

6.1 Average Rates of Change Worksheet with solutions

This document was composed and formatted by Annie Bernatchez.

The course material originated from www.jensenmath.ca

1) Calculate the average rate of change for the function $g(x) = 4x^2 - 5x + 1$ over each interval.

a) $2 \leq x \leq 4$ $m = \frac{g(4) - g(2)}{4 - 2}$ $= \frac{45 - 7}{2}$ $= 19$	b) $2 \leq x \leq 3$ $m = \frac{g(3) - g(2)}{3 - 2}$ $= \frac{22 - 7}{1}$ $= 15$	c) $2 \leq x \leq 2.5$ $m = \frac{g(2.5) - g(2)}{2.5 - 2}$ $= \frac{13.5 - 7}{0.5}$ $= 13$
d) $2 \leq x \leq 2.25$ $m = \frac{g(2.25) - g(2)}{2.25 - 2}$ $= \frac{10 - 7}{0.25}$ $= 12$	e) $2 \leq x \leq 2.1$ $m = \frac{g(2.1) - g(2)}{2.1 - 2}$ $= \frac{8.14 - 7}{0.1}$ $= 11.4$	f) $2 \leq x \leq 2.01$ $m = \frac{g(2.01) - g(2)}{2.01 - 2}$ $= \frac{7.1104 - 7}{0.01}$ $= 11.04$

2) An emergency flare is shot into the air. Its height, in meters, above the ground at various times in its flight is given in the following table:

Time (s)	0.0	0.5	1.0	1.5	2.0	2.5	3.0	3.5	4.0
Height (m)	2.00	15.75	27.00	35.75	42.00	45.75	47.00	45.75	42.00

Determine the average rate of change in the height of the flare during each interval

a) $1.0 \leq t \leq 2.0$ $m = \frac{h(2) - h(1)}{2 - 1}$ $m = \frac{42 - 27}{1}$ $m = 15 \text{ m/s}$	b) $3.0 \leq t \leq 4.0$ $m = \frac{h(4) - h(3)}{4 - 3}$ $= \frac{42 - 47}{1}$ $= -5 \text{ m/s}$
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3) What is the average rate of change in the values of the function $f(x) = 4x$ from $x = 2$ to $x = 6$? What about from $x = 2$ to $x = 26$? What do your results indicate about $f(x)$?

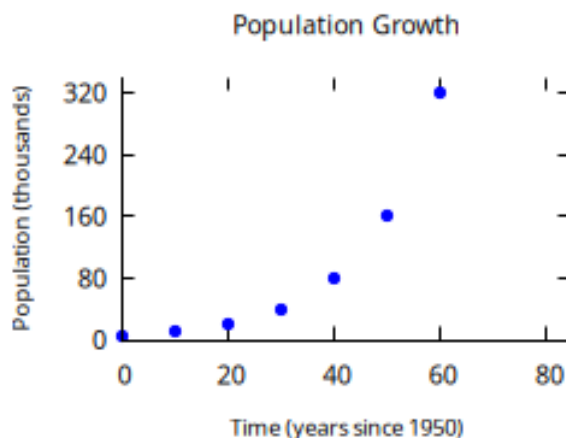
$$2 \leq x \leq 6: m = \frac{f(6) - f(2)}{6 - 2} = \frac{24 - 8}{4} = 4$$

$$2 \leq x \leq 26: m = \frac{f(26) - f(2)}{26 - 2} = \frac{104 - 8}{24} = 4$$

The average rate of change will always be 4 because it is a linear function with a slope of 4.

4) The population of a city has continued to grow since 1950. The population, P , in thousands, and the time t , in years, since 1950 are given in the table below and in the graph.

Time (years)	0	10	20	30	40	50	60
Population (thousands)	5	10	20	40	80	160	320



a) Calculate the average rate of change in the population for the following intervals of time.

i) $0 \leq t \leq 20$ $m = \frac{P(20) - P(0)}{20 - 0}$ $= \frac{20\,000 - 5000}{20}$ $= 750 \text{ ppl/year}$	ii) $20 \leq t \leq 40$ $m = \frac{P(40) - P(20)}{40 - 20}$ $= \frac{80\,000 - 20\,000}{20}$ $= 3000 \text{ ppl/year}$
iii) $40 \leq t \leq 60$ $m = \frac{P(60) - P(40)}{60 - 40}$ $= \frac{320\,000 - 80\,000}{20}$ $= 12\,000 \text{ ppl/year}$	iv) $0 \leq t \leq 60$ $m = \frac{P(60) - P(0)}{60 - 0}$ $= \frac{320\,000 - 5000}{60}$ $= 5250 \text{ ppl/year}$

b) Is the population growth constant?

No, the rate of growth increases as time increases.

5) A company that sells sweatshirts finds that the profit can be modelled by $P(s) = -0.30s^2 + 3.5s + 11.15$, where $P(s)$ is the profit, in thousands of dollars, and s is the number of sweatshirts sold (expressed in thousands).

a) Calculate the average rate of change in the profit for the following intervals.

i) $1 \leq s \leq 2$ $m = \frac{P(2) - P(1)}{2 - 1}$ $= \frac{16.95 - 14.35}{1}$ $= \$2.60/\text{sweatshirt}$	ii) $2 \leq s \leq 3$ $m = \frac{P(3) - P(2)}{3 - 2}$ $= \frac{18.95 - 16.95}{1}$ $= \$2.00/\text{sweatshirt}$
iii) $3 \leq s \leq 4$ $m = \frac{P(4) - P(3)}{4 - 3}$ $= \frac{20.35 - 18.95}{1}$ $= \$1.40/\text{sweatshirt}$	iv) $4 \leq s \leq 5$ $m = \frac{P(5) - P(4)}{5 - 4}$ $= \frac{21.15 - 20.35}{1}$ $= \$0.80/\text{sweatshirt}$

b) As the number of sweatshirts sold increases, what do you notice about the average rate of change in profit on each sweatshirt? What does this mean?

The rate of change is positive but decreasing. Profits are going up but at a decreasing rate.

c) Predict if the rate of change in profit will stay positive. Explain what this means.

$$x_{\text{vertex}} = \frac{-b}{2a} = \frac{-3.5}{2(-0.3)} \approx 5.83$$

No; at around 6000 sweatshirts sold, profits will start to decrease.

Answer Key

1) a) 19 b) 15 c) 13 d) 12 e) 11.4 f) 11.04

2) a) 15 m/s b) -5 m/s

3) 4; 4; the average rate of change is always 4 because the function is linear, with a slope of 4.

4) a) i) 750 ppl/year ii) 3000 ppl/year iii) 12 000 ppl/year iv) 5250 ppl/year b) no, the rate of growth increases as the time increases.

5) a) i) \$2.60/sweatshirt ii) \$2.00/sweatshirt iii) \$1.40/sweatshirt iv) \$0.80/sweatshirt b) The rate of change is still positive, but it is decreasing. This means that the profit is still increasing, but at a decreasing rate. c) No; after 6000 sweatshirts are sold, the rate of change becomes negative. This means that the profit begins to decrease after 6000 sweatshirts are sold.