

5.5 Résoudre des équations trigonométriques linéaires – Feuille de travail avec solutions

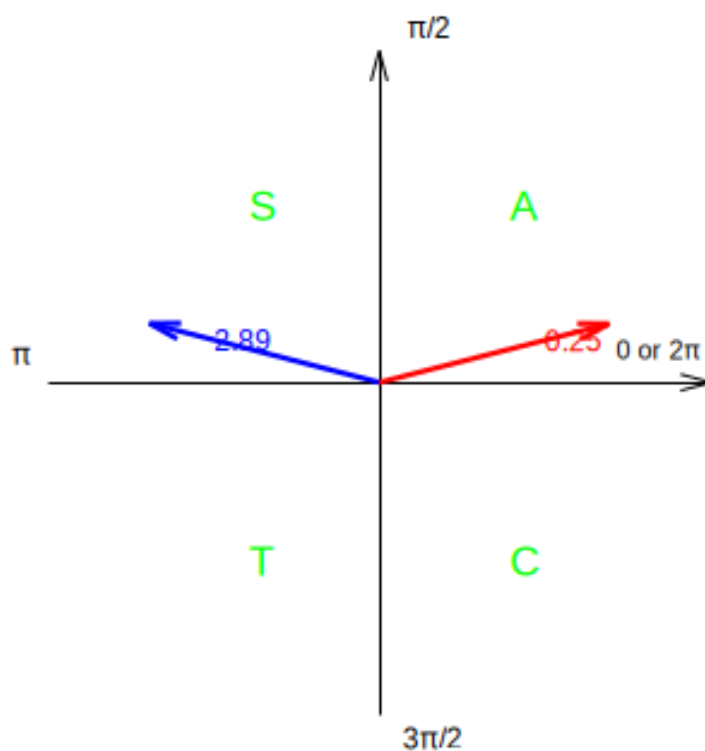
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Le contenu du cours provient de www.jensenmath.ca

1) Déterminer les solutions approximatives de chaque équation dans l'intervalle $0 \leq x \leq 2\pi$, au centième de radian près.

