

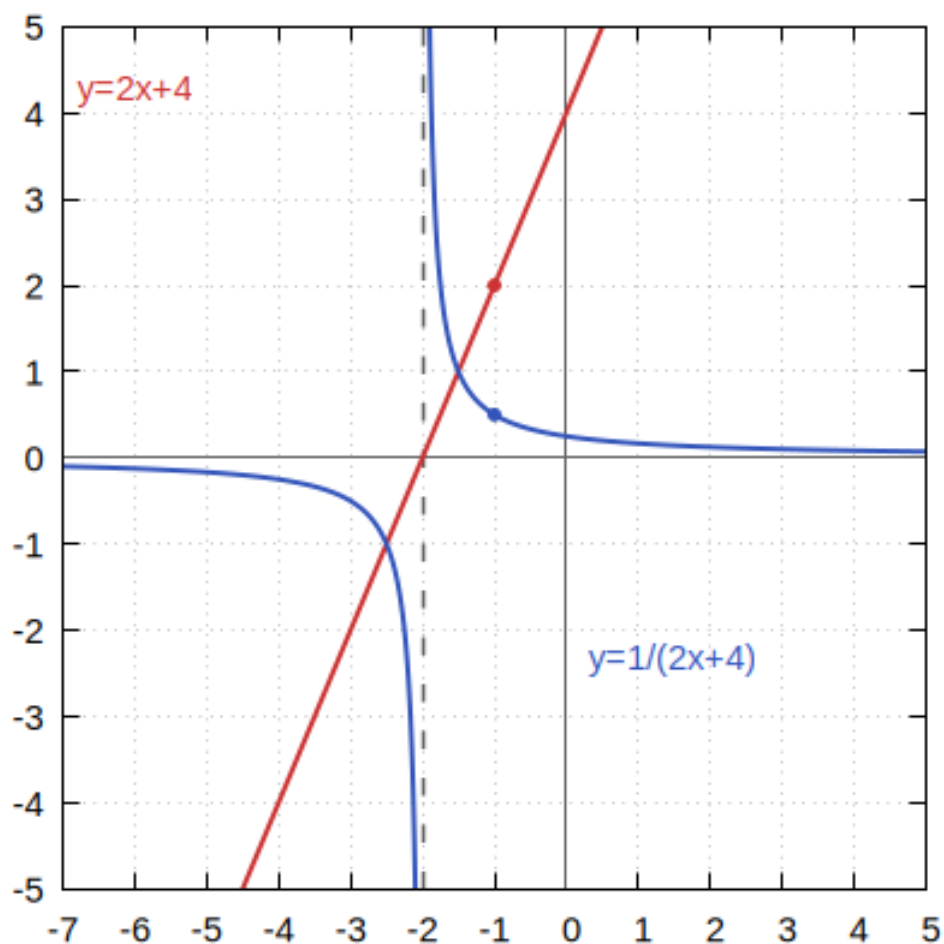
3.1/3.2 Reciprocal of Linear and Quadratic Functions Lesson with solutions

This document was composed and formatted by Annie Bernatchez.

The course material originated from www.jensenmath.ca

Part 1: Analyze the Reciprocal of a Linear Function

Example 1:



a) Draw the horizontal and vertical asymptotes for the reciprocal function and state what they are

$HA : y = 0$

$VA : x = -2$

b) What graphical characteristic in the reciprocal function does the zero (x-int) of the original function correspond to?

The vertical asymptote of the reciprocal function passes through the x-intercept of the linear function.

c) When the original function is increasing, what is happening to the reciprocal function?

It is DECREASING.

d) What are the y-coordinates of the points of intersection?

