

5.7 Solve Quadratic Trigonometric Equations Worksheet with solutions

This document was composed and formatted by Annie Bernatchez.

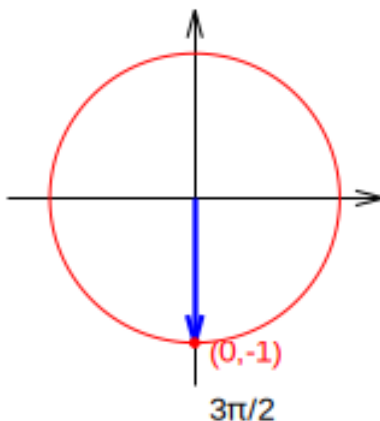
The course material originated from www.jensenmath.ca

1) Solve $\sin^2 x - 2\sin x - 3 = 0$ on the interval $0 \leq x \leq 2\pi$

$$(\sin x - 3)(\sin x + 1) = 0$$

$$\sin x - 3 = 0 \rightarrow \sin x = 3 \rightarrow \text{No solution}$$

$$\sin x + 1 = 0 \rightarrow \sin x = -1$$

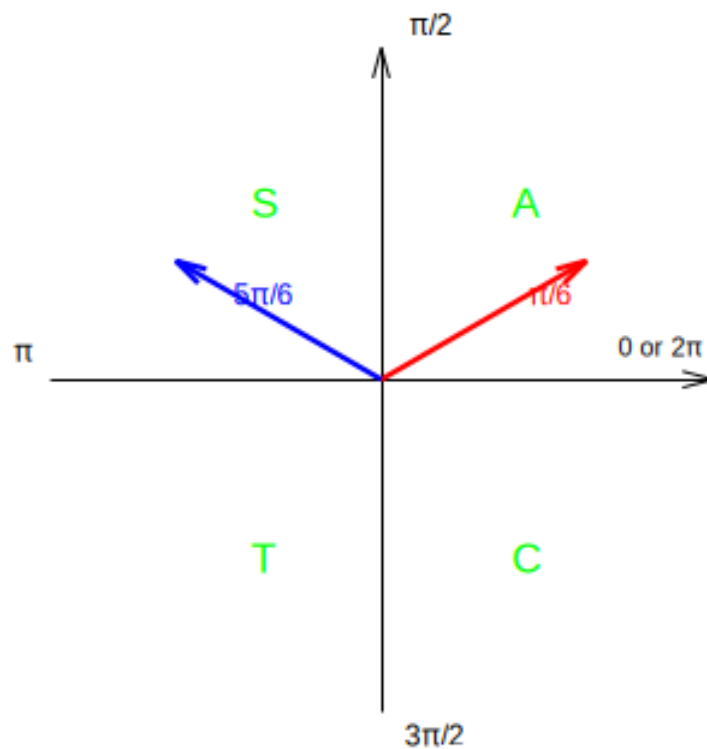


$$x = \frac{3\pi}{2}$$

2) Solve $\csc^2 x - \csc x - 2 = 0$ on the interval $0 \leq x \leq 2\pi$

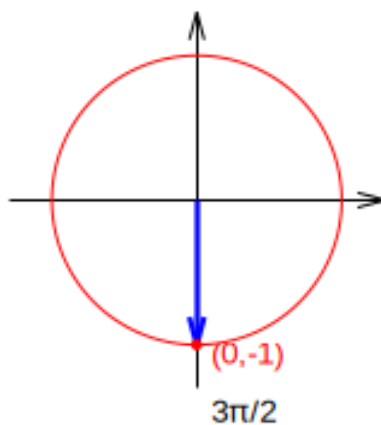
$$(\csc x - 2)(\csc x + 1) = 0$$

$$\csc x - 2 = 0 \rightarrow \csc x = 2 \rightarrow \sin x = \frac{1}{2}$$



$$x_1 = \frac{\pi}{6}, x_2 = \pi - \frac{\pi}{6} = \frac{5\pi}{6}$$

$$\csc x + 1 = 0 \rightarrow \csc x = -1 \rightarrow \sin x = -1$$



$$x_3 = \frac{3\pi}{2}$$

$$\text{Answer: } x = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{3\pi}{2}$$

