

5.5 Solve Linear Trigonometric Equations Worksheet with solutions

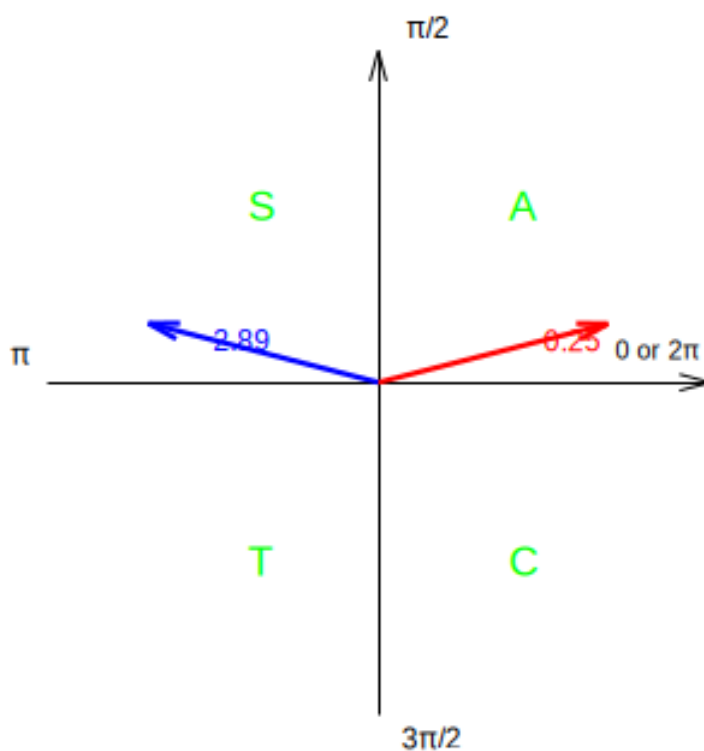
This document was composed and formatted by Annie Bernatchez.

The course material originated from www.jensenmath.ca

1) Determine approximate solutions for each equation in the interval $0 \leq x \leq 2\pi$, to the nearest hundredth of a radian.