a) $\sin x - \frac{1}{4} = 0$

$$\sin x = \frac{1}{4}$$

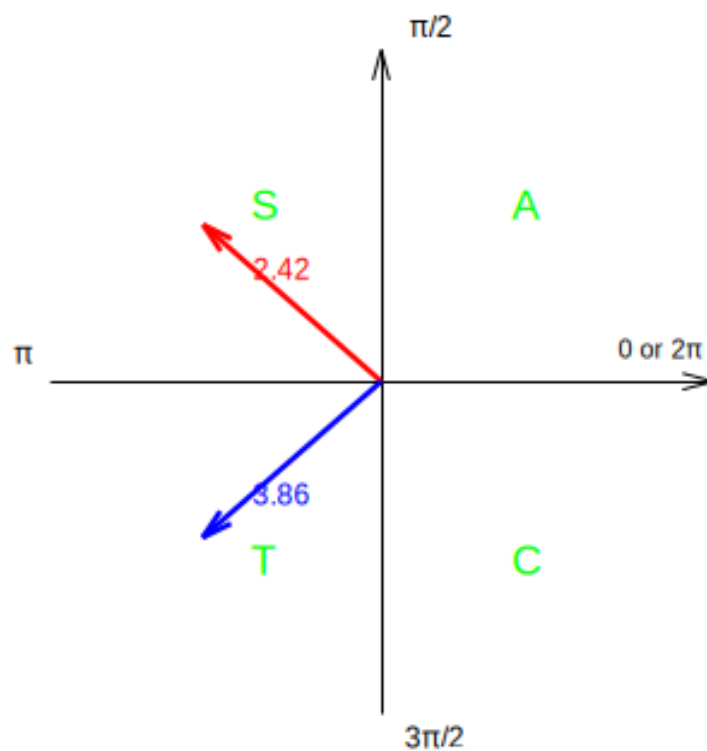


$$x_1 = \sin^{-1}\left(\frac{1}{4}\right) \approx 0.25$$

$$x_2 = \pi - 0.25 \approx 2.89$$

b) $\cos x + 0.75 = 0$

$$\cos x = -0.75$$



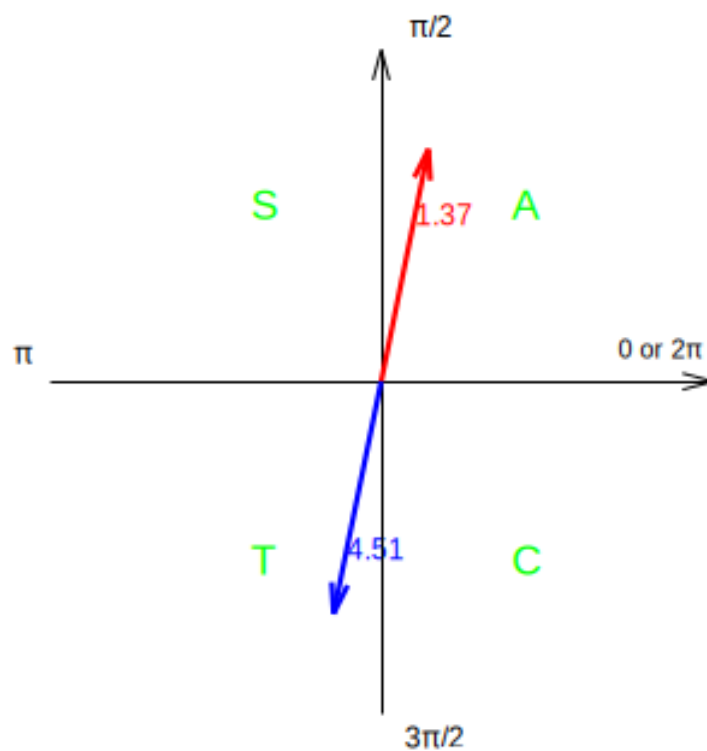
$$x_1 = \cos^{-1}(-0.75) \approx 2.42$$

$$\beta = \pi - 2.42 = 0.72$$

$$x_2 = \pi + \beta = \pi + 0.72 \approx 3.86$$

$$\text{c) } \tan x - 5 = 0$$

$$\tan x = 5$$

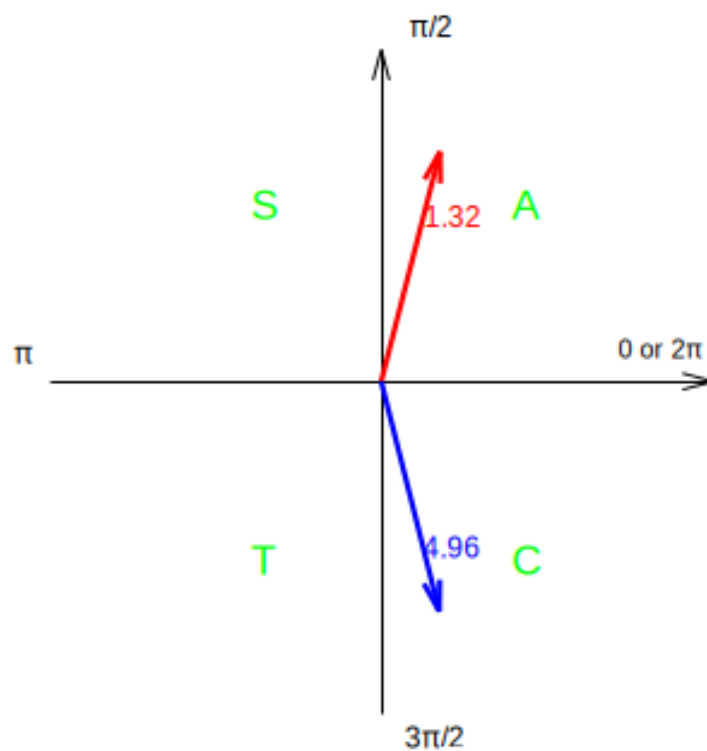


$$x_1 = \tan^{-1}(5) \approx 1.37$$

$$x_2 = \pi + 1.37 \approx 4.51$$

d) $\sec x - 4 = 0$

$$\sec x = 4 \rightarrow \cos x = \frac{1}{4}$$

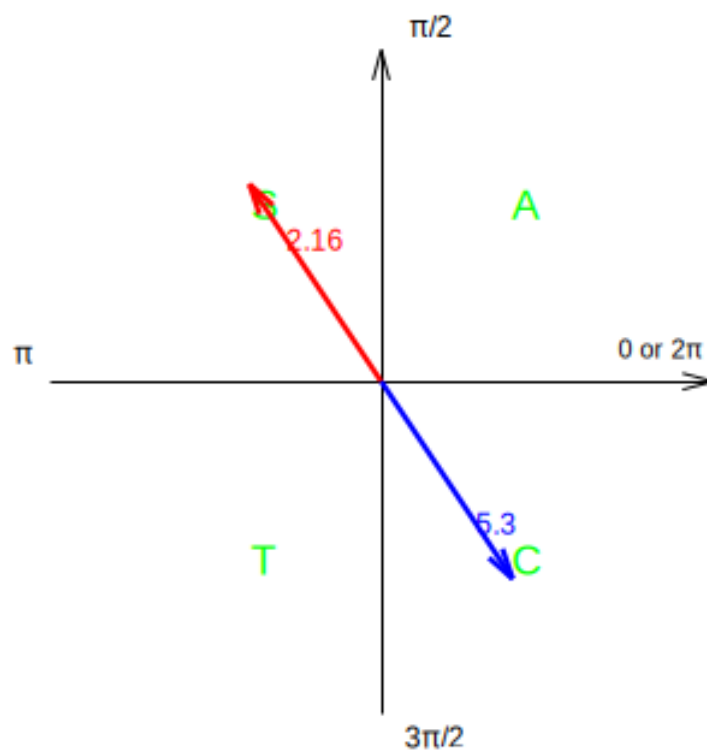


$$x_1 = \cos^{-1}\left(\frac{1}{4}\right) \approx 1.32$$

$$x_2 = 2\pi - 1.32 \approx 4.96$$

e) $3 \cot x + 2 = 0$

$$\cot x = -\frac{2}{3} \rightarrow \tan x = -\frac{3}{2}$$

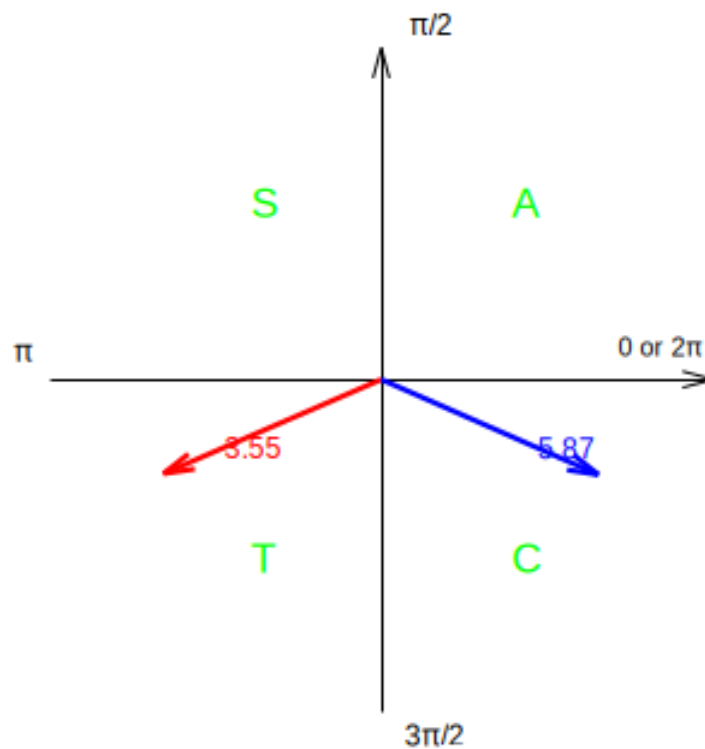


$$x_1 = \tan^{-1}\left(-\frac{3}{2}\right) = -0.98 + 2\pi \approx 5.3$$

$$x_2 = \pi - 0.98 \approx 2.16$$

f) $2\csc x + 5 = 0$