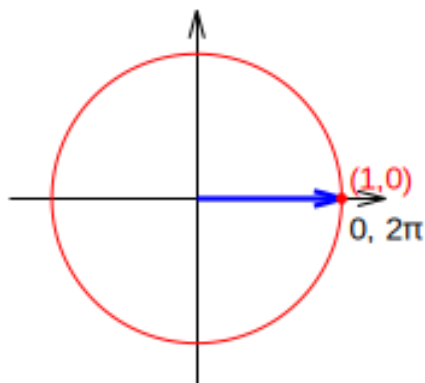
3) Solve $2\sec^2 x - \sec x - 1 = 0$ on the interval $0 \leq x \leq 2\pi$

$$2\sec^2 x - 2\sec x + \sec x - 1 = 0$$

$$2\sec x(\sec x - 1) + 1(\sec x - 1) = 0$$

$$(\sec x - 1)(2\sec x + 1) = 0$$

$$\sec x - 1 = 0 \rightarrow \sec x = 1 \rightarrow \cos x = 1$$



$$x_1 = 0, x_2 = 2\pi$$

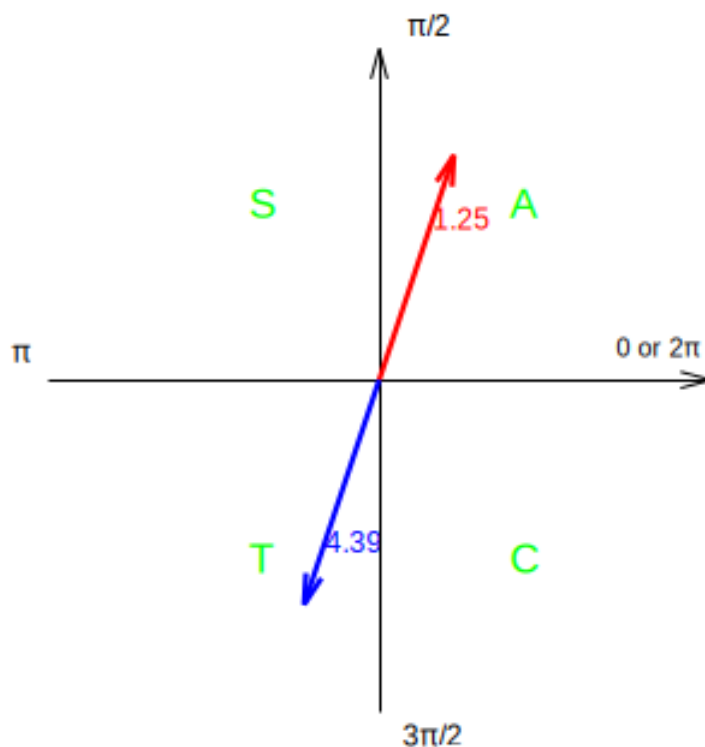
$$2\sec x + 1 = 0 \rightarrow \sec x = -\frac{1}{2} \rightarrow \cos x = -2 \rightarrow \text{No solution}$$

Answer: $x = 0, 2\pi$

4) Solve $\tan^2 x - \tan x - 6 = 0$ **on the interval** $0 \leq x \leq 2\pi$. **Round answers to the nearest hundredth of a radian.**

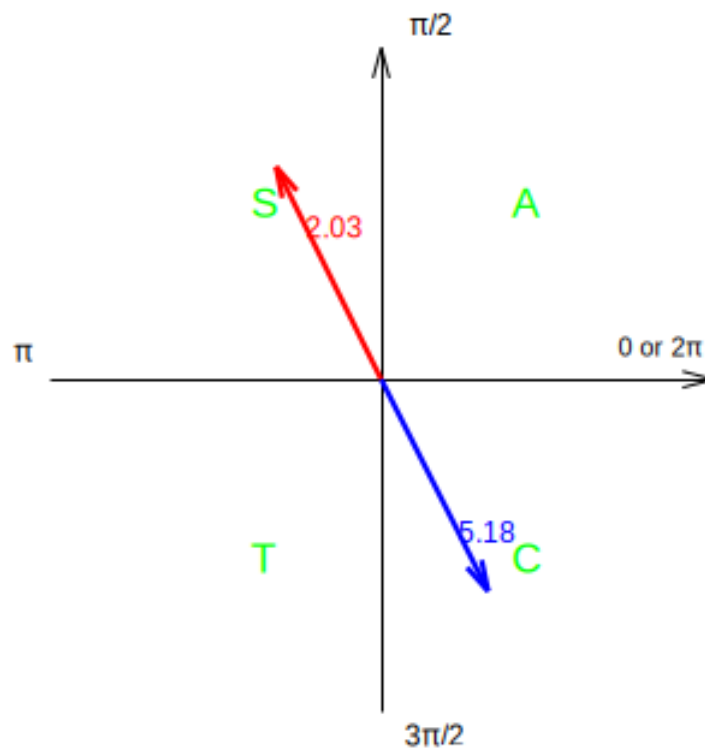
$$(\tan x - 3)(\tan x + 2) = 0$$

$$\tan x = 3 \rightarrow x_1 = \tan^{-1}(3) \approx 1.25$$



$$x_1 \approx 1.25, x_2 \approx \pi + 1.25 \approx 4.39$$

$$\tan x = -2 \rightarrow x_3 = \tan^{-1}(-2) \approx -1.107 + 2\pi \approx 5.18$$