a) $\sin x - \frac{1}{4} = 0$

$$\sin x = \frac{1}{4}$$

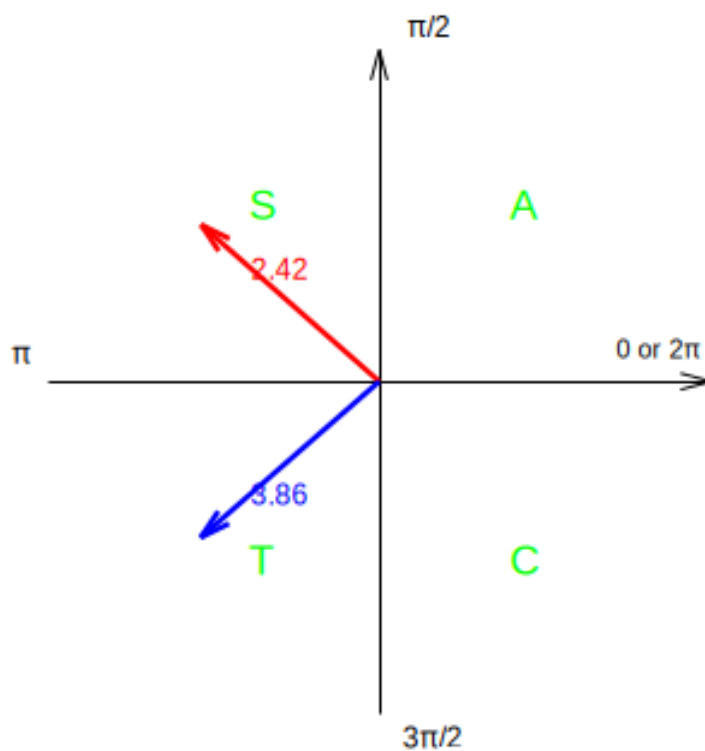


$$x_1 = \sin^{-1}\left(\frac{1}{4}\right) \approx 0.25$$

$$x_2 = \pi - 0.25 \approx 2.89$$

b) $\cos x + 0.75 = 0$

$$\cos x = -0.75$$



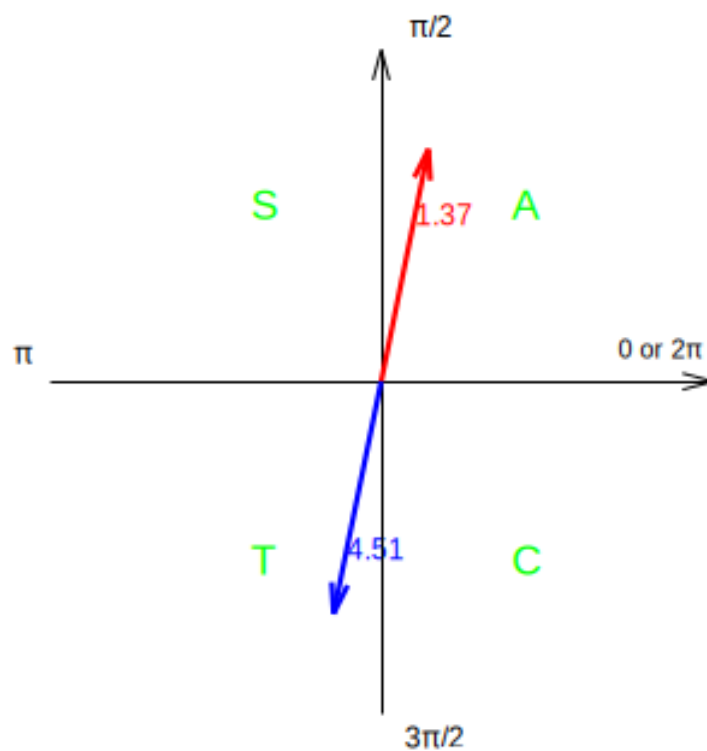
$$x_1 = \cos^{-1}(-0.75) \approx 2.42$$

$$\beta = \pi - 2.42 = 0.72$$

$$x_2 = \pi + \beta = \pi + 0.72 \approx 3.86$$

c) $\tan x - 5 = 0$

$$\tan x = 5$$

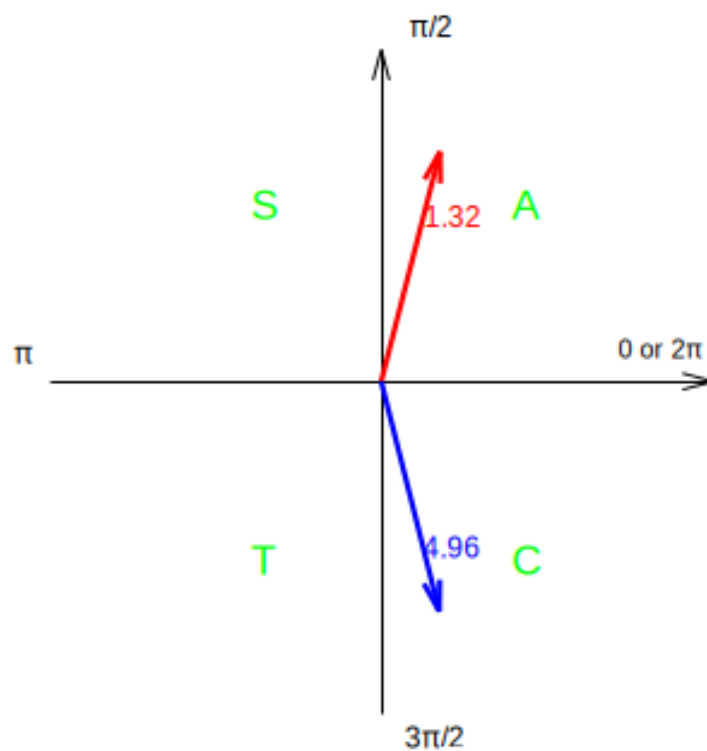


$$x_1 = \tan^{-1}(5) \approx 1.37$$

$$x_2 = \pi + 1.37 \approx 4.51$$

d) $\sec x - 4 = 0$

