

5.6 Solve Double Angle Trigonometric Equations Worksheet with solutions

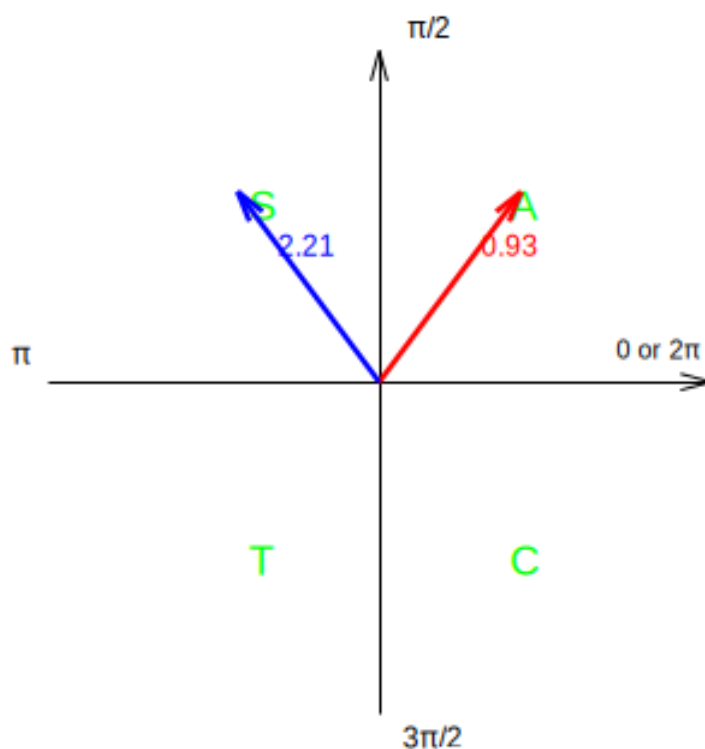
This document was composed and formatted by Annie Bernatchez.

The course material originated from www.jensenmath.ca

Determine solutions for each equation in the interval $0 \leq x \leq 2\pi$, to the nearest hundredth of a radian. Give exact answers where possible.

a) $\sin(2x) - 0.8 = 0$

Let $\theta = 2x$. $\sin \theta = 0.8$



$$\theta_1 = \sin^{-1}(0.8) \approx 0.93$$

$$\theta_2 = \pi - 0.93 \approx 2.21$$

$$2x = \theta \rightarrow 2x = 0.93 \rightarrow x_1 \approx 0.46, \text{ and } 2x = 2.21 \rightarrow x_2 \approx 1.11$$

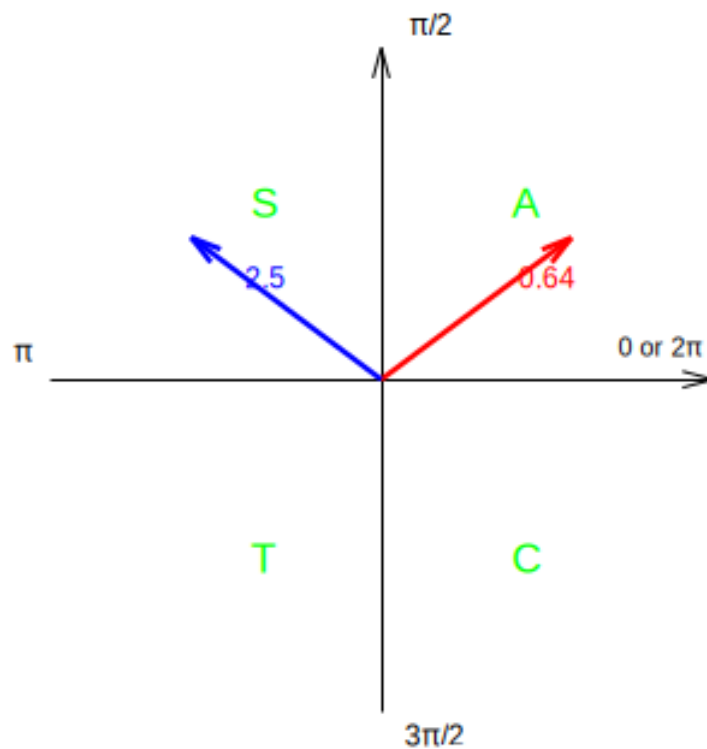
Add the period of π to find other solutions:

