

3.3 Quotient of Linear Functions Lesson with solutions

This document was composed and formatted by Annie Bernatchez.

The course material originated from www.jensenmath.ca

Part 1: Key Features of the Quotient of Linear Functions

Features of $f(x) = \frac{ax + b}{cx + d}$

- If an x value is a zero of the denominator ONLY, this results in a vertical asymptote
 - Equation of vertical asymptote is $x = \frac{-d}{c}$
- If an x value is a zero of the numerator AND denominator, this results in a **hole** in the graph NOT a vertical asymptote
- There is a horizontal asymptote at the ratio of the leading coefficients
 - Equation of horizontal asymptote is $y = \frac{a}{c}$
- Forms a **Hyperbola**: the two branches of the graph of the function are equidistant from the point of intersection of the vertical and horizontal asymptotes
 - Once you know the shape of one branch, you can translate the points to graph the other branch
- You can find the x -intercept by setting $y = 0$ and solving for x
 - This results in $\left(\frac{-b}{a}, 0\right)$
- You can find the y -intercept by setting $x = 0$ and solving for y
 - This results in $\left(0, \frac{b}{d}\right)$

Part 2: Graphing a Quotient of Linear Functions

Example 1: Graph each of the following functions

a) $f(x) = \frac{x-3}{x+2}$

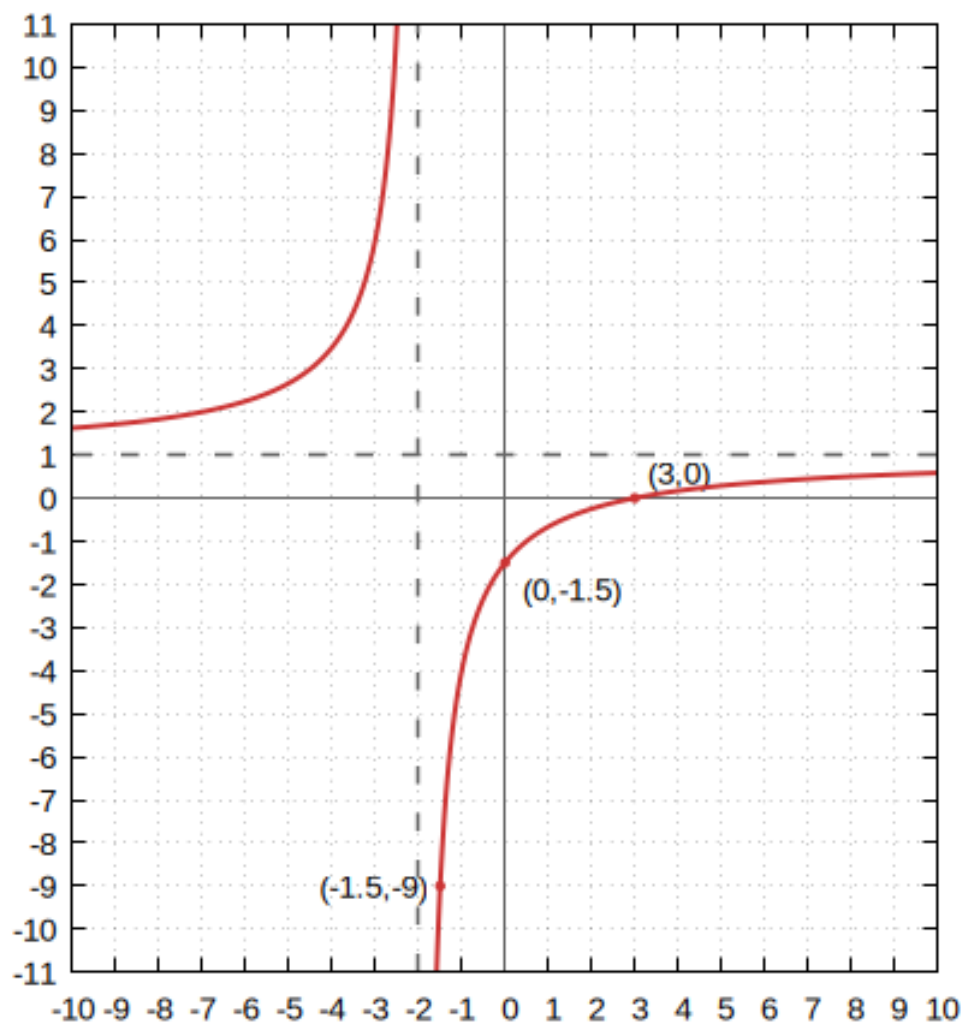
VA : $x + 2 = 0 \Rightarrow x = -2$

HA : $\frac{1}{1} = 1 \Rightarrow y = 1$

x -int : $0 = \frac{x-3}{x+2} \Rightarrow 0 = x-3 \Rightarrow x = 3$, SO $(3, 0)$

y -int : $f(0) = \frac{0-3}{0+2} = -\frac{3}{2}$, SO $(0, -1.5)$

Another point: $f(-1.5) = \frac{-1.5-3}{-1.5+2} = \frac{-4.5}{0.5} = -9$, SO $(-1.5, -9)$



b) $g(x) = \frac{2x-3}{x-1}$

VA : $x - 1 = 0 \Rightarrow x = 1$

HA : $\frac{2}{1} = 2 \Rightarrow y = 2$

x-int : $0 = \frac{2x-3}{x-1} \Rightarrow 0 = 2x-3 \Rightarrow x = \frac{3}{2}$, SO (1.5, 0)

y-int : $g(0) = \frac{2(0)-3}{0-1} = 3$, SO (0, 3)

Other points: $g(2) = \frac{2(2)-3}{2-1} = \frac{1}{1} = 1$, SO (2, 1); $g(3) = \frac{2(3)-3}{3-1} = \frac{3}{2}$, SO (3, 1.5)