$$\sec x = 4 \rightarrow \cos x = \frac{1}{4}$$

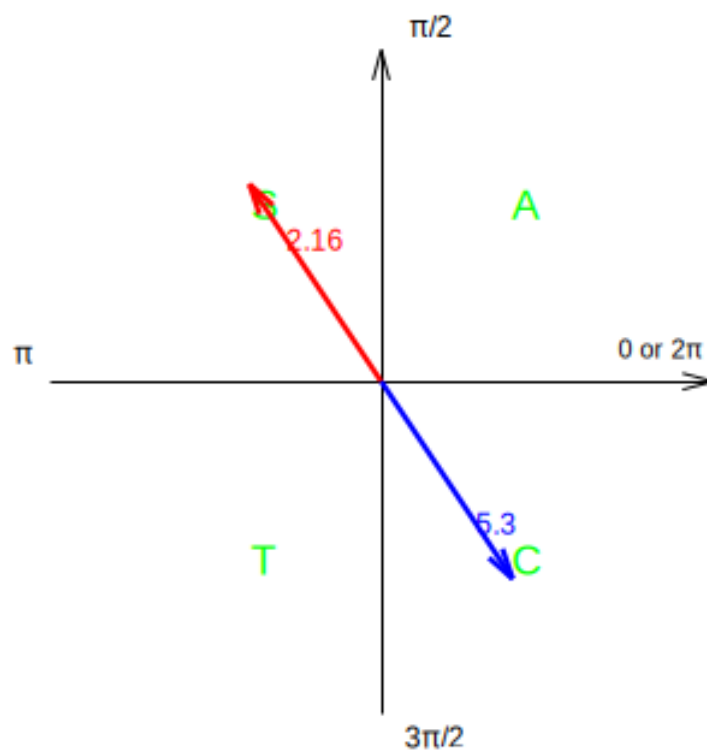


$$x_1 = \cos^{-1}\left(\frac{1}{4}\right) \approx 1.32$$

$$x_2 = 2\pi - 1.32 \approx 4.96$$

e) $3 \cot x + 2 = 0$

$$\cot x = -\frac{2}{3} \rightarrow \tan x = -\frac{3}{2}$$

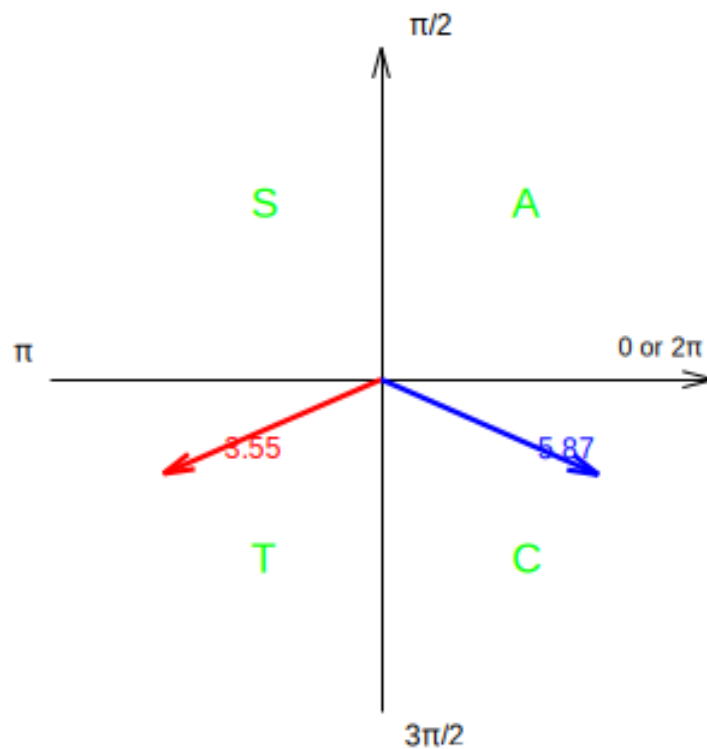


$$x_1 = \tan^{-1}\left(-\frac{3}{2}\right) = -0.98 + 2\pi \approx 5.3$$

$$x_2 = \pi - 0.98 \approx 2.16$$

f) $2\csc x + 5 = 0$

$$\csc x = -\frac{5}{2} \rightarrow \sin x = -\frac{2}{5}$$



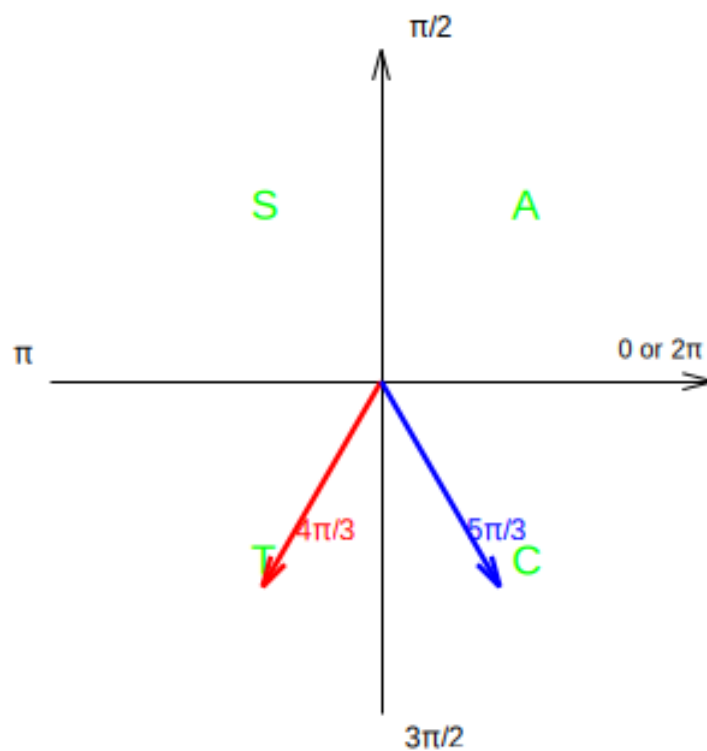
$$x_1 = \sin^{-1}\left(-\frac{2}{5}\right) = -0.41 + 2\pi \approx 5.87$$