$$x_3 = x_1 + \pi \approx 3.61$$

$$x_4 = x_2 + \pi \approx 4.25$$

b) $5\sin(2x) - 3 = 0$

Let $\theta = 2x$. $\sin \theta = \frac{3}{5}$



$$\theta_1 = \sin^{-1}\left(\frac{3}{5}\right) \approx 0.64$$

$$\theta_2 = \pi - 0.64 \approx 2.5$$

$$2x = \theta \rightarrow 2x = 0.64 \rightarrow x_1 \approx 0.32, \text{ and } 2x = 2.5 \rightarrow x_2 \approx 1.25$$

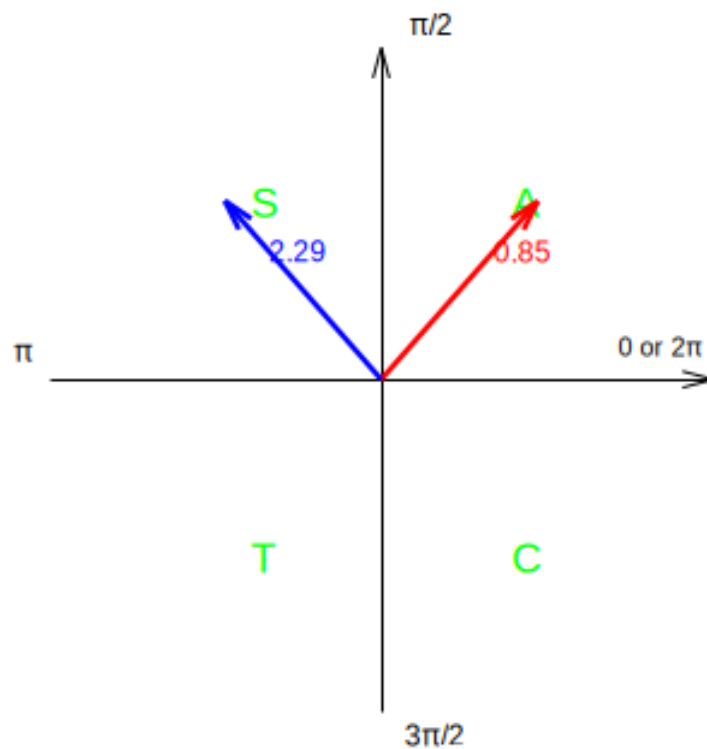
Add the period of π to find other solutions:

$$x_3 = x_1 + \pi \approx 3.46$$

$$x_4 = x_2 + \pi \approx 4.39$$

$$\text{c) } -4\sin(2x) + 3 = 0$$

$$\text{Let } \theta = 2x. \sin \theta = \frac{3}{4}$$



$$\theta_1 = \sin^{-1}\left(\frac{3}{4}\right) \approx 0.85$$

$$\theta_2 = \pi - 0.85 \approx 2.29$$

$$2x = \theta \rightarrow 2x = 0.85 \rightarrow x_1 \approx 0.42, \text{ and } 2x = 2.29 \rightarrow x_2 \approx 1.15$$