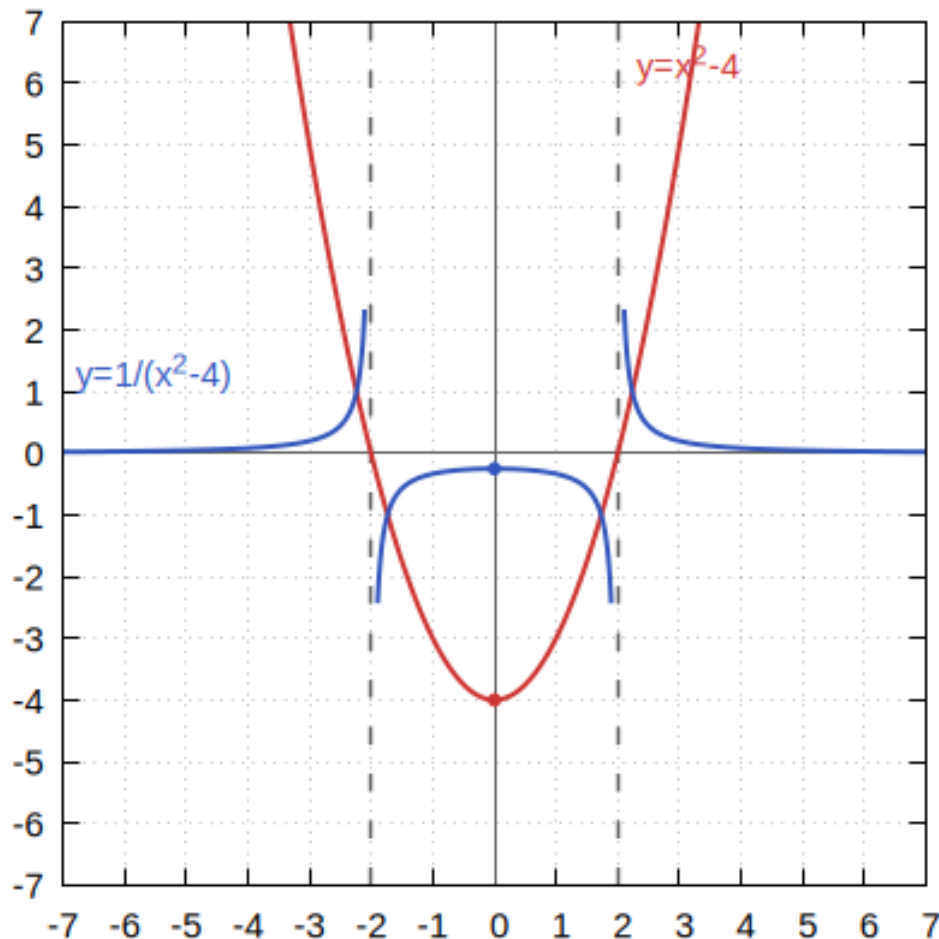
1 and -1. This is because the reciprocal of each of those does not change their value.

e) Label a point on the graph of both functions at $x = 2$. What do you notice about the y values of each point?

The y -value of the reciprocal function is the reciprocal of the y -value of the linear function.

Part 2: Analyze the Reciprocal of a Quadratic Function

Example 2:



a) Draw the horizontal and vertical asymptotes for the reciprocal function and state what they are

$VA : x = 2$ and $x = -2$

$HA : y = 0$

b) What graphical characteristic in the reciprocal function do the zeros (x -int) of the original function correspond to?

The vertical asymptotes of the reciprocal function pass through the x -intercepts of the quadratic.

c) When the original function is decreasing, what is happening to the reciprocal function?

It is INCREASING.

d) What are the y -coordinates of the points of intersection?

1 and -1

f) Label the local min or max point on each function. What do you notice about them?

They have the same x -coordinate, which is exactly half way between the vertical asymptotes.

The quadratic has a local min but the reciprocal has a local max.

Properties of Reciprocal Functions

- All the y -coordinates of the reciprocal function are the reciprocals of the y -coordinates of the original function
- The graph of the reciprocal function has a vertical asymptote at the x -intercepts (zeros) of the original
 - This is because it makes the denominator of the reciprocal $= 0$
- $y = 0$ will always be a horizontal asymptote
- The reciprocal function has the same positive/negative intervals as the original function
- Intervals of increase on the original function are intervals of decrease on the reciprocal
- Intervals of decrease on the original function are intervals of increase on the reciprocal
- If 1 is in the range of the original function, this is where the functions will intersect
- If the original function has a local min point, the reciprocal will have a local max at the same x -value (and vice versa)

Part 3: Graphing Reciprocal Functions

Process:

- Find key features of the function in the denominator and graph it using a table of values
- Create a table of values for the reciprocal function by keeping the same x values but using the reciprocal of all y values
- Draw vertical asymptotes at any point that is a zero of the original linear/quadratic function
 - Reciprocal of 0 is undefined
- If the numerator is something other than 1, multiply the y -values by this stretch factor