$$\csc x = -\frac{5}{2} \rightarrow \sin x = -\frac{2}{5}$$



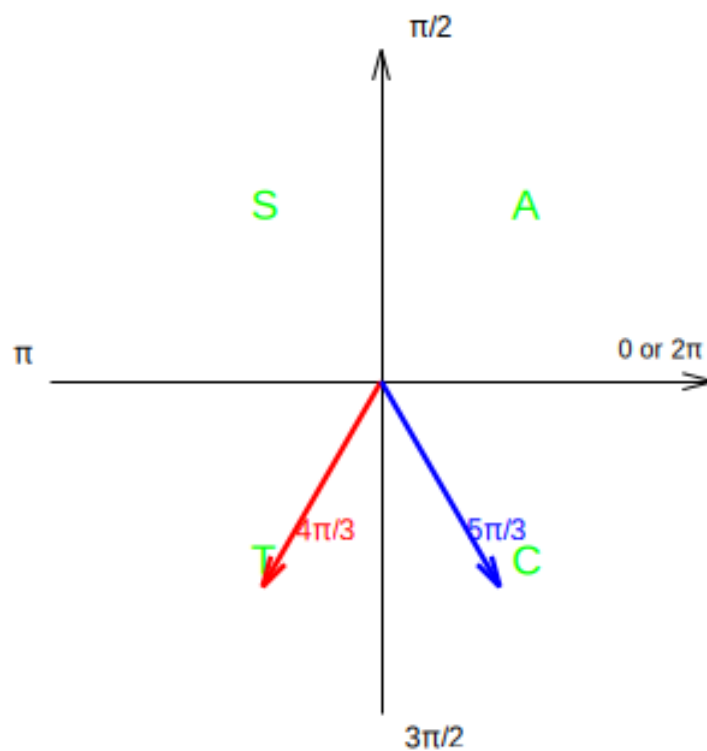
$$x_1 = \sin^{-1}\left(-\frac{2}{5}\right) = -0.41 + 2\pi \approx 5.87$$

$$x_2 = \pi + 0.41 \approx 3.55$$

2) Déterminer les solutions exactes de chaque équation dans l'intervalle $0 \leq x \leq 2\pi$.

a) $\sin x + \frac{\sqrt{3}}{2} = 0$

$\sin x = -\frac{\sqrt{3}}{2}$. Du triangle spécial : $\sin \frac{\pi}{3} = \frac{\sqrt{3}}{2}$. Placer $\frac{\pi}{3}$ aux quadrants 3 et 4.

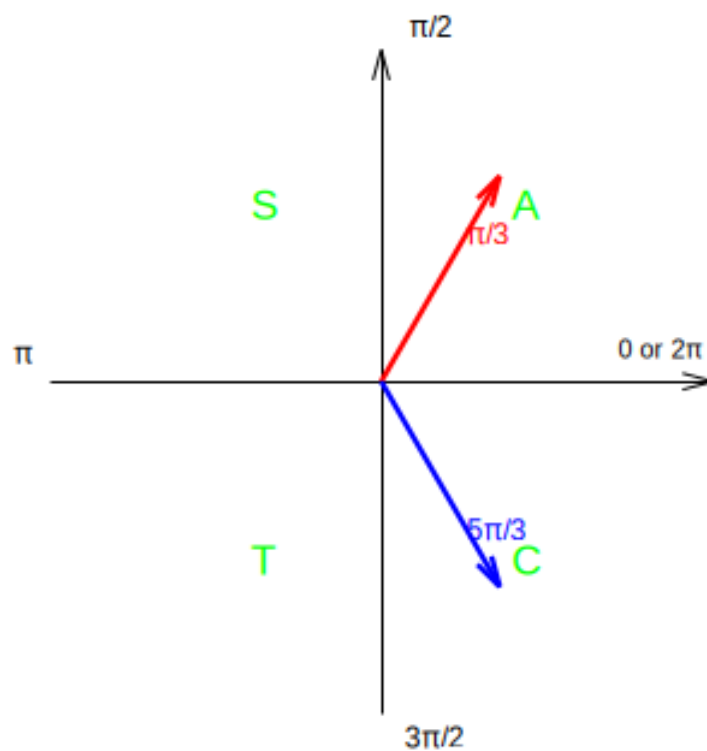


$$x_1 = 2\pi - \frac{\pi}{3} = \frac{5\pi}{3}$$

$$x_2 = \pi + \frac{\pi}{3} = \frac{4\pi}{3}$$

b) $\cos x - 0.5 = 0$

$\cos x = \frac{1}{2}$. Du triangle spécial : $\cos \frac{\pi}{3} = \frac{1}{2}$. Placer $\frac{\pi}{3}$ aux quadrants 1 et 4.

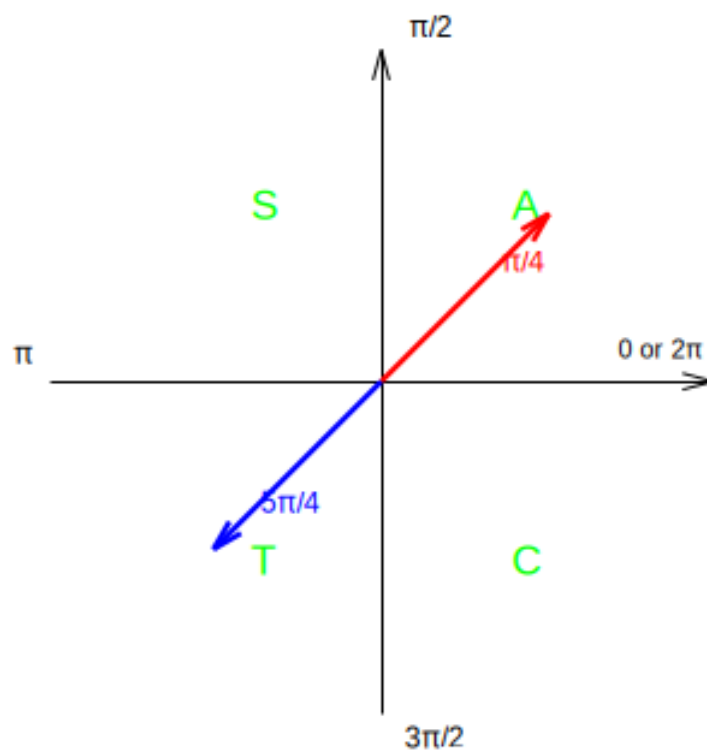


$$x_1 = \frac{\pi}{3}$$

$$x_2 = 2\pi - \frac{\pi}{3} = \frac{5\pi}{3}$$

c) $\tan x - 1 = 0$

$\tan x = 1$. Du triangle spécial : $\tan \frac{\pi}{4} = 1$. Placer $\frac{\pi}{4}$ aux quadrants 1 et 3.