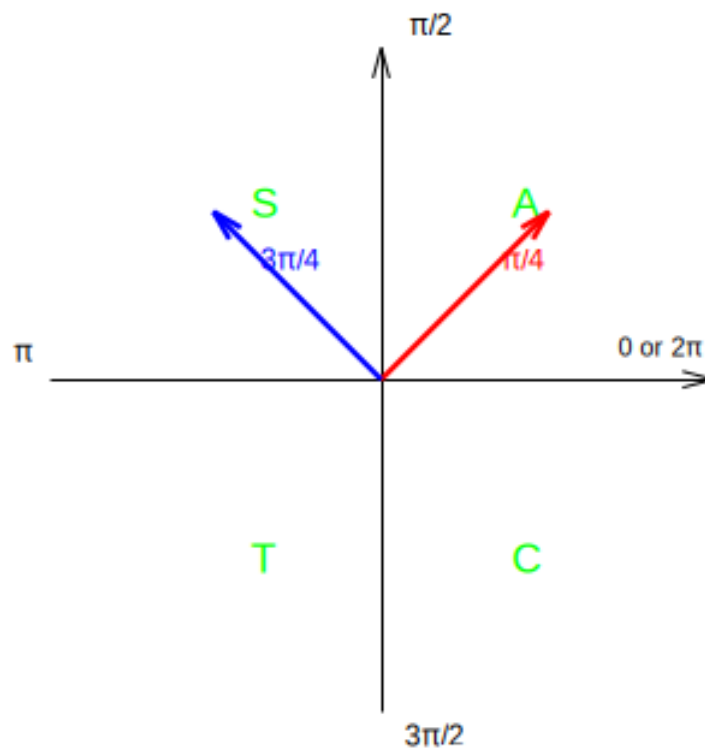
Add the period of π to find other solutions:

$$x_3 = x_1 + \pi \approx 3.57$$

$$x_4 = x_2 + \pi \approx 4.29$$

d) $\sin(2x) = \frac{1}{\sqrt{2}}$

Let $\theta = 2x$. $\sin \theta = \frac{1}{\sqrt{2}}$. From special triangle: $\sin \frac{\pi}{4} = \frac{1}{\sqrt{2}}$. Place in Q1 and Q2.



$$\theta_1 = \frac{\pi}{4}, \theta_2 = \pi - \frac{\pi}{4} = \frac{3\pi}{4}$$

$$2x = \theta \rightarrow 2x = \frac{\pi}{4} \rightarrow x_1 = \frac{\pi}{8}, \text{ and } 2x = \frac{3\pi}{4} \rightarrow x_2 = \frac{3\pi}{8}$$

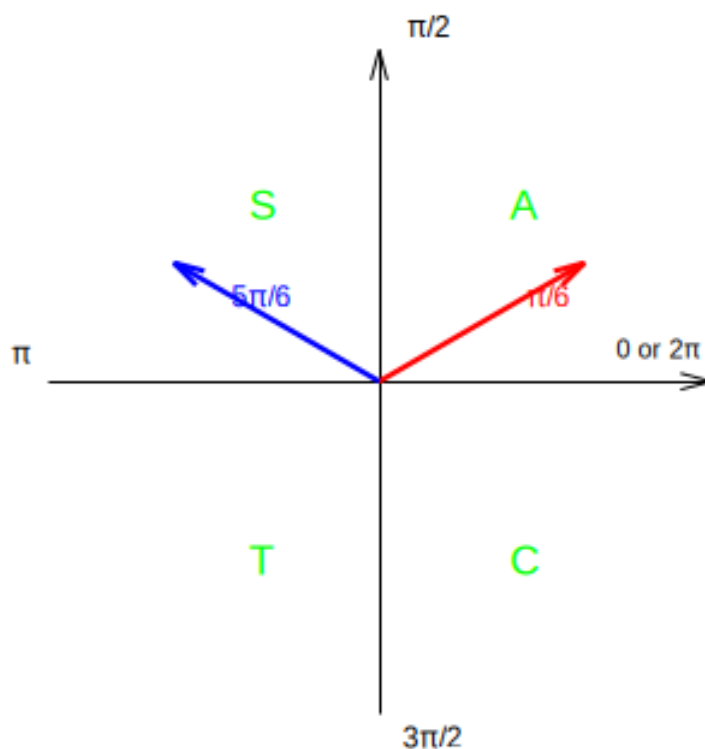
Add the period of π to find other solutions:

