

## 5.6 Résoudre des équations trigonométriques à angle double – Feuille de travail avec solutions

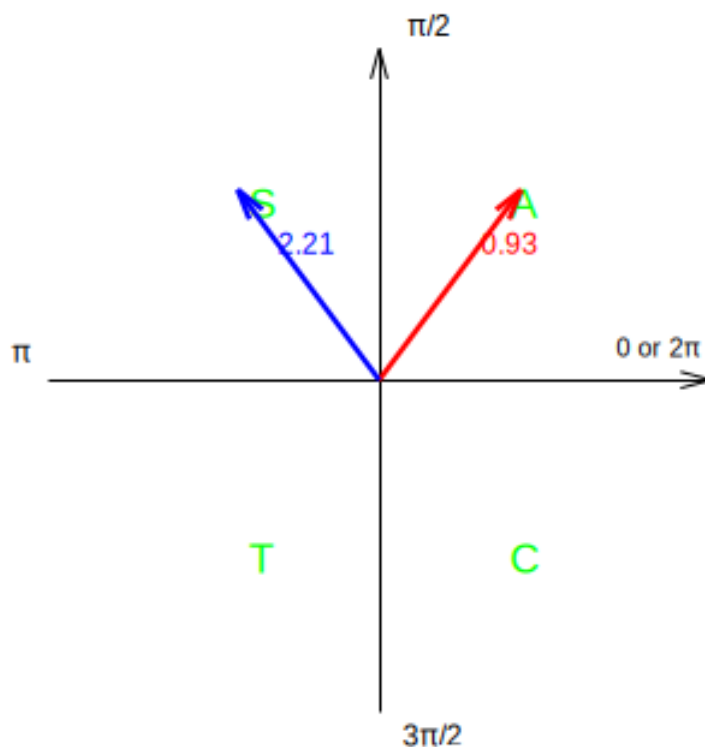
Ce document a été composé et mis en forme par Annie Bernatchez.

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Déterminer les solutions de chaque équation dans l'intervalle  $0 \leq x \leq 2\pi$ , au centième de radian près. Donner des réponses exactes lorsque c'est possible.

a)  $\sin(2x) - 0.8 = 0$

Soit  $\theta = 2x$ .  $\sin \theta = 0.8$



$$\theta_1 = \sin^{-1}(0.8) \approx 0.93$$

$$\theta_2 = \pi - 0.93 \approx 2.21$$

$$2x = \theta \rightarrow 2x = 0.93 \rightarrow x_1 \approx 0.46, \text{ et } 2x = 2.21 \rightarrow x_2 \approx 1.11$$

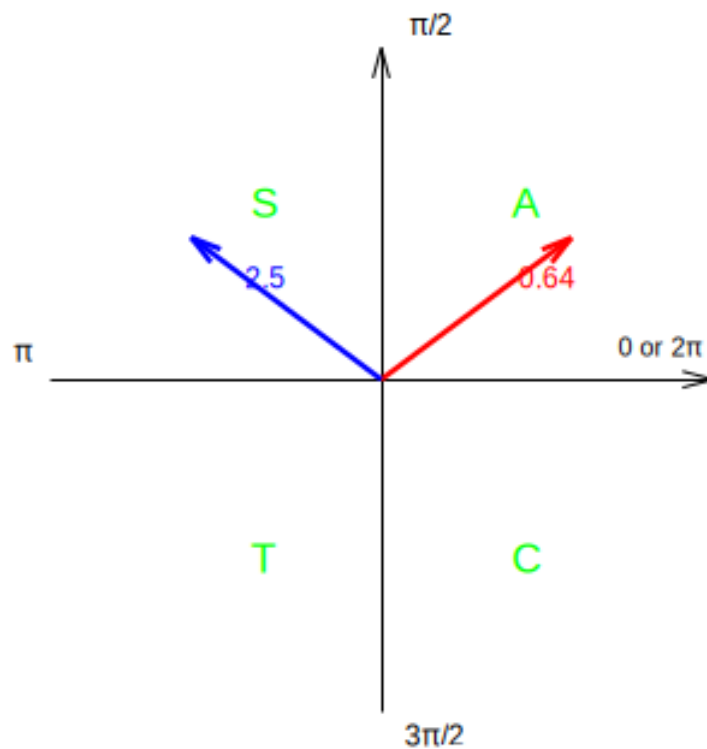
Ajouter la période  $\pi$  pour trouver d'autres solutions :