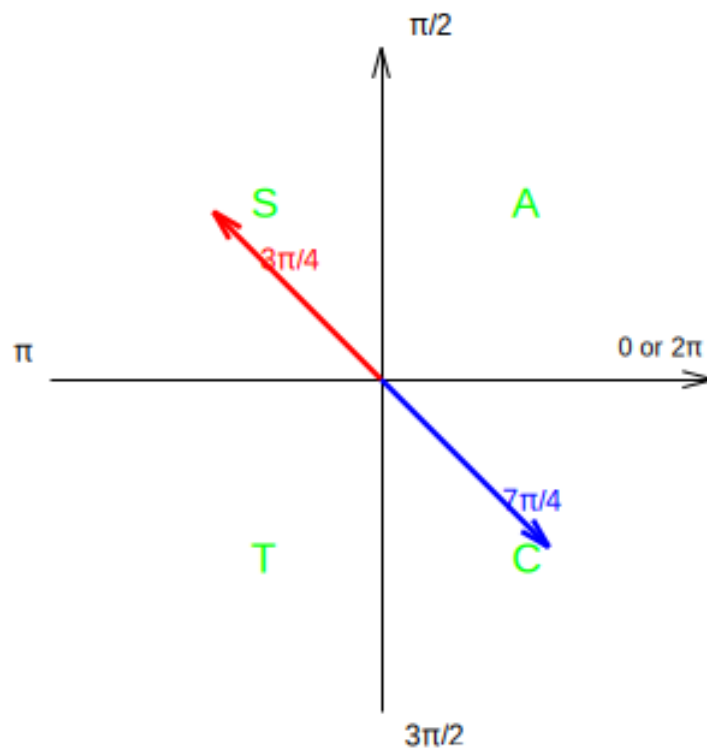


$$x_1 = \frac{\pi}{4}$$

$$x_2 = \pi + \frac{\pi}{4} = \frac{5\pi}{4}$$

d) $\cot x + 1 = 0$

$\cot x = -1 \rightarrow \tan x = -1$. Du triangle spécial : $\tan \frac{\pi}{4} = 1$. Placer $\frac{\pi}{4}$ aux quadrants 2 et 4.



$$x_1 = \pi - \frac{\pi}{4} = \frac{3\pi}{4}$$

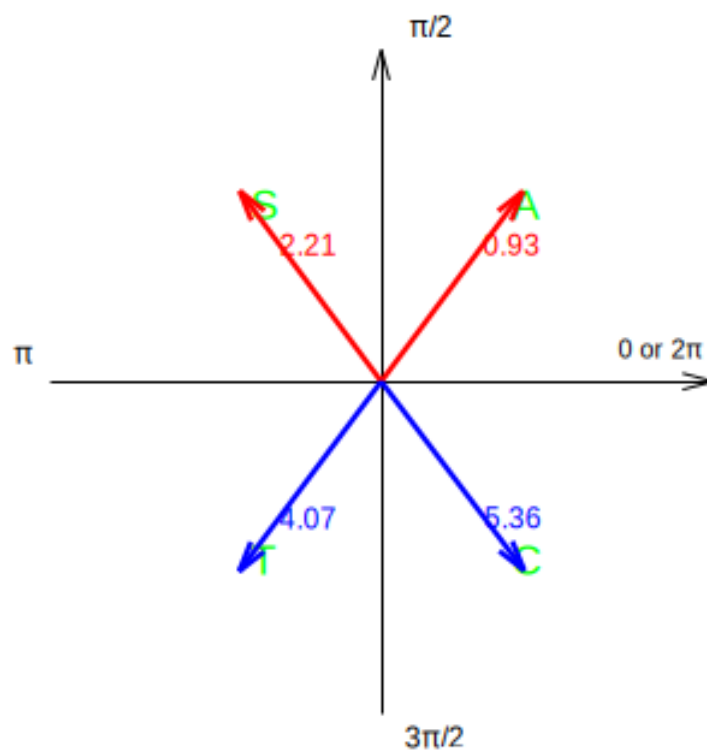
$$x_2 = 2\pi - \frac{\pi}{4} = \frac{7\pi}{4}$$

3) Déterminer les solutions approximatives de chaque équation dans l'intervalle $0 \leq x \leq 2\pi$, au centième de radian près.

a) $\sin^2 x - 0.64 = 0$

$$\sin^2 x = 0.64$$

$$\sin x = \pm \sqrt{0.64} = \pm 0.8$$



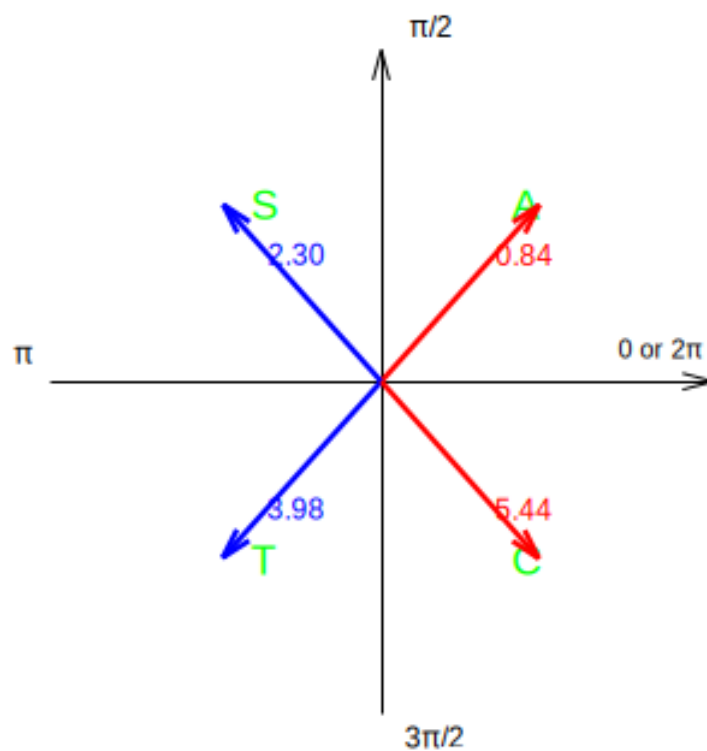
$$\sin x = 0.8 \rightarrow x_1 = \sin^{-1}(0.8) \approx 0.93, x_2 = \pi - 0.93 \approx 2.21$$