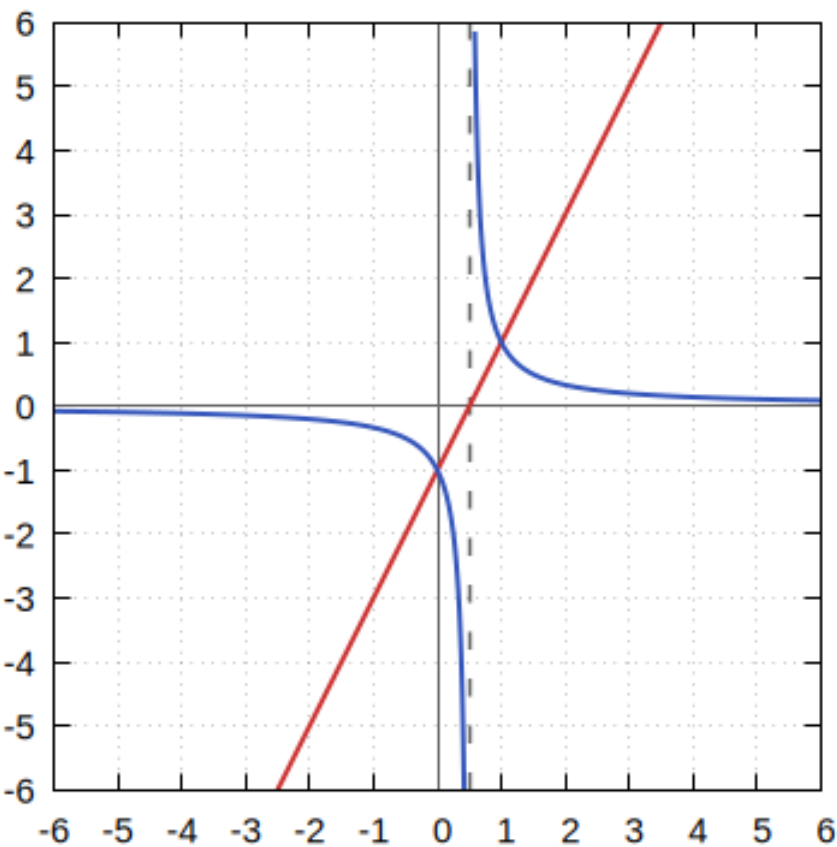
Example 3: Graph each of the following reciprocal functions. Start by graphing the function in the denominator.

a) $y = \frac{1}{2x - 1}$

$y = 2x - 1$, x -int : $(\frac{1}{2}, 0)$

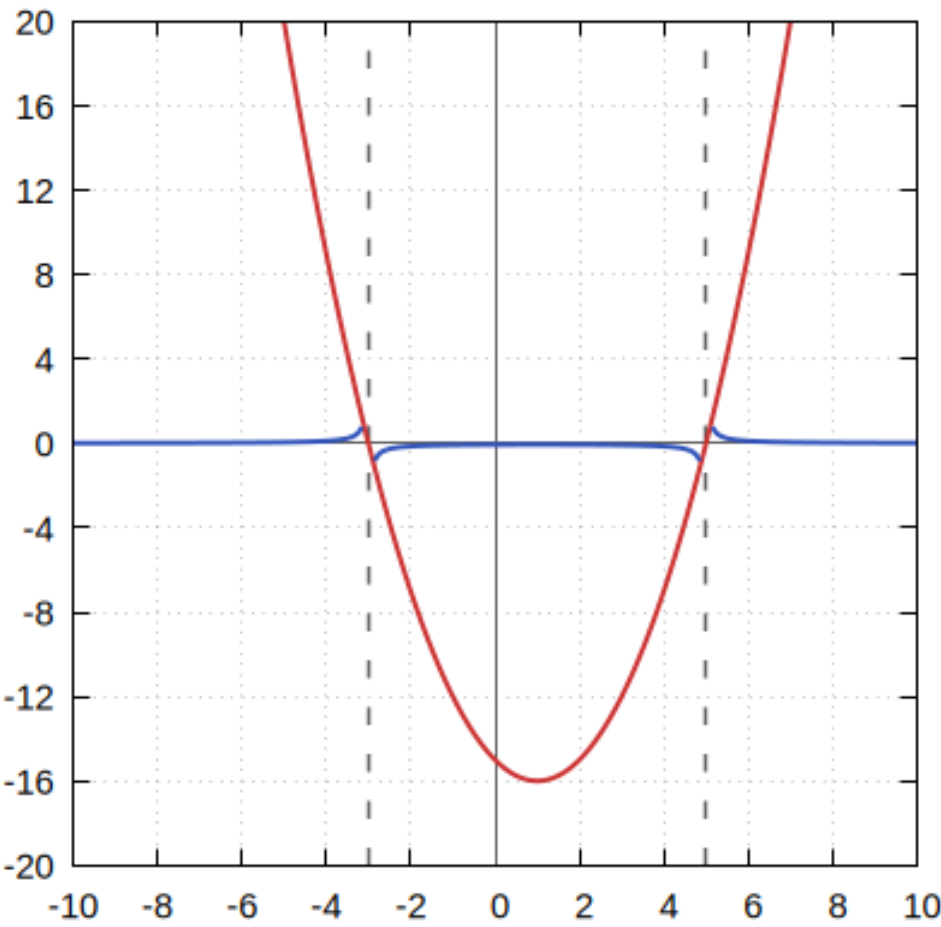
$VA : x = \frac{1}{2}$, $HA : y = 0$

x	$y = 2x - 1$	$y = \frac{1}{2x - 1}$
-2	-5	-0.2
-1	-3	-0.33
0	-1	-1
0.5	0	undefined
1	1	1
2	3	0.33
3	5	0.2