$$x_2 = \pi + 0.41 \approx 3.55$$

2) Determine exact solutions for each equation in the interval $0 \leq x \leq 2\pi$.

a) $\sin x + \frac{\sqrt{3}}{2} = 0$

$\sin x = -\frac{\sqrt{3}}{2}$. From special triangle: $\sin \frac{\pi}{3} = \frac{\sqrt{3}}{2}$. Place $\frac{\pi}{3}$ in Q3 and Q4.

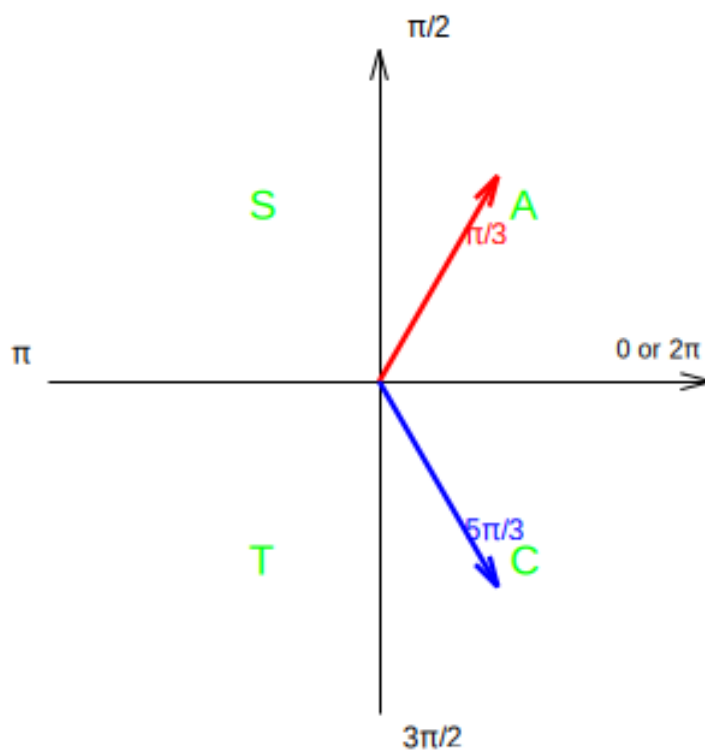


$$x_1 = 2\pi - \frac{\pi}{3} = \frac{5\pi}{3}$$

$$x_2 = \pi + \frac{\pi}{3} = \frac{4\pi}{3}$$

b) $\cos x - 0.5 = 0$

$\cos x = \frac{1}{2}$. From special triangle: $\cos \frac{\pi}{3} = \frac{1}{2}$. Place $\frac{\pi}{3}$ in Q1 and Q4.