$$\sin x = -0.8 \rightarrow x_3 = \sin^{-1}(-0.8) = -0.93 + 2\pi \approx 5.36, x_4 = \pi + 0.93 \approx 4.07$$

b) $\cos^2 x - \frac{4}{9} = 0$

$$\cos^2 x = \frac{4}{9}$$

$$\cos x = \pm \sqrt{\frac{4}{9}} = \pm \frac{2}{3}$$



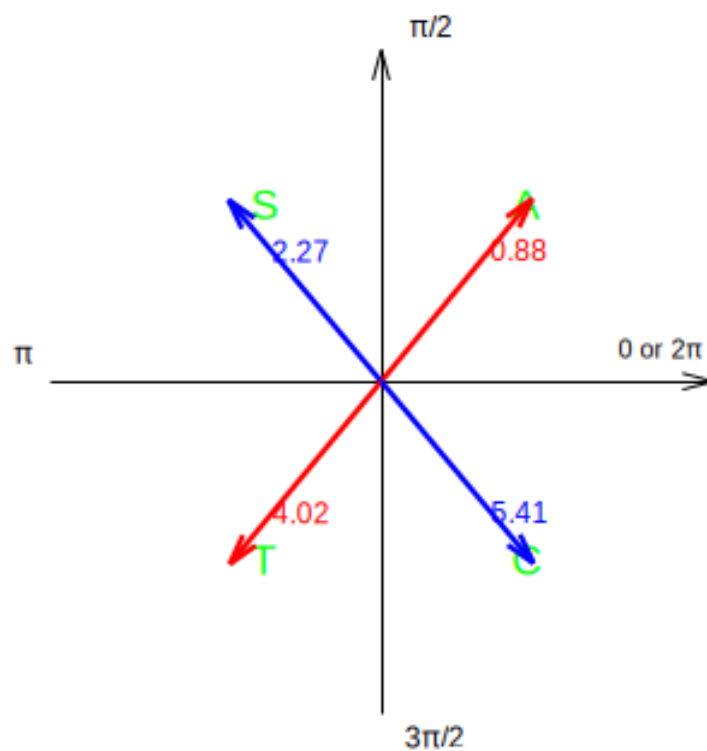
$$\cos x = \frac{2}{3} \rightarrow x_1 = \cos^{-1}\left(\frac{2}{3}\right) \approx 0.84, x_2 = 2\pi - 0.84 \approx 5.44$$

$$\cos x = -\frac{2}{3} \rightarrow x_3 = \cos^{-1}\left(-\frac{2}{3}\right) \approx 2.30, x_4 = \pi + 0.84 \approx 3.98$$

c) $\tan^2 x - 1.44 = 0$

$$\tan^2 x = 1.44$$

$$\tan x = \pm \sqrt{1.44} = \pm 1.2$$



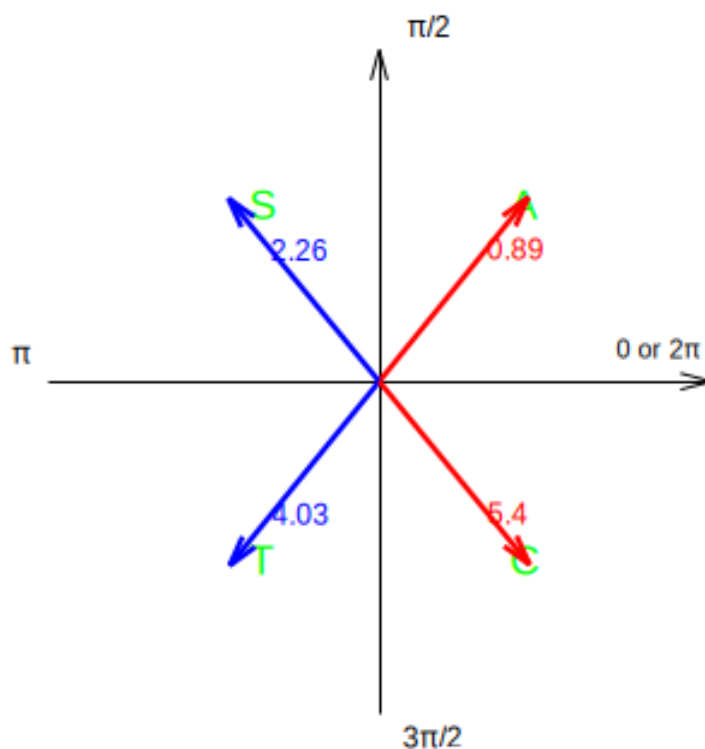
$$\tan x = 1.2 \rightarrow x_1 = \tan^{-1}(1.2) \approx 0.88, x_2 = \pi + 0.88 \approx 4.02$$

$$\tan x = -1.2 \rightarrow x_3 = \pi - 0.88 \approx 2.27, x_4 = 2\pi - 0.88 \approx 5.41$$

