation and graph for

a)  $(f \times g)(x)$

$$(f \times g)(x) = f(x)g(x) = (x+3)(x^2+8x+15) = (x+3)(x+3)(x+5) = (x+3)^2(x+5)$$

$x$ -intercepts at  $-3$  (order 2) and  $-5$  (order 1). Extends from Q3 to Q1.



b)  $(f \div g)(x)$

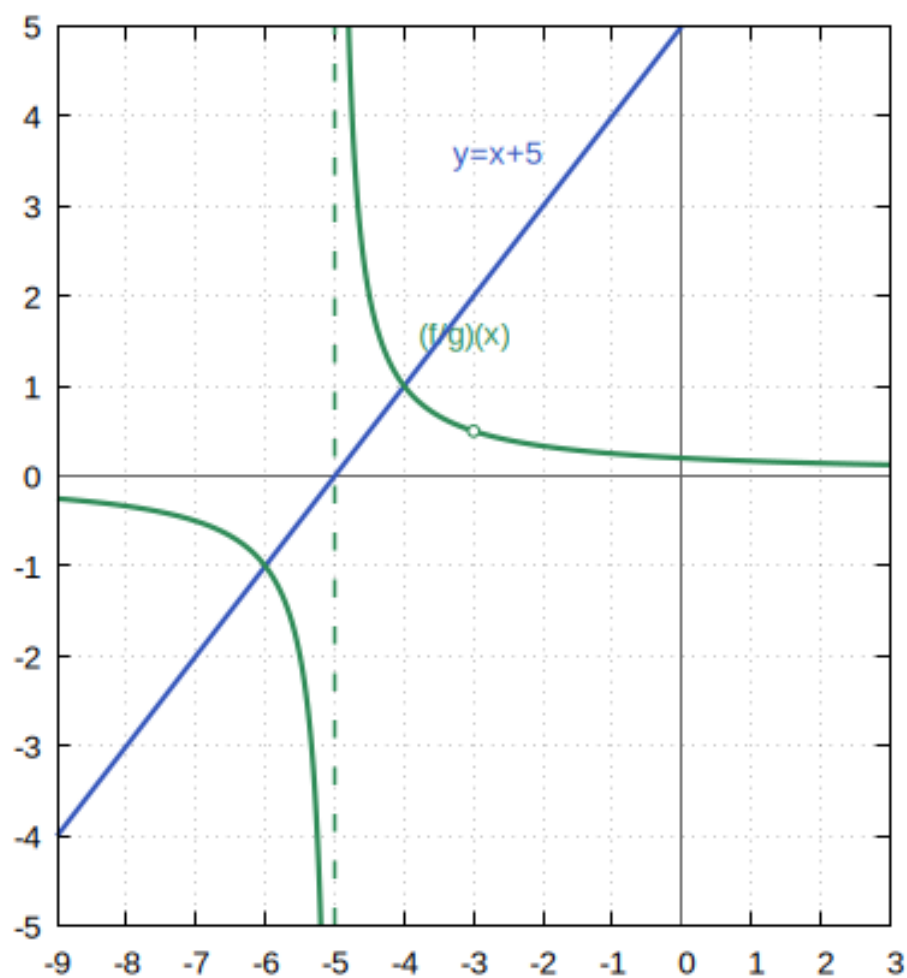
$$(f \div g)(x) = \frac{f(x)}{g(x)} = \frac{x+3}{(x+3)(x+5)} = \frac{1}{x+5}; x \neq -5, -3$$

$VA: x = -5$

$HA: y = 0$

Hole at  $x = -3$

**Note:** always a HA at  $y = 0$  when the denominator is a higher degree than the numerator



c) State the domain and range of both functions

$(f \times g)(x)$ :  $D : \{x \in \mathbb{R}\}$ ,  $R : \{y \in \mathbb{R}\}$

$(f \div g)(x)$ :  $D : \{x \in \mathbb{R} | x \neq -5, -3\}$ ,  $R : \{y \in \mathbb{R} | y \neq 0, 0.5\}$