$$x_3 \approx 5.18, x_4 \approx \pi - 1.107 \approx 2.03$$

Answer: $x \approx 1.25, 2.03, 4.39, 5.18$

5) Solve $6\cos^2 x + 5\cos x - 6 = 0$ **on the interval** $0 \leq x \leq 2\pi$

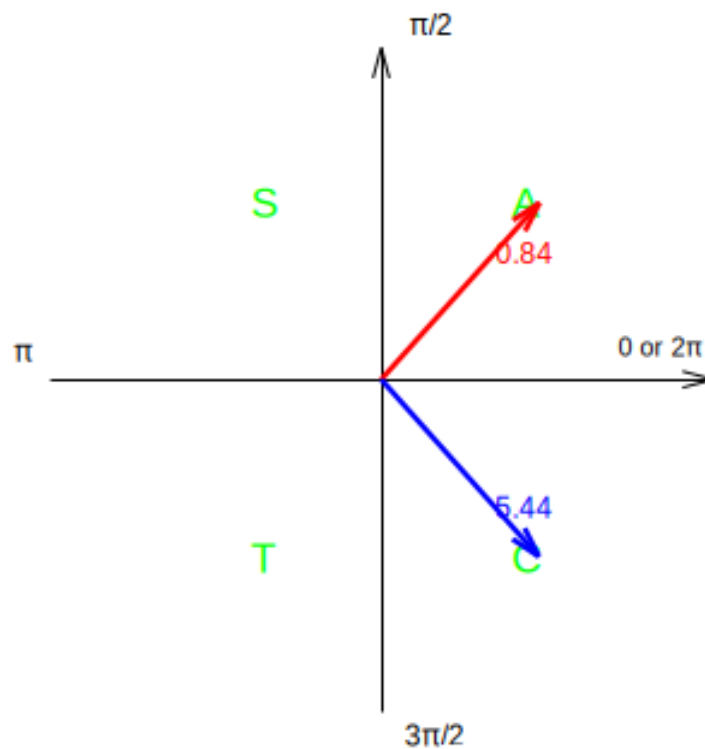
$$6\cos^2 x + 9\cos x - 4\cos x - 6 = 0$$

$$3\cos x(2\cos x + 3) - 2(2\cos x + 3) = 0$$

$$(2\cos x + 3)(3\cos x - 2) = 0$$

$$2\cos x + 3 = 0 \rightarrow \cos x = -\frac{3}{2} \rightarrow \text{No solution}$$

$$3\cos x - 2 = 0 \rightarrow \cos x = \frac{2}{3} \rightarrow x_1 = \cos^{-1}\left(\frac{2}{3}\right) \approx 0.84$$



$$x_1 \approx 0.84, x_2 \approx 2\pi - 0.84 \approx 5.44$$

Answer: $x \approx 0.84, 5.44$

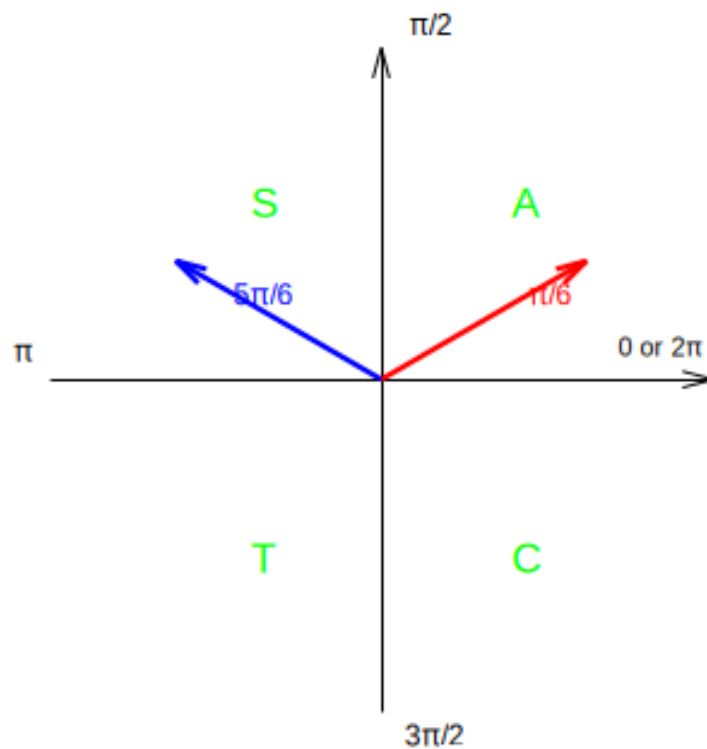
6) Solve $3\csc^2 x - 5\csc x - 2 = 0$ on the interval $0 \leq x \leq 2\pi$