d) $\sec^2 x - 2.5 = 0$

$$\sec^2 x = 2.5 \rightarrow \cos^2 x = \frac{1}{2.5}$$

$$\cos x = \pm \sqrt{\frac{1}{2.5}} \approx \pm 0.63$$



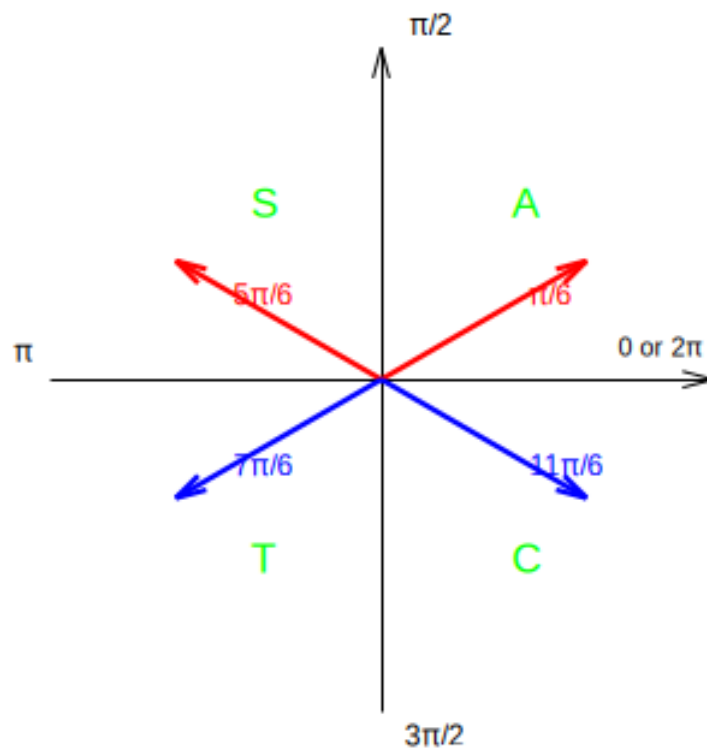
$$\cos x = 0.63 \rightarrow x_1 = \cos^{-1}(0.63) \approx 0.89, x_2 = 2\pi - 0.89 \approx 5.4$$

$$\cos x = -0.63 \rightarrow x_3 = \cos^{-1}(-0.63) \approx 2.26, x_4 = \pi + 0.89 \approx 4.03$$

4) Déterminer les solutions exactes de chaque équation dans l'intervalle $0 \leq x \leq 2\pi$.

a) $\sin^2 x - \frac{1}{4} = 0$

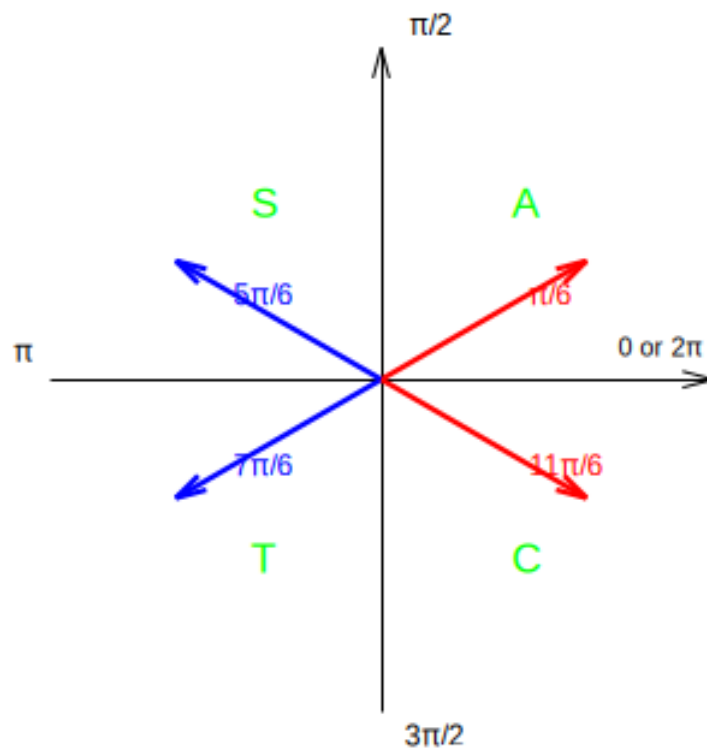
$\sin^2 x = \frac{1}{4} \rightarrow \sin x = \pm \frac{1}{2}$. Du triangle spécial : $\sin \frac{\pi}{6} = \frac{1}{2}$. Placer $\frac{\pi}{6}$ aux quadrants 1 et 2 pour $\sin x = \frac{1}{2}$, et aux quadrants 3 et 4 pour $\sin x = -\frac{1}{2}$.



$$x_1 = \frac{\pi}{6}, x_2 = \frac{5\pi}{6}, x_3 = \pi + \frac{\pi}{6} = \frac{7\pi}{6}, x_4 = 2\pi - \frac{\pi}{6} = \frac{11\pi}{6}$$

b) $\cos^2 x - \frac{3}{4} = 0$

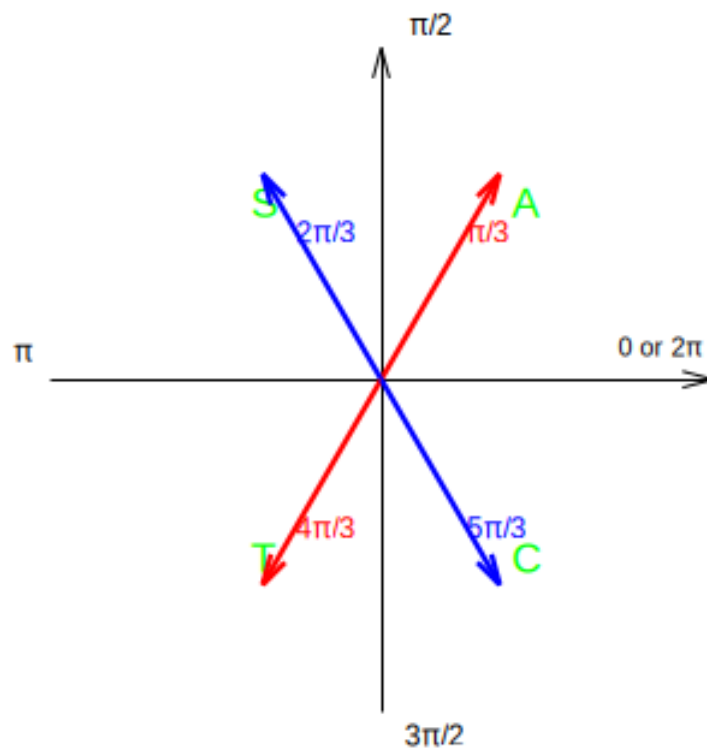
$\cos^2 x = \frac{3}{4} \rightarrow \cos x = \pm \frac{\sqrt{3}}{2}$. Du triangle spécial : $\cos \frac{\pi}{6} = \frac{\sqrt{3}}{2}$. Placer $\frac{\pi}{6}$ aux quadrants 1 et 4 pour $\cos x = \frac{\sqrt{3}}{2}$, et aux quadrants 2 et 3 pour $\cos x = -\frac{\sqrt{3}}{2}$.