$$x_3 = x_1 + \pi = \frac{\pi}{8} + \pi = \frac{9\pi}{8}$$

$$x_4 = x_2 + \pi = \frac{3\pi}{8} + \pi = \frac{11\pi}{8}$$

$$\text{e) } \sin(4x) = \frac{1}{2}$$

Let $\theta = 4x$. $\sin \theta = \frac{1}{2}$. From special triangle: $\sin \frac{\pi}{6} = \frac{1}{2}$. Place in Q1 and Q2.



$$\theta_1 = \frac{\pi}{6}, \theta_2 = \pi - \frac{\pi}{6} = \frac{5\pi}{6}$$

$$4x = \theta \rightarrow 4x = \frac{\pi}{6} \rightarrow x_1 = \frac{\pi}{24}, \text{ and } 4x = \frac{5\pi}{6} \rightarrow x_2 = \frac{5\pi}{24}$$

Add the period of $\frac{\pi}{2} = \frac{12\pi}{24}$ to find other solutions:

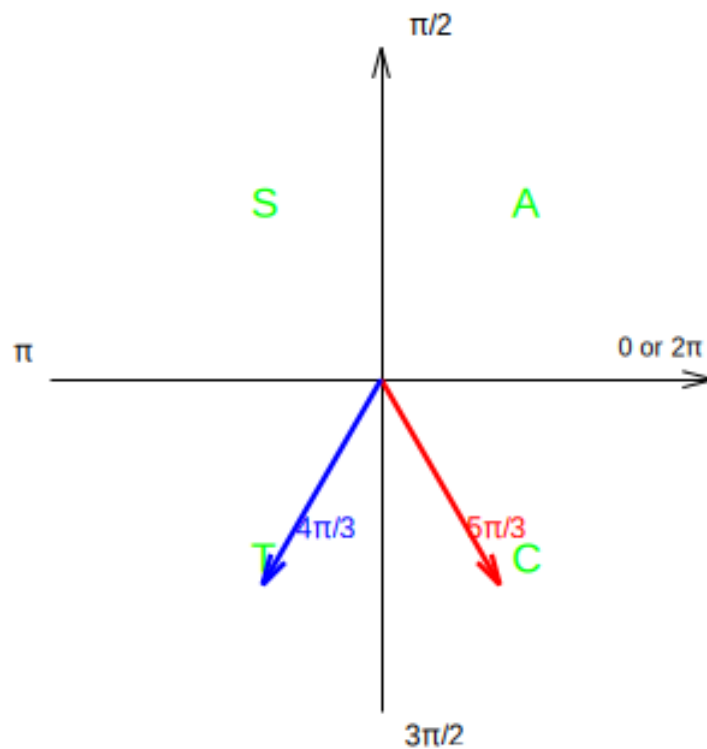
$$x_3 = x_1 + \frac{\pi}{2} = \frac{13\pi}{24}, x_4 = x_2 + \frac{\pi}{2} = \frac{17\pi}{24}$$

$$x_5 = x_3 + \frac{\pi}{2} = \frac{25\pi}{24}, x_6 = x_4 + \frac{\pi}{2} = \frac{29\pi}{24}$$

$$x_7 = x_5 + \frac{\pi}{2} = \frac{37\pi}{24}, x_8 = x_6 + \frac{\pi}{2} = \frac{41\pi}{24}$$

$$f) \sin(3x) = -\frac{\sqrt{3}}{2}$$

Let $3x = \theta$. $\sin \theta = -\frac{\sqrt{3}}{2}$. From special triangle: $\sin \frac{\pi}{3} = \frac{\sqrt{3}}{2}$. Place in Q3 and Q4.