$$3\csc^2 x - 6\csc x + 1\csc x - 2 = 0$$

$$3\csc x(\csc x - 2) + 1(\csc x - 2) = 0$$

$$(\csc x - 2)(3\csc x + 1) = 0$$

$$\csc x = 2 \rightarrow \sin x = \frac{1}{2}$$



$$x_1 = \frac{\pi}{6}, x_2 = \pi - \frac{\pi}{6} = \frac{5\pi}{6}$$

$$\csc x = -\frac{1}{3} \rightarrow \sin x = -3 \rightarrow \text{No solution}$$

$$\text{Answer: } x = \frac{\pi}{6}, \frac{5\pi}{6}$$

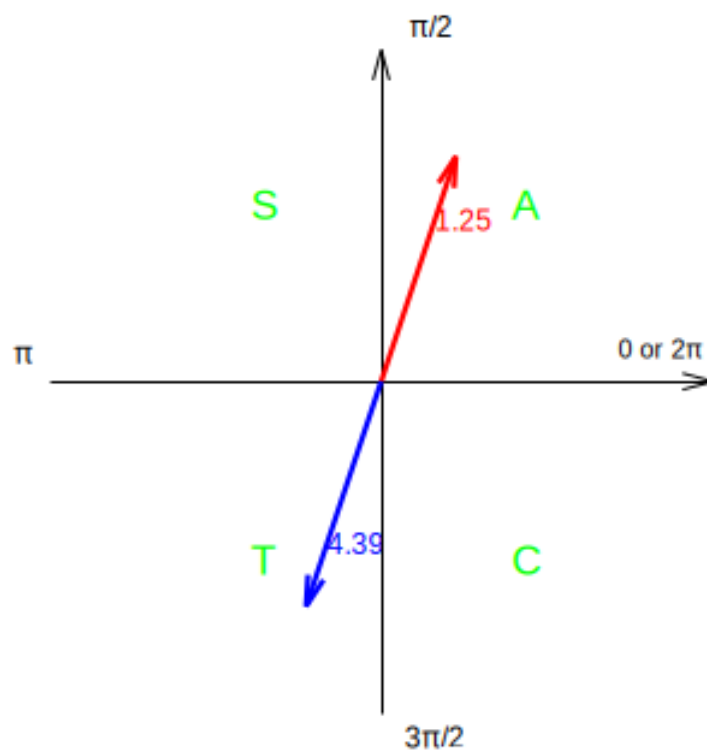
7) Solve $2\tan^2 x - 5\tan x - 3 = 0$ on the interval $0 \leq x \leq 2\pi$

$$2\tan^2 x - 6\tan x + 1\tan x - 3 = 0$$

$$2\tan x(\tan x - 3) + 1(\tan x - 3) = 0$$

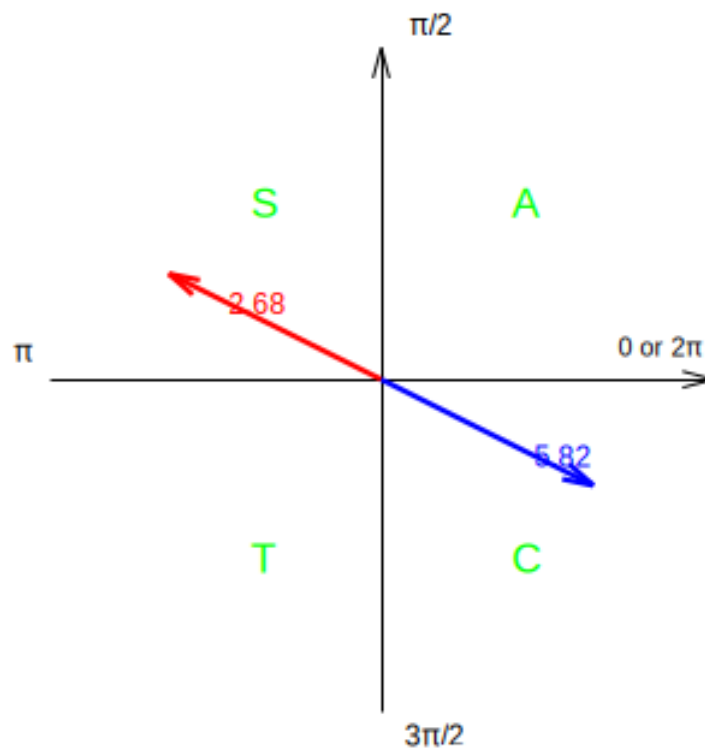
$$(\tan x - 3)(2\tan x + 1) = 0$$

$$\tan x = 3 \rightarrow x_1 \approx \tan^{-1}(3) \approx 1.25$$



$$x_1 \approx 1.25, x_2 \approx \pi + 1.25 \approx 4.39$$

$$\tan x = -\frac{1}{2} \rightarrow \beta = \tan^{-1}\left(\frac{1}{2}\right) \approx 0.46$$