$$x_1 = \frac{\pi}{6}, x_2 = \pi - \frac{\pi}{6} = \frac{5\pi}{6}, x_3 = \pi + \frac{\pi}{6} = \frac{7\pi}{6}, x_4 = 2\pi - \frac{\pi}{6} = \frac{11\pi}{6}$$

c) $\tan^2 x - 3 = 0$

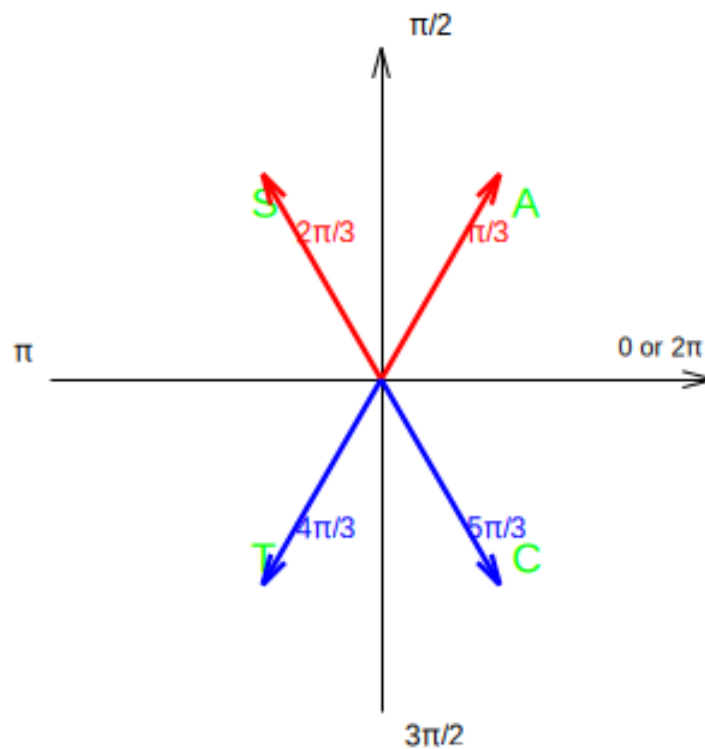
$\tan^2 x = 3 \rightarrow \tan x = \pm \sqrt{3}$. Du triangle spécial : $\tan \frac{\pi}{3} = \sqrt{3}$. Placer $\frac{\pi}{3}$ aux quadrants 1 et 3 pour $\tan x = \sqrt{3}$, et aux quadrants 2 et 4 pour $\tan x = -\sqrt{3}$.



$$x_1 = \frac{\pi}{3}, x_2 = \pi - \frac{\pi}{3} = \frac{2\pi}{3}, x_3 = \pi + \frac{\pi}{3} = \frac{4\pi}{3}, x_4 = 2\pi - \frac{\pi}{3} = \frac{5\pi}{3}$$

d) $3\csc^2 x - 4 = 0$

$\csc^2 x = \frac{4}{3} \rightarrow \sin^2 x = \frac{3}{4} \rightarrow \sin x = \pm \frac{\sqrt{3}}{2}$. Du triangle spécial : $\sin \frac{\pi}{3} = \frac{\sqrt{3}}{2}$. Placer $\frac{\pi}{3}$ aux quadrants 1 et 2 pour $\sin x = \frac{\sqrt{3}}{2}$, et aux quadrants 3 et 4 pour $\sin x = -\frac{\sqrt{3}}{2}$.

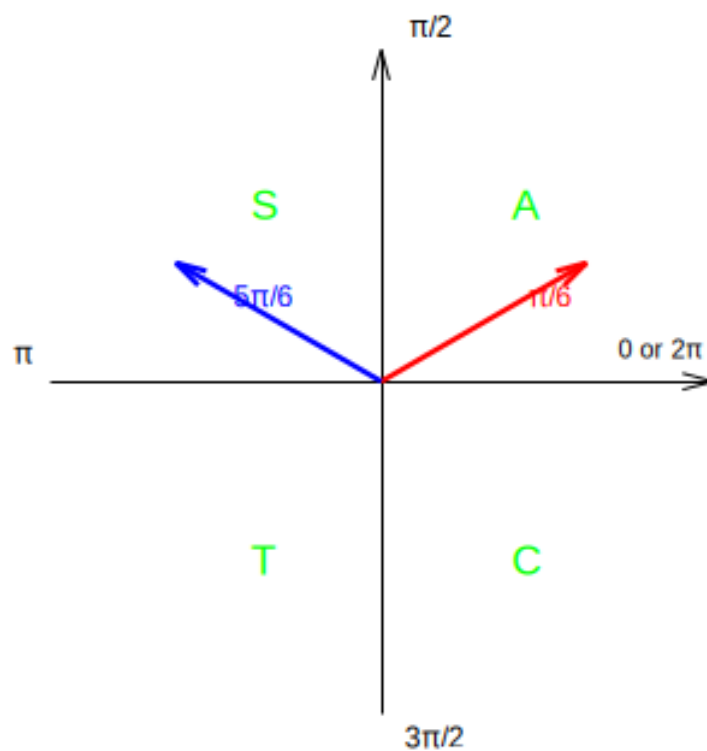


$$x_1 = \frac{\pi}{3}, x_2 = \pi - \frac{\pi}{3} = \frac{2\pi}{3}, x_3 = \pi + \frac{\pi}{3} = \frac{4\pi}{3}, x_4 = 2\pi - \frac{\pi}{3} = \frac{5\pi}{3}$$

5) Déterminer les solutions de chaque équation dans l'intervalle $0 \leq x \leq 2\pi$.

