

6.4 Limits Worksheet with solutions

This document was composed and formatted by Annie Bernatchez.

The course material originated from www.jensenmath.ca

1) Evaluate each limit

a) $\lim_{x \rightarrow 2} \frac{3x}{x^2 + 2}$ $= \frac{3(2)}{(2)^2 + 2}$ $= \frac{6}{6}$ $= 1$	b) $\lim_{x \rightarrow -1} (x^4 + x^3 + x^2)$ $= (-1)^4 + (-1)^3 + (-1)^2$ $= 1 - 1 + 1$ $= 1$	c) $\lim_{x \rightarrow 9} \left(\sqrt{x} + \frac{1}{\sqrt{x}} \right)^2$ $= \left(\sqrt{9} + \frac{1}{\sqrt{9}} \right)^2$ $= \left(3 + \frac{1}{3} \right)^2 = \left(\frac{10}{3} \right)^2$ $= \frac{100}{9}$
--	--	---

2) Evaluate the limit of each

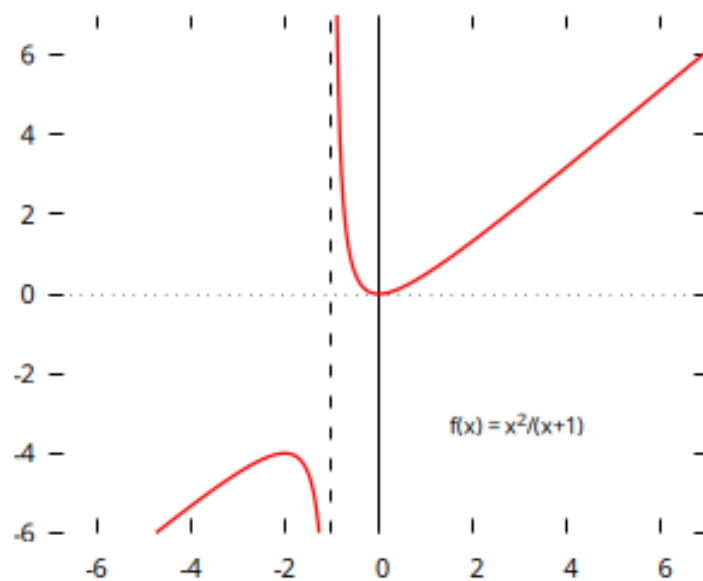
a) $\lim_{x \rightarrow 2} \frac{4 - x^2}{2 - x}$ $= \lim_{x \rightarrow 2} \frac{(2 - x)(2 + x)}{2 - x}$ $= 2 + 2$ $= 4$	b) $\lim_{x \rightarrow -1} \frac{2x^2 + 5x + 3}{x + 1}$ $= \lim_{x \rightarrow -1} \frac{(x + 1)(2x + 3)}{x + 1}$ $= 2(-1) + 3$ $= 1$	c) $\lim_{x \rightarrow 3} \frac{x^3 - 27}{x - 3}$ $= \lim_{x \rightarrow 3} \frac{(x - 3)(x^2 + 3x + 9)}{x - 3}$ $= (3)^2 + 3(3) + 9$ $= 27$
d) $\lim_{x \rightarrow 4} \frac{16 - x^2}{x^3 + 64}$ $= \lim_{x \rightarrow 4} \frac{(4 - x)(4 + x)}{(x + 4)(x^2 - 4x + 16)}$ $= \frac{4 - 4}{(4)^2 - 4(4) + 16}$ $= \frac{0}{16}$ $= 0$	e) $\lim_{x \rightarrow 4} \frac{x^2 - 16}{x^2 - 5x + 6}$ $= \lim_{x \rightarrow 4} \frac{(x - 4)(x + 4)}{(x - 2)(x - 3)}$ $= \frac{(4 - 4)(4 + 4)}{(4 - 2)(4 - 3)}$ $= \frac{0(8)}{2(1)}$ $= 0$	f) $\lim_{x \rightarrow -1} \frac{x^2 + x}{x + 1}$ $= \lim_{x \rightarrow -1} \frac{x(x + 1)}{x + 1}$ $= -1$

3) Complete the following table and use results to estimate $\lim_{x \rightarrow 2} \frac{x - 2}{x^2 - x - 2}$

x	1.9	1.99	1.999	2.001	2.01	2.1
$\frac{x - 2}{x^2 - x - 2}$	0.3448	0.3344	0.3334	0.3332	0.3322	0.3226

$$\lim_{x \rightarrow 2} \frac{x - 2}{x^2 - x - 2} \approx \frac{1}{3}$$

4) Use the graph to find the following limits:



a) $\lim_{x \rightarrow -1^+} \frac{x^2}{x+1}$

$= \infty$

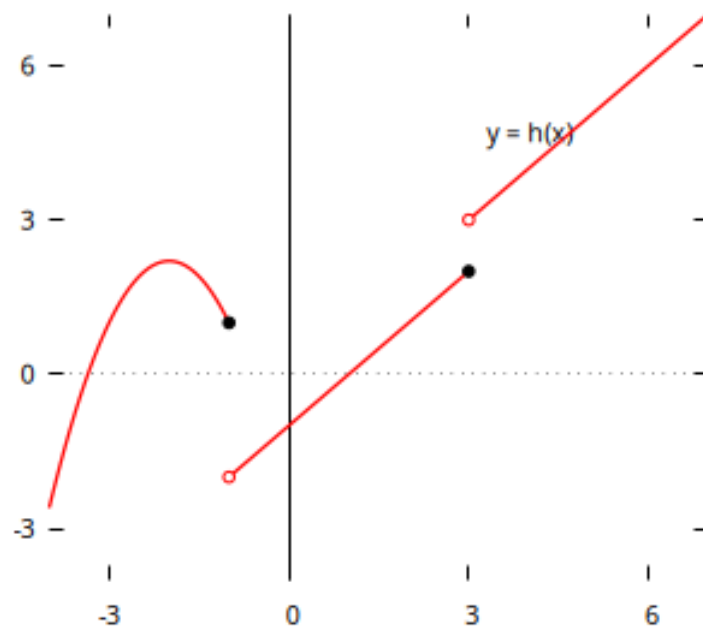
b) $\lim_{x \rightarrow -1^-} \frac{x^2}{x+1}$

$= -\infty$

c) $\lim_{x \rightarrow -1} \frac{x^2}{x+1}$

does not exist

5) Use the graph to determine the following limits



a) $\lim_{x \rightarrow 1^+} h(x)$

$= -2$

b) $\lim_{x \rightarrow 1^-} h(x)$

$= 1$

c) $\lim_{x \rightarrow -1} h(x)$

does not exist

d) $\lim_{x \rightarrow 3^+} h(x)$

$= 3$

e) $\lim_{x \rightarrow 3^-} h(x)$

$= 2$

f) $\lim_{x \rightarrow 3} h(x)$

does not exist

Answer Key

1) a) 1 b) 1 c) $\frac{100}{9}$

2) a) 4 b) 1 c) 27 d) 0 e) 0 f) -1

3) $\frac{1}{3}$

4) a) ∞ b) $-\infty$ c) does not exist

5) a) -2 b) 1 c) does not exist d) 3 e) 2 f) does not exist