$$x_3 = \pi - 0.46 \approx 2.68, x_4 = 2\pi - 0.46 \approx 5.82$$

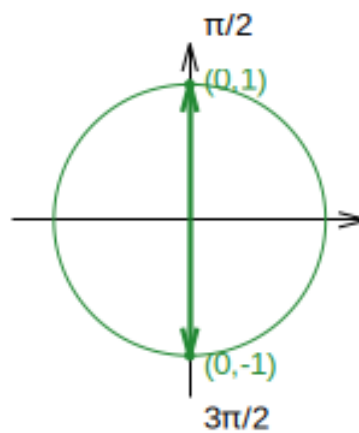
Answers: $x \approx 1.25, 2.68, 4.39, 5.82$

8) Solve $\cot x \csc^2 x = 2 \cot x$ on the interval $0 \leq x \leq 2\pi$

$$0 = 2 \cot x - \cot x \csc^2 x$$

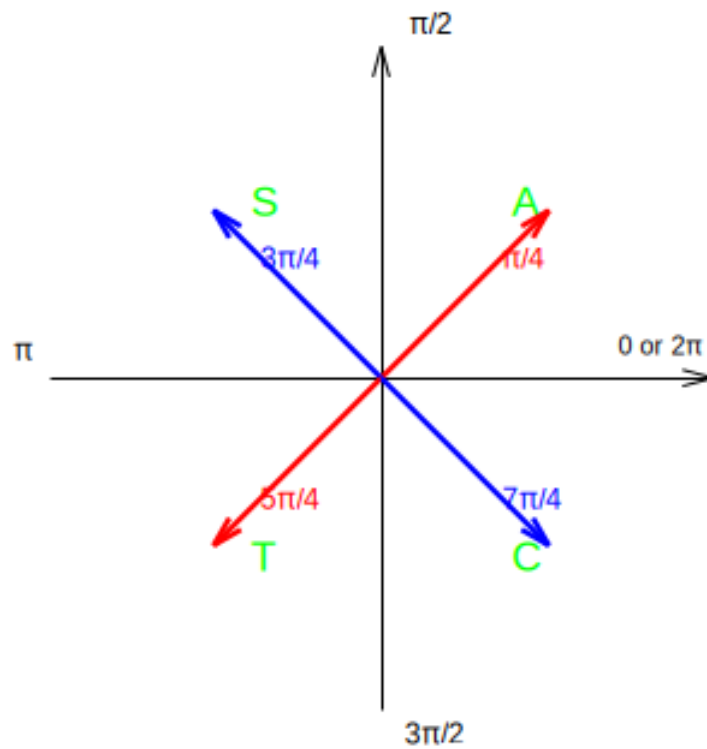
$$0 = \cot x (2 - \csc^2 x)$$

$$\cot x = 0 \rightarrow \tan x \text{ undefined}$$



$$x_1 = \frac{\pi}{2}, x_2 = \frac{3\pi}{2}$$

$$2 - \csc^2 x = 0 \rightarrow \csc^2 x = 2 \rightarrow \sin^2 x = \frac{1}{2} \rightarrow \sin x = \pm \frac{1}{\sqrt{2}}$$



$$x_3 = \frac{\pi}{4}, x_4 = \frac{3\pi}{4}, x_5 = \frac{5\pi}{4}, x_6 = \frac{7\pi}{4}$$

$$\text{Answer: } x = \frac{\pi}{4}, \frac{\pi}{2}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{3\pi}{2}, \frac{7\pi}{4}$$