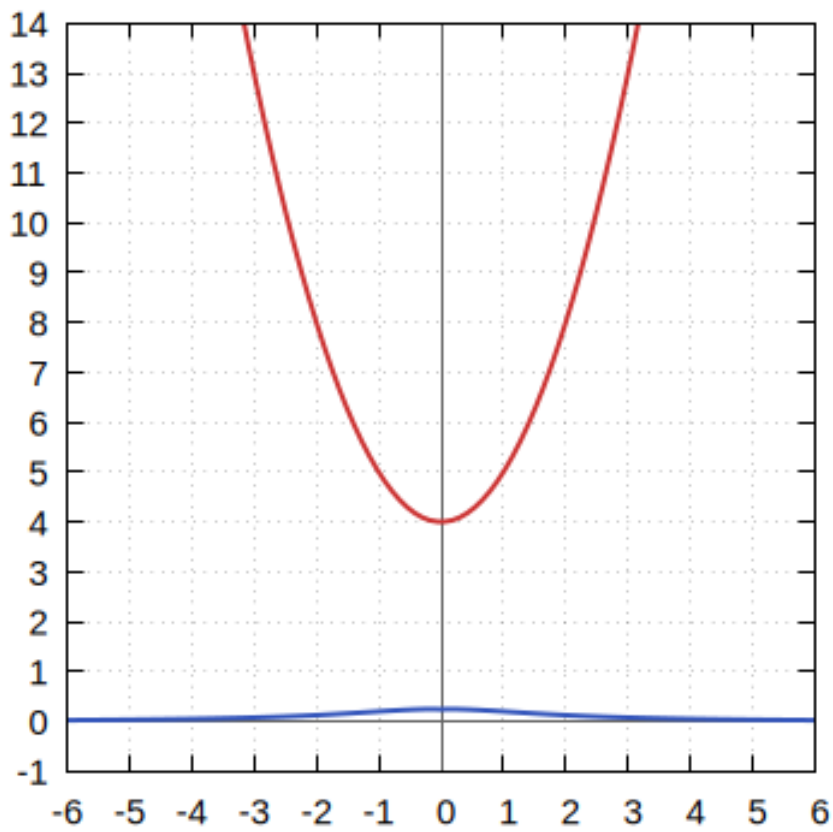
b) $y = \frac{1}{x^2 - 2x - 15} = \frac{1}{(x - 5)(x + 3)}$
 $y = x^2 - 2x - 15 = (x - 5)(x + 3)$, x-int : (5, 0) and (- 3, 0)
x-vertex = $-\frac{b}{2a} = -\frac{-2}{2(1)} = 1$
VA : $x = 5$ and $x = -3$, HA : $y = 0$

x	$y = x^2 - 2x - 15$	$y = \frac{1}{x^2 - 2x - 15}$
-5	20	0.05
-3	0	undefined
-1	-12	-0.08
1	-16	-0.0625
3	-12	-0.08
5	0	undefined
7	20	0.05



c) $y = \frac{1}{x^2 + 4}$
 $y = x^2 + 4$. Since $x^2 + 4 \neq 0$ for any real x , there is no x -int, so VA : none.
 x -vert : $x = -\frac{b}{2a} = -\frac{0}{2(1)} = 0$, HA : $y = 0$

x	$y = x^2 + 4$	$y = \frac{1}{x^2 + 4}$
-3	13	0.08
-2	8	0.125
-1	5	0.2
0	4	0.25
1	5	0.2
2	8	0.125
3	13	0.08



d) $y = \frac{2}{x^2 - 6x + 9} = 2 \left[\frac{1}{(x - 3)^2} \right]$

$y = x^2 - 6x + 9 = (x - 3)^2$, x-int : (3, 0), x-vertex = 3

VA : x = 3, HA : y = 0

x	$y = x^2 - 6x + 9$	$y = \frac{2}{x^2 - 6x + 9}$
0	9	0.22
1	4	0.5
2	1	2
3	0	undefined
4	1	2
5	4	0.5
6	9	0.22