$$x_3 = x_1 + \pi \approx 3.61$$

$$x_4 = x_2 + \pi \approx 4.25$$

b)  $5 \sin(2x) - 3 = 0$

Soit  $\theta = 2x$ .  $\sin \theta = \frac{3}{5}$



$$\theta_1 = \sin^{-1}\left(\frac{3}{5}\right) \approx 0.64$$

$$\theta_2 = \pi - 0.64 \approx 2.5$$

$$2x = \theta \rightarrow 2x = 0.64 \rightarrow x_1 \approx 0.32, \text{ et } 2x = 2.5 \rightarrow x_2 \approx 1.25$$

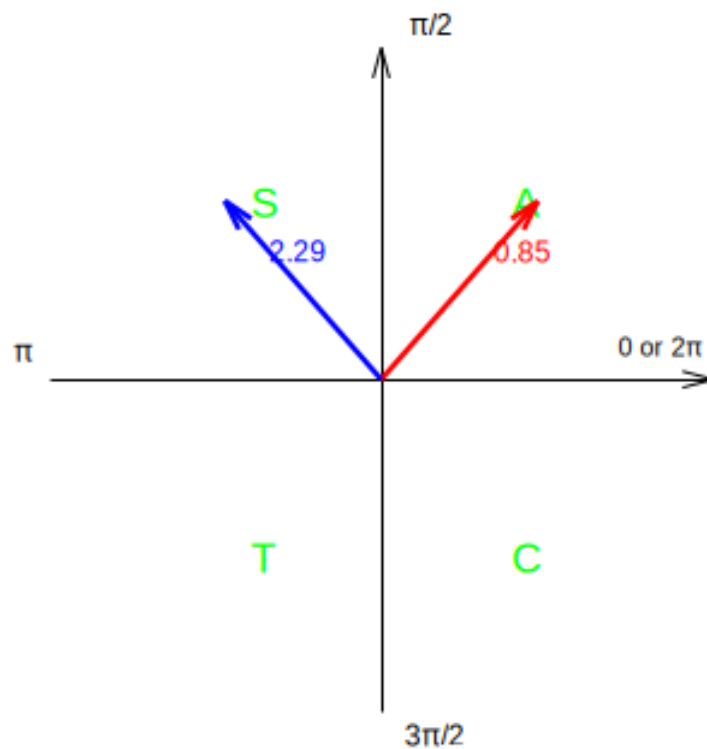
Ajouter la période  $\pi$  pour trouver d'autres solutions :

$$x_3 = x_1 + \pi \approx 3.46$$

$$x_4 = x_2 + \pi \approx 4.39$$

$$\text{c) } -4\sin(2x) + 3 = 0$$

$$\text{Soit } \theta = 2x. \quad \sin \theta = \frac{3}{4}$$



$$\theta_1 = \sin^{-1}\left(\frac{3}{4}\right) \approx 0.85$$

$$\theta_2 = \pi - 0.85 \approx 2.29$$

$$2x = \theta \rightarrow 2x = 0.85 \rightarrow x_1 \approx 0.42, \text{ et } 2x = 2.29 \rightarrow x_2 \approx 1.15$$