9) Solve for θ to the nearest hundredth, where $0 \leq \theta \leq 2\pi$

a) $3 \tan^2 \theta - 2 \tan \theta = 1$

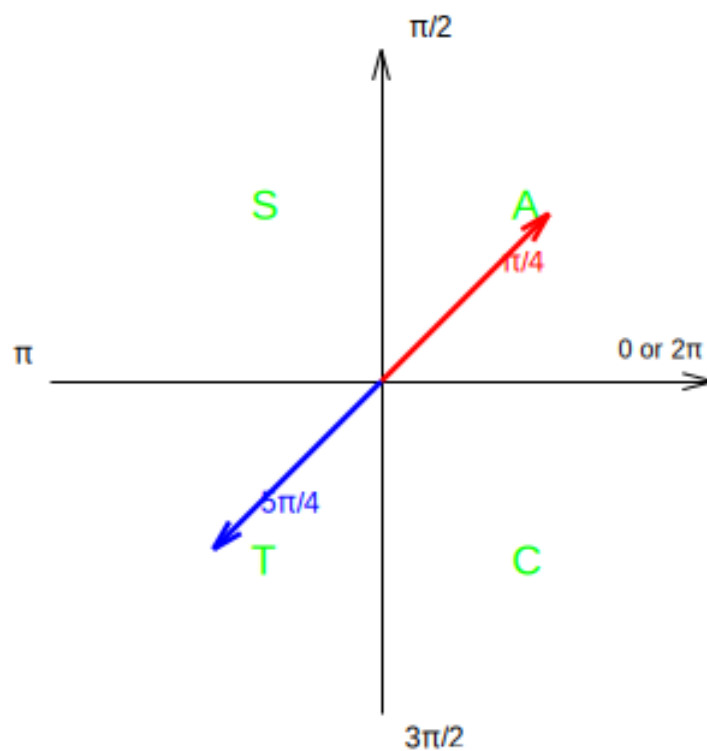
$$3 \tan^2 \theta - 2 \tan \theta - 1 = 0$$

$$3 \tan^2 \theta - 3 \tan \theta + 1 \tan \theta - 1 = 0$$

$$3 \tan \theta (\tan \theta - 1) + 1 (\tan \theta - 1) = 0$$

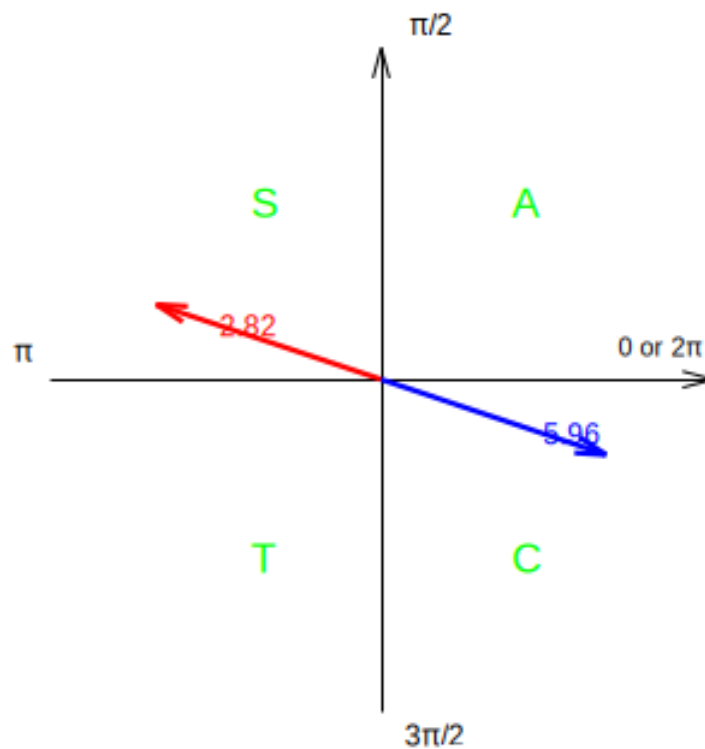
$$(\tan \theta - 1)(3 \tan \theta + 1) = 0$$

$$\tan \theta = 1$$



$$\theta_1 = \frac{\pi}{4}, \theta_2 = \pi + \frac{\pi}{4} = \frac{5\pi}{4}$$

$$\tan \theta = -\frac{1}{3} \rightarrow \theta_3 = \tan^{-1}\left(-\frac{1}{3}\right) \approx -0.32 + 2\pi \approx 5.96$$