$$\theta_1 = 2\pi - \frac{\pi}{3} = \frac{5\pi}{3}, \theta_2 = \pi + \frac{\pi}{3} = \frac{4\pi}{3}$$

$$3x = \theta \rightarrow 3x = \frac{5\pi}{3} \rightarrow x_1 = \frac{5\pi}{9}, \text{ and } 3x = \frac{4\pi}{3} \rightarrow x_2 = \frac{4\pi}{9}$$

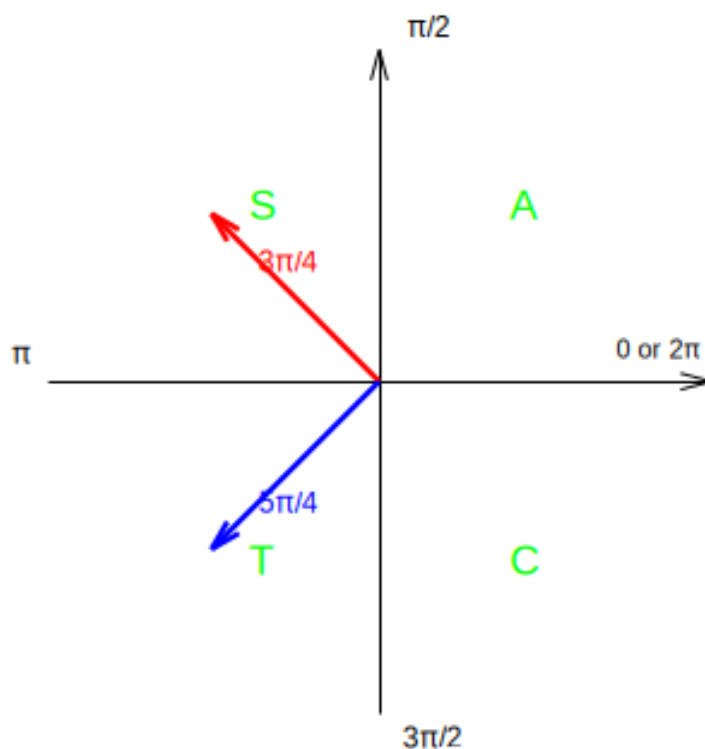
Add the period of $\frac{2\pi}{3} = \frac{6\pi}{9}$ to find other solutions:

$$x_3 = x_1 + \frac{2\pi}{3} = \frac{11\pi}{9}, x_4 = x_2 + \frac{2\pi}{3} = \frac{10\pi}{9}$$

$$x_5 = x_3 + \frac{2\pi}{3} = \frac{17\pi}{9}, x_6 = x_4 + \frac{2\pi}{3} = \frac{16\pi}{9}$$

$$\text{g) } \cos(4x) = -\frac{1}{\sqrt{2}}$$

Let $\theta = 4x$. $\cos \theta = -\frac{1}{\sqrt{2}}$. From special triangle: $\cos \frac{\pi}{4} = \frac{1}{\sqrt{2}}$. Place in Q2 and Q3.



$$\theta_1 = \pi - \frac{\pi}{4} = \frac{3\pi}{4}, \theta_2 = \pi + \frac{\pi}{4} = \frac{5\pi}{4}$$

$$4x = \theta \rightarrow 4x = \frac{3\pi}{4} \rightarrow x_1 = \frac{3\pi}{16}, \text{ and } 4x = \frac{5\pi}{4} \rightarrow x_2 = \frac{5\pi}{16}$$

Add the period of $\frac{\pi}{2} = \frac{8\pi}{16}$ to find other solutions:

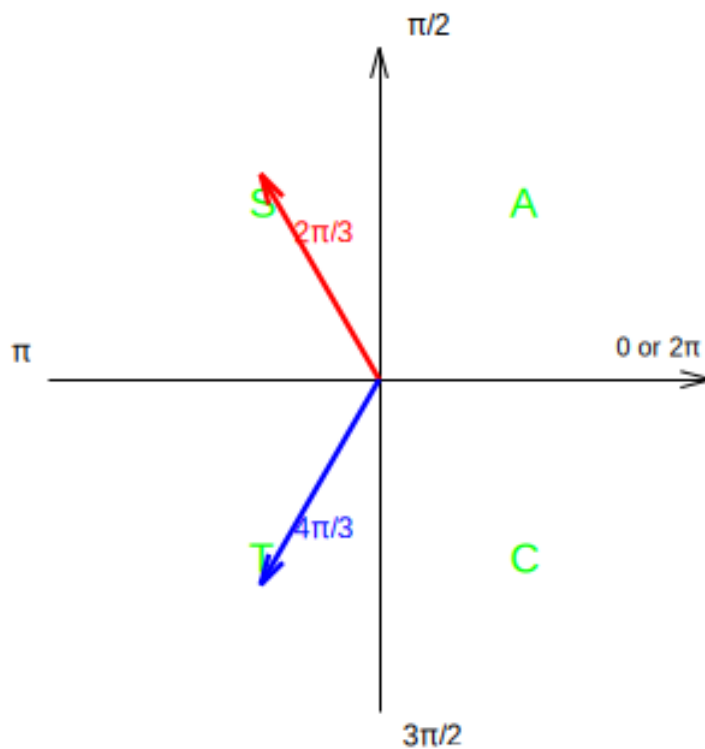
$$x_3 = x_1 + \frac{\pi}{2} = \frac{11\pi}{16}, x_4 = x_2 + \frac{\pi}{2} = \frac{13\pi}{16}$$

$$x_5 = x_3 + \frac{\pi}{2} = \frac{19\pi}{16}, x_6 = x_4 + \frac{\pi}{2} = \frac{21\pi}{16}$$

$$x_7 = x_5 + \frac{\pi}{2} = \frac{27\pi}{16}, x_8 = x_6 + \frac{\pi}{2} = \frac{29\pi}{16}$$

h) $\cos(2x) = -\frac{1}{2}$

Let $\theta = 2x$. $\cos \theta = -\frac{1}{2}$. From special triangle: $\cos \frac{\pi}{3} = \frac{1}{2}$. Place in Q2 and Q3.



$$\theta_1 = \pi - \frac{\pi}{3} = \frac{2\pi}{3}, \theta_2 = \pi + \frac{\pi}{3} = \frac{4\pi}{3}$$

$$2x = \theta \rightarrow 2x = \frac{2\pi}{3} \rightarrow x_1 = \frac{\pi}{3}, \text{ and } 2x = \frac{4\pi}{3} \rightarrow x_2 = \frac{2\pi}{3}$$

Add the period of $\pi = \frac{3\pi}{3}$ to find other solutions:

$$x_3 = x_1 + \pi = \frac{4\pi}{3}, x_4 = x_2 + \pi = \frac{5\pi}{3}$$

Answer Key

a) 0.46, 1.11, 3.61, 4.25

b) 0.32, 1.25, 3.46, 4.39

c) 0.42, 1.15, 3.57, 4.29

d) $\frac{\pi}{8}, \frac{3\pi}{8}, \frac{9\pi}{8}, \frac{11\pi}{8}$

e) $\frac{\pi}{24}, \frac{5\pi}{24}, \frac{13\pi}{24}, \frac{17\pi}{24}, \frac{25\pi}{24}, \frac{29\pi}{24}, \frac{37\pi}{24}, \frac{41\pi}{24}$

f) $\frac{4\pi}{9}, \frac{5\pi}{9}, \frac{10\pi}{9}, \frac{11\pi}{9}, \frac{16\pi}{9}, \frac{17\pi}{9}$

g) $\frac{3\pi}{16}, \frac{5\pi}{16}, \frac{11\pi}{16}, \frac{13\pi}{16}, \frac{19\pi}{16}, \frac{21\pi}{16}, \frac{27\pi}{16}, \frac{29\pi}{16}$

h) $\frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{3}$