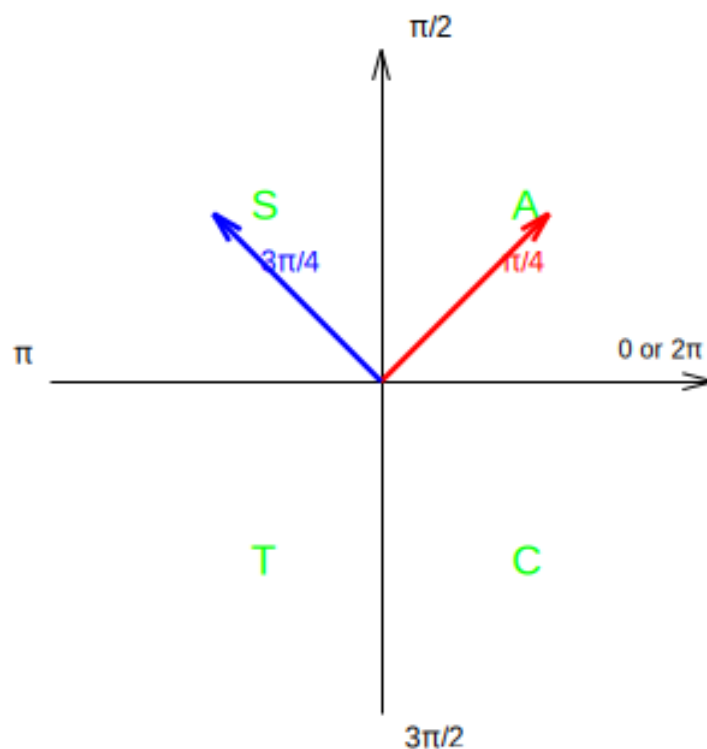
Ajouter la période  $\pi$  pour trouver d'autres solutions :

$$x_3 = x_1 + \pi \approx 3.57$$

$$x_4 = x_2 + \pi \approx 4.29$$

$$\text{d) } \sin(2x) = \frac{1}{\sqrt{2}}$$

Soit  $\theta = 2x$ .  $\sin \theta = \frac{1}{\sqrt{2}}$ . Du triangle spécial :  $\sin \frac{\pi}{4} = \frac{1}{\sqrt{2}}$ . Placer aux quadrants 1 et 2.



$$\theta_1 = \frac{\pi}{4}, \theta_2 = \pi - \frac{\pi}{4} = \frac{3\pi}{4}$$

$$2x = \theta \rightarrow 2x = \frac{\pi}{4} \rightarrow x_1 = \frac{\pi}{8}, \text{ et } 2x = \frac{3\pi}{4} \rightarrow x_2 = \frac{3\pi}{8}$$

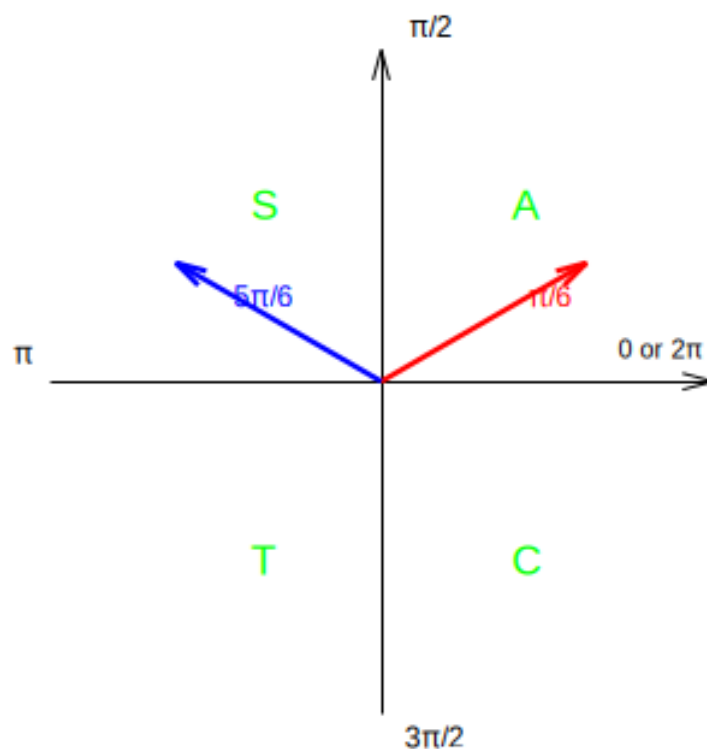
Ajouter la période  $\pi$  pour trouver d'autres solutions :