$$\theta_3 \approx 5.96, \theta_4 = \pi - 0.32 \approx 2.82$$

Answer: $\theta = \frac{\pi}{4}, \frac{5\pi}{4}, 2.82, 5.96$

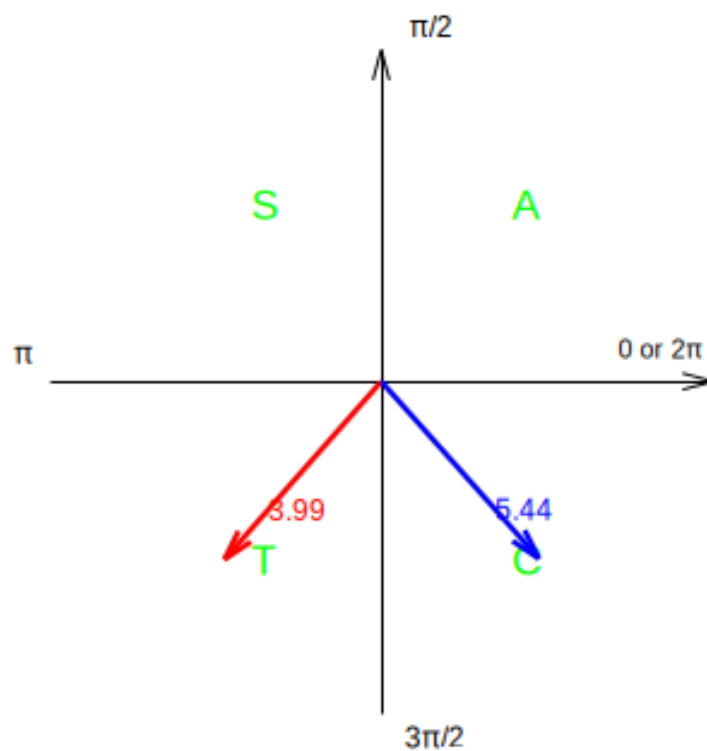
b) $12\sin^2\theta + \sin\theta - 6 = 0$

$$12\sin^2\theta + 9\sin\theta - 8\sin\theta - 6 = 0$$

$$3\sin\theta(4\sin\theta + 3) - 2(4\sin\theta + 3) = 0$$

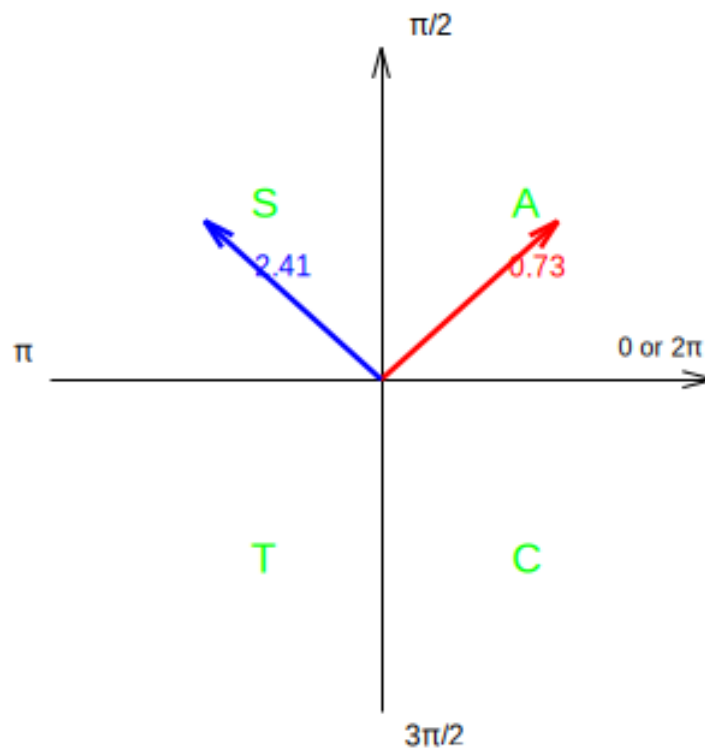
$$(4\sin\theta + 3)(3\sin\theta - 2) = 0$$

$$\sin\theta = -\frac{3}{4} \rightarrow \theta_1 = \sin^{-1}\left(-\frac{3}{4}\right) \approx -0.85 + 2\pi \approx 5.44$$



$$\theta_1 \approx 5.44, \theta_2 = \pi + 0.85 \approx 3.99$$

$$\sin \theta = \frac{2}{3} \rightarrow \theta_3 = \sin^{-1} \left(\frac{2}{3} \right) \approx 0.73$$