a) $3 \sin x = \sin x + 1$

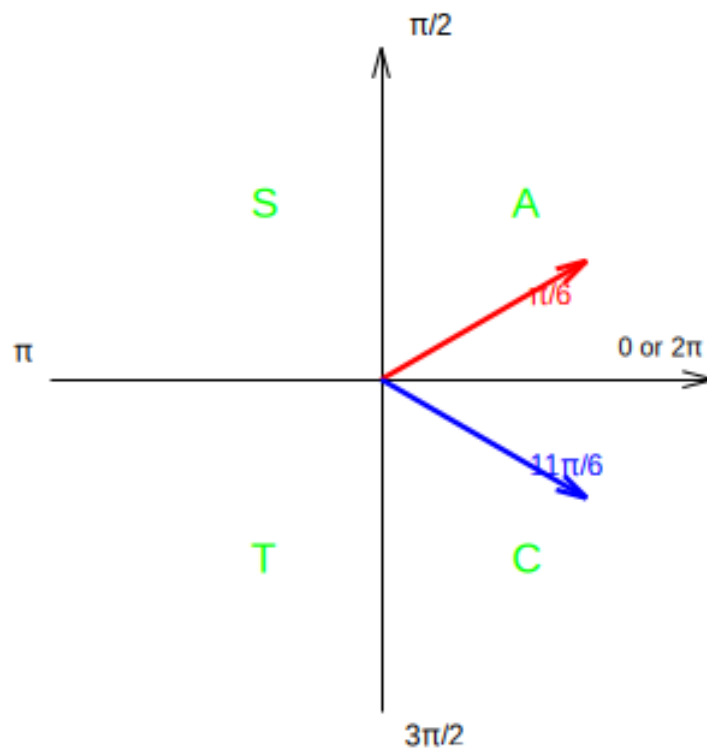
$2 \sin x = 1 \rightarrow \sin x = \frac{1}{2}$. Du triangle spécial : $\sin \frac{\pi}{6} = \frac{1}{2}$. Placer aux quadrants 1 et 2.



$$x_1 = \frac{\pi}{6}, x_2 = \pi - \frac{\pi}{6} = \frac{5\pi}{6}$$

b) $5 \cos x - \sqrt{3} = 3 \cos x$

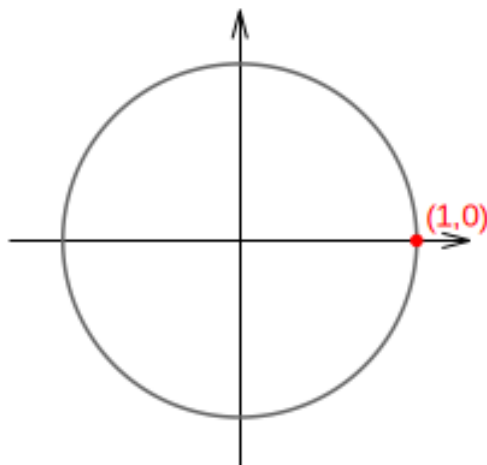
$2 \cos x = \sqrt{3} \rightarrow \cos x = \frac{\sqrt{3}}{2}$. Du triangle spécial : $\cos \frac{\pi}{6} = \frac{\sqrt{3}}{2}$. Placer aux quadrants 1 et 4.



$$x_1 = \frac{\pi}{6}, x_2 = 2\pi - \frac{\pi}{6} = \frac{11\pi}{6}$$

c) $7 \sec x = 7$

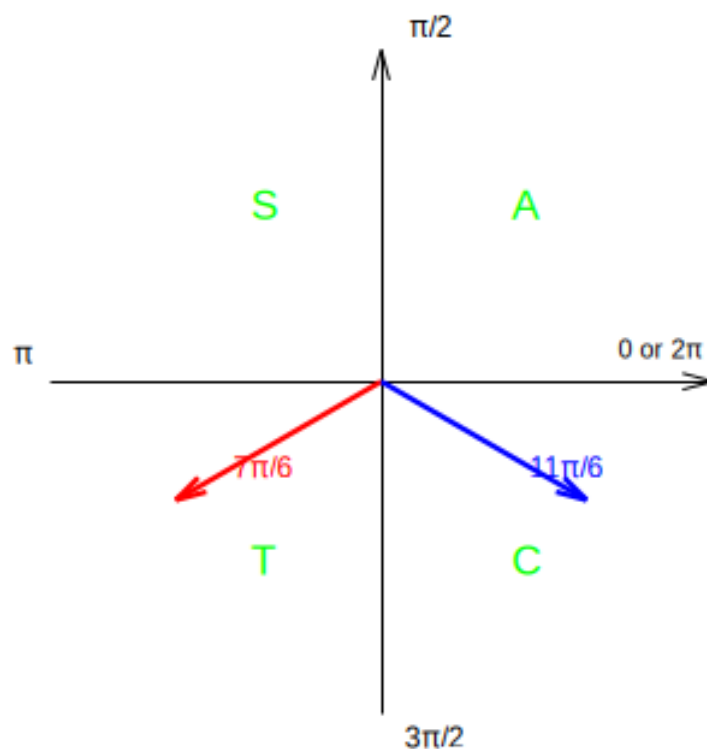
$\sec x = 1 \rightarrow \cos x = 1$. Utiliser le cercle trigonométrique, où chaque point est $(\cos x, \sin x)$.



$$x_1 = 0, x_2 = 2\pi$$

d) $2 \csc x + 17 = 15 + \csc x$

$\csc x = -2 \rightarrow \sin x = -\frac{1}{2}$. Du triangle spécial : $\sin \frac{\pi}{6} = \frac{1}{2}$. Placer $\frac{\pi}{6}$ aux quadrants 3 et 4.



$$x_1 = \pi + \frac{\pi}{6} = \frac{7\pi}{6}, \quad x_2 = 2\pi - \frac{\pi}{6} = \frac{11\pi}{6}$$

Corrigé

1) a) 0.25, 2.89 b) 2.42, 3.86 c) 1.37, 4.51 d) 1.32, 4.97 e) 2.16, 5.3 f) 3.55, 5.87

2) a) $\frac{4\pi}{3}, \frac{5\pi}{3}$ b) $\frac{\pi}{3}, \frac{5\pi}{3}$ c) $\frac{\pi}{4}, \frac{5\pi}{4}$ d) $\frac{3\pi}{4}, \frac{7\pi}{4}$

3) a) 0.93, 2.21, 4.07, 5.36 b) 0.84, 2.3, 3.98, 5.44 c) 0.88, 2.27, 4.02, 5.41 d) 0.89, 2.26, 4.03, 5.4

4) a) $\frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6}$ b) $\frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6}$ c) $\frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{3}$ d) $\frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{3}$

5) a) $\frac{\pi}{6}, \frac{5\pi}{6}$ b) $\frac{\pi}{6}, \frac{11\pi}{6}$ c) 0 ou 2π d) 3.67 ou 5.76