$$x_3 = x_1 + \pi = \frac{\pi}{8} + \pi = \frac{9\pi}{8}$$

$$x_4 = x_2 + \pi = \frac{3\pi}{8} + \pi = \frac{11\pi}{8}$$

$$\text{e) } \sin(4x) = \frac{1}{2}$$

Soit  $\theta = 4x$ .  $\sin \theta = \frac{1}{2}$ . Du triangle spécial :  $\sin \frac{\pi}{6} = \frac{1}{2}$ . Placer aux quadrants 1 et 2.



$$\theta_1 = \frac{\pi}{6}, \theta_2 = \pi - \frac{\pi}{6} = \frac{5\pi}{6}$$

$$4x = \theta \rightarrow 4x = \frac{\pi}{6} \rightarrow x_1 = \frac{\pi}{24}, \text{ et } 4x = \frac{5\pi}{6} \rightarrow x_2 = \frac{5\pi}{24}$$

Ajouter la période  $\frac{\pi}{2} = \frac{12\pi}{24}$  pour trouver d'autres solutions :

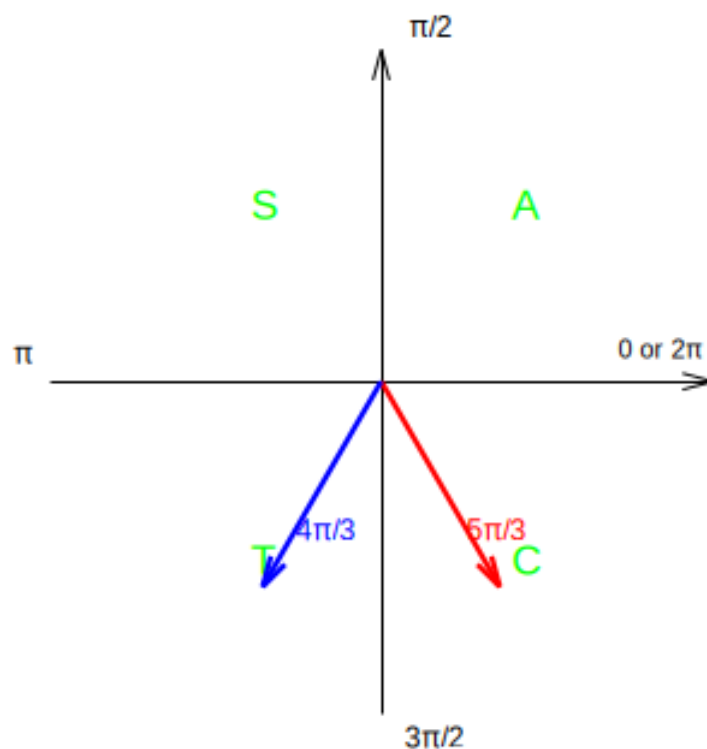
$$x_3 = x_1 + \frac{\pi}{2} = \frac{13\pi}{24}, x_4 = x_2 + \frac{\pi}{2} = \frac{17\pi}{24}$$

$$x_5 = x_3 + \frac{\pi}{2} = \frac{25\pi}{24}, x_6 = x_4 + \frac{\pi}{2} = \frac{29\pi}{24}$$

$$x_7 = x_5 + \frac{\pi}{2} = \frac{37\pi}{24}, x_8 = x_6 + \frac{\pi}{2} = \frac{41\pi}{24}$$

$$\text{f) } \sin(3x) = -\frac{\sqrt{3}}{2}$$

Soit  $3x = \theta$ .  $\sin \theta = -\frac{\sqrt{3}}{2}$ . Du triangle spécial :  $\sin \frac{\pi}{3} = \frac{\sqrt{3}}{2}$ . Placer aux quadrants 3 et 4.



