

6.3 Newton Quotient Worksheet with solutions

This document was composed and formatted by Annie Bernatchez.

The course material originated from www.jensenmath.ca

1) Find the equation of the derivative for each of the following functions. Also, find the instantaneous rate of change for the function when $x = 4$ and $x = -1$.

a) $f(x) = 3x - 8$ $f'(x) = \lim_{h \rightarrow 0} \frac{3(x+h) - 8 - (3x - 8)}{h}$ $= \lim_{h \rightarrow 0} \frac{3x + 3h - 8 - 3x + 8}{h}$ $= \lim_{h \rightarrow 0} \frac{3h}{h}$ $f'(x) = 3$ $f'(4) = 3, f'(-1) = 3$	b) $y = 20x + x^2$ $f'(x) = \lim_{h \rightarrow 0} \frac{20(x+h) + (x+h)^2 - (20x + x^2)}{h}$ $= \lim_{h \rightarrow 0} \frac{20h + 2xh + h^2}{h}$ $f'(x) = 2x + 20$ $f'(4) = 28, f'(-1) = 18$
c) $y = 2x^3 + 4$ $f'(x) = \lim_{h \rightarrow 0} \frac{2(x+h)^3 + 4 - (2x^3 + 4)}{h}$ $= \lim_{h \rightarrow 0} \frac{6x^2h + 6xh^2 + 2h^3}{h}$ $f'(x) = 6x^2$ $f'(4) = 96, f'(-1) = 6$	d) $f(x) = x^2 - 9x + 17$ $f'(x) = \lim_{h \rightarrow 0} \frac{(x+h)^2 - 9(x+h) + 17 - (x^2 - 9x + 17)}{h}$ $= \lim_{h \rightarrow 0} \frac{2xh + h^2 - 9h}{h}$ $f'(x) = 2x - 9$ $f'(4) = -1, f'(-1) = -11$
e) $f(x) = \frac{x(x+1)}{2}$ $f'(x) = \lim_{h \rightarrow 0} \frac{\frac{(x+h)(x+h+1)}{2} - \frac{x(x+1)}{2}}{h}$ $= \lim_{h \rightarrow 0} \frac{2xh + h^2 + h}{2h}$ $f'(x) = x + \frac{1}{2}$ $f'(4) = \frac{9}{2}, f'(-1) = -\frac{1}{2}$	f) $f(x) = \frac{1}{x}$ $f'(x) = \lim_{h \rightarrow 0} \frac{\frac{1}{x+h} - \frac{1}{x}}{h}$ $= \lim_{h \rightarrow 0} \frac{-1}{x(x+h)}$ $f'(x) = -\frac{1}{x^2}$ $f'(4) = -\frac{1}{16}, f'(-1) = -1$

2) State whether the functions are increasing, decreasing, or neither when $x = 4$ for each function in #1. How do you know?

a, b, c and e are increasing functions when $x = 4$ since the instantaneous rate of change is positive

d and f are decreasing when $x = 4$

3)a) State the derivative of $f(x) = x^3$

$$f'(x) = \lim_{h \rightarrow 0} \frac{(x+h)^3 - x^3}{h}$$

$$= \lim_{h \rightarrow 0} \frac{3x^2h + 3xh^2 + h^3}{h}$$

$$f'(x) = 3x^2$$

b) Evaluate $f'(-6)$

$$f'(-6) = 3(-6)^2 = 108$$

c) Determine the equation of the tangent line at $x = 6$

$$f(6) = (6)^3 = 216, \text{ so point } (6, 216) \text{ is on the tangent line}$$

$$f'(6) = 3(6)^2 = 108, \text{ so slope of the tangent line is } 108$$

$$y = mx + b$$

$$216 = 108(6) + b$$

$$b = -432$$

$$y = 108x - 432$$

Answer Key

1) a) $f'(x) = 3$, $f'(4) = 3$, $f'(-1) = 3$ **b)** $f'(x) = 20 + 2x$, $f'(4) = 28$, $f'(-1) = 18$

3. $f'(x) = 6x^2$, $f'(4) = 96$, $f'(-1) = 6$

4. $f'(x) = 2x - 9$, $f'(4) = -1$, $f'(-1) = -11$

e) $f'(x) = x + \frac{1}{2}$, $f'(4) = \frac{9}{2}$, $f'(-1) = -\frac{1}{2}$ **f)** $f'(x) = -\frac{1}{x^2}$, $f'(4) = -\frac{1}{16}$, $f'(-1) = -1$

2) a, b, c and e are increasing functions when $x = 4$ since the instantaneous rate of change is positive

d and f are decreasing when $x = 4$

3) a) $f'(x) = 3x^2$ **b)** 108 **c)** $y = 108x - 432$