$$\theta_3 \approx 0.73, \theta_4 = \pi - 0.73 \approx 2.41$$

Answer: $\theta \approx 0.73, 2.41, 3.99, 5.44$

c) $5 \cos(2\theta) - \cos \theta + 3 = 0$

$$5(2 \cos^2 \theta - 1) - \cos \theta + 3 = 0$$

$$10 \cos^2 \theta - 5 - \cos \theta + 3 = 0$$

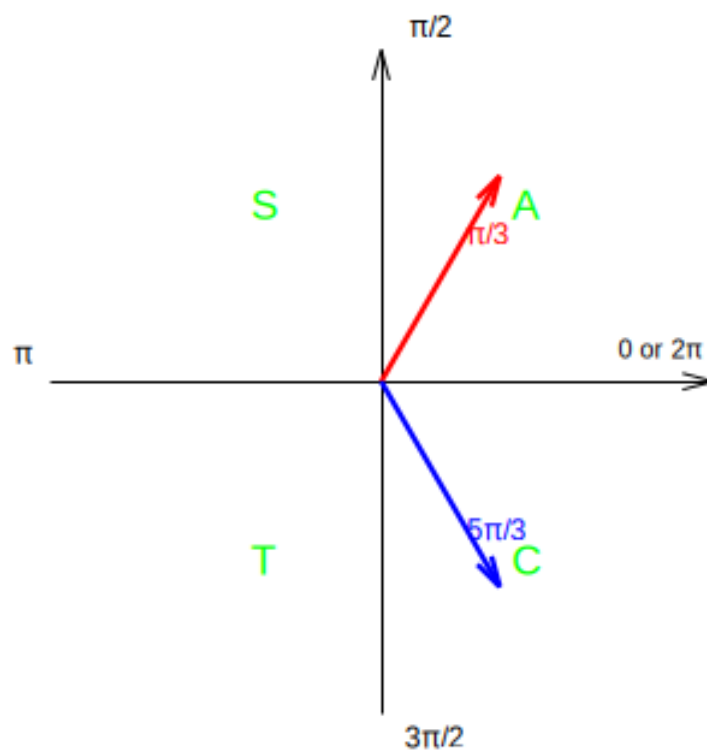
$$10 \cos^2 \theta - \cos \theta - 2 = 0$$

$$10 \cos^2 \theta - 5 \cos \theta + 4 \cos \theta - 2 = 0$$

$$5 \cos \theta (2 \cos \theta - 1) + 2(2 \cos \theta - 1) = 0$$

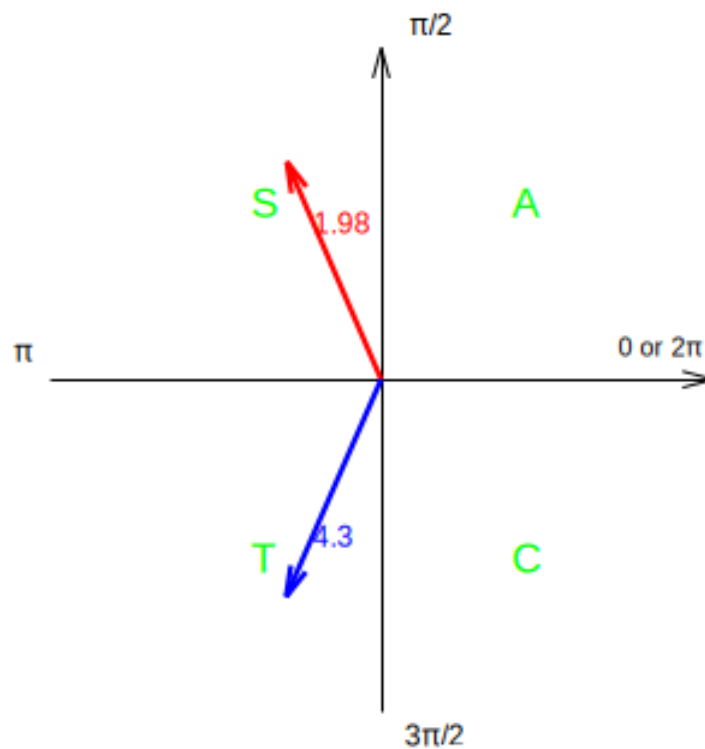
$$(2 \cos \theta - 1)(5 \cos \theta + 2) = 0$$

$$\cos \theta = \frac{1}{2}$$



$$\theta_1 = \frac{\pi}{3}, \theta_2 = 2\pi - \frac{\pi}{3} = \frac{5\pi}{3}$$

$$\cos \theta = -\frac{2}{5} \rightarrow \theta_3 = \cos^{-1}\left(-\frac{2}{5}\right) \approx 1.98$$



$$\theta_3 \approx 1.98, \theta_4 \approx \pi + 1.16 \approx 4.3$$

$$\text{Answer: } \theta = \frac{\pi}{3}, 1.98, 4.3, \frac{5\pi}{3}$$

Answer Key

1) $\frac{3\pi}{2}$ 2) $\frac{\pi}{6}, \frac{5\pi}{6}, \frac{3\pi}{2}$ 3) $0, 2\pi$ 4) $1.25, 2.03, 4.39, 5.18$ 5) $0.84, 5.44$ 6) $\frac{\pi}{6}, \frac{5\pi}{6}$ 7) $1.25, 2.68, 4.39, 5.82$

8) $\frac{\pi}{4}, \frac{\pi}{2}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{3\pi}{2}, \frac{7\pi}{4}$

9) a) $\frac{\pi}{4}, 2.82, \frac{5\pi}{4}, 5.96$ b) $0.73, 2.41, 3.99, 5.44$ c) $\frac{\pi}{3}, 1.98, 4.3, \frac{5\pi}{3}$