$$\theta_1 = 2\pi - \frac{\pi}{3} = \frac{5\pi}{3}, \theta_2 = \pi + \frac{\pi}{3} = \frac{4\pi}{3}$$

$$3x = \theta \rightarrow 3x = \frac{5\pi}{3} \rightarrow x_1 = \frac{5\pi}{9}, \text{ et } 3x = \frac{4\pi}{3} \rightarrow x_2 = \frac{4\pi}{9}$$

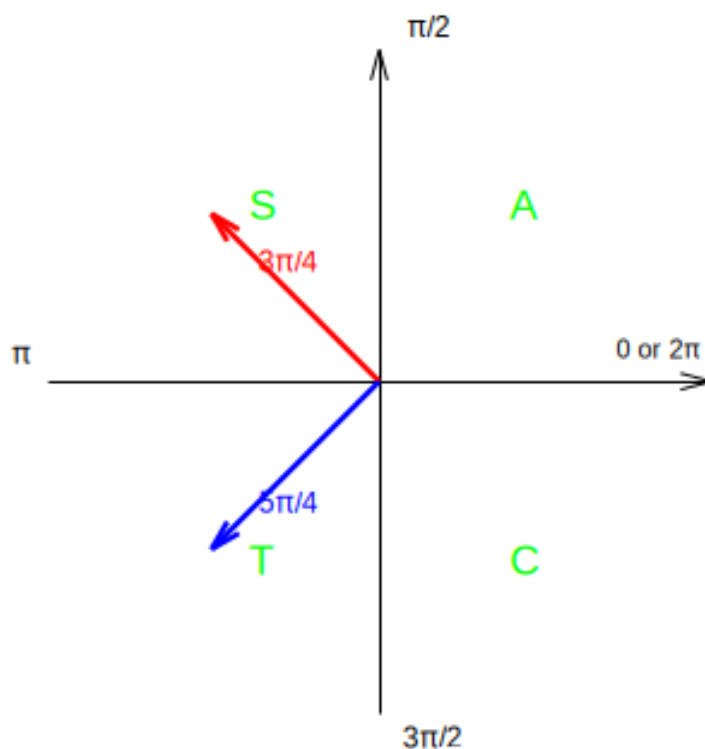
Ajouter la période  $\frac{2\pi}{3} = \frac{6\pi}{9}$  pour trouver d'autres solutions :

$$x_3 = x_1 + \frac{2\pi}{3} = \frac{11\pi}{9}, x_4 = x_2 + \frac{2\pi}{3} = \frac{10\pi}{9}$$

$$x_5 = x_3 + \frac{2\pi}{3} = \frac{17\pi}{9}, x_6 = x_4 + \frac{2\pi}{3} = \frac{16\pi}{9}$$

$$g) \cos(4x) = -\frac{1}{\sqrt{2}}$$

Soit  $\theta = 4x$ .  $\cos \theta = -\frac{1}{\sqrt{2}}$ . Du triangle spécial :  $\cos \frac{\pi}{4} = \frac{1}{\sqrt{2}}$ . Placer aux quadrants 2 et 3.



$$\theta_1 = \pi - \frac{\pi}{4} = \frac{3\pi}{4}, \theta_2 = \pi + \frac{\pi}{4} = \frac{5\pi}{4}$$

$$4x = \theta \rightarrow 4x = \frac{3\pi}{4} \rightarrow x_1 = \frac{3\pi}{16}, \text{ et } 4x = \frac{5\pi}{4} \rightarrow x_2 = \frac{5\pi}{16}$$

Ajouter la période  $\frac{\pi}{2} = \frac{8\pi}{16}$  pour trouver d'autres solutions :

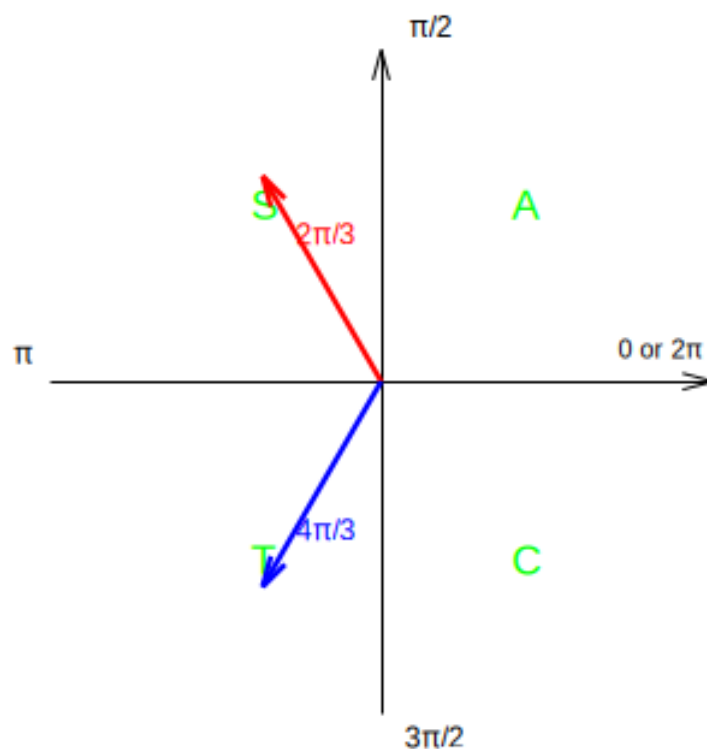
$$x_3 = x_1 + \frac{\pi}{2} = \frac{11\pi}{16}, x_4 = x_2 + \frac{\pi}{2} = \frac{13\pi}{16}$$

$$x_5 = x_3 + \frac{\pi}{2} = \frac{19\pi}{16}, x_6 = x_4 + \frac{\pi}{2} = \frac{21\pi}{16}$$

$$x_7 = x_5 + \frac{\pi}{2} = \frac{27\pi}{16}, x_8 = x_6 + \frac{\pi}{2} = \frac{29\pi}{16}$$

h)  $\cos(2x) = -\frac{1}{2}$

Soit  $\theta = 2x$ .  $\cos \theta = -\frac{1}{2}$ . Du triangle spécial :  $\cos \frac{\pi}{3} = \frac{1}{2}$ . Placer aux quadrants 2 et 3.



$$\theta_1 = \pi - \frac{\pi}{3} = \frac{2\pi}{3}, \theta_2 = \pi + \frac{\pi}{3} = \frac{4\pi}{3}$$

$$2x = \theta \rightarrow 2x = \frac{2\pi}{3} \rightarrow x_1 = \frac{\pi}{3}, \text{ et } 2x = \frac{4\pi}{3} \rightarrow x_2 = \frac{2\pi}{3}$$

Ajouter la période  $\pi = \frac{3\pi}{3}$  pour trouver d'autres solutions :

$$x_3 = x_1 + \pi = \frac{4\pi}{3}, x_4 = x_2 + \pi = \frac{5\pi}{3}$$

## Corrigé

**a)** 0.46, 1.11, 3.61, 4.25

**b)** 0.32, 1.25, 3.46, 4.39

**c)** 0.42, 1.15, 3.57, 4.29

**d)**  $\frac{\pi}{8}, \frac{3\pi}{8}, \frac{9\pi}{8}, \frac{11\pi}{8}$

**e)**  $\frac{\pi}{24}, \frac{5\pi}{24}, \frac{13\pi}{24}, \frac{17\pi}{24}, \frac{25\pi}{24}, \frac{29\pi}{24}, \frac{37\pi}{24}, \frac{41\pi}{24}$

**f)**  $\frac{4\pi}{9}, \frac{5\pi}{9}, \frac{10\pi}{9}, \frac{11\pi}{9}, \frac{16\pi}{9}, \frac{17\pi}{9}$

**g)**  $\frac{3\pi}{16}, \frac{5\pi}{16}, \frac{11\pi}{16}, \frac{13\pi}{16}, \frac{19\pi}{16}, \frac{21\pi}{16}, \frac{27\pi}{16}, \frac{29\pi}{16}$

**h)**  $\frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{3}$