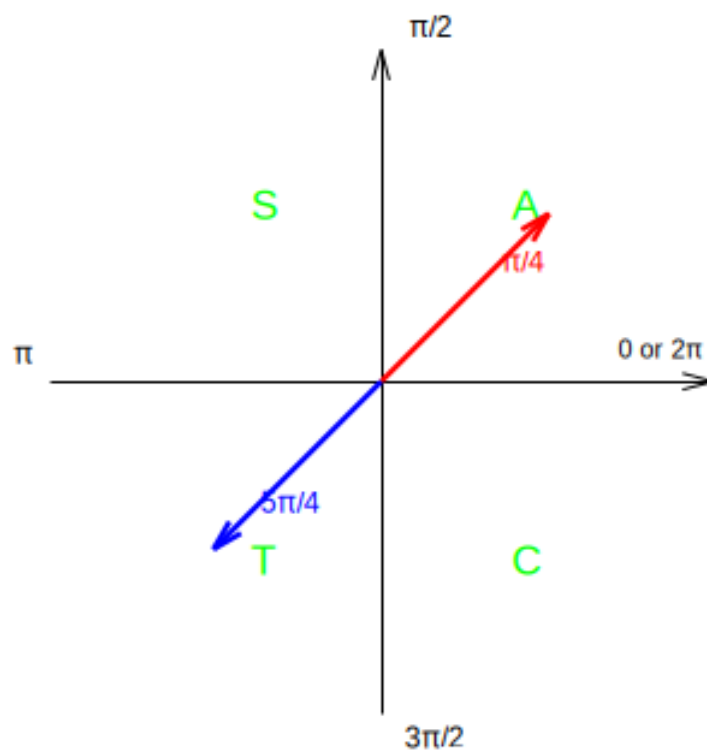


$$x_1 = \frac{\pi}{3}$$

$$x_2 = 2\pi - \frac{\pi}{3} = \frac{5\pi}{3}$$

c) $\tan x - 1 = 0$

$\tan x = 1$. From special triangle: $\tan \frac{\pi}{4} = 1$. Place $\frac{\pi}{4}$ in Q1 and Q3.

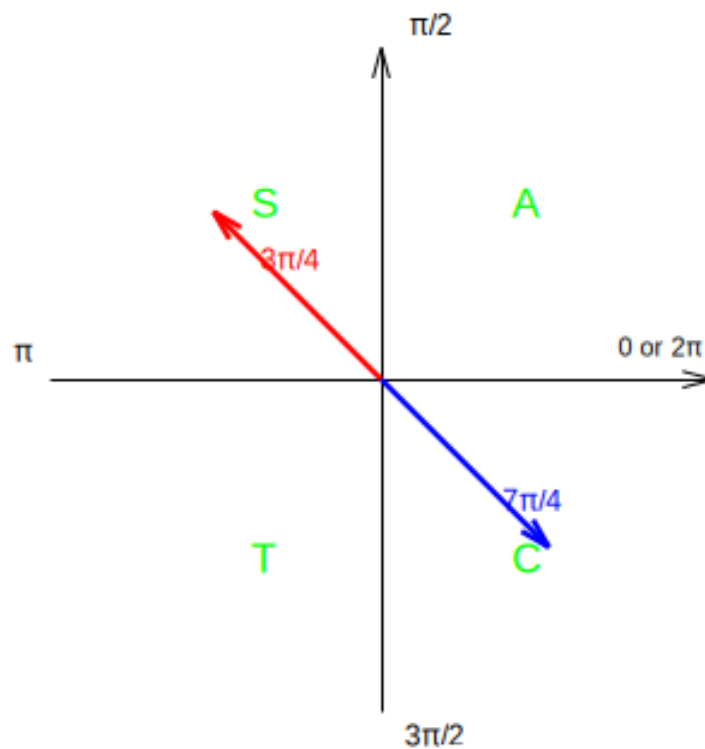


$$x_1 = \frac{\pi}{4}$$

$$x_2 = \pi + \frac{\pi}{4} = \frac{5\pi}{4}$$

d) $\cot x + 1 = 0$

$\cot x = -1 \rightarrow \tan x = -1$. From special triangle: $\tan \frac{\pi}{4} = 1$. Place $\frac{\pi}{4}$ in Q2 and Q4.



$$x_1 = \pi - \frac{\pi}{4} = \frac{3\pi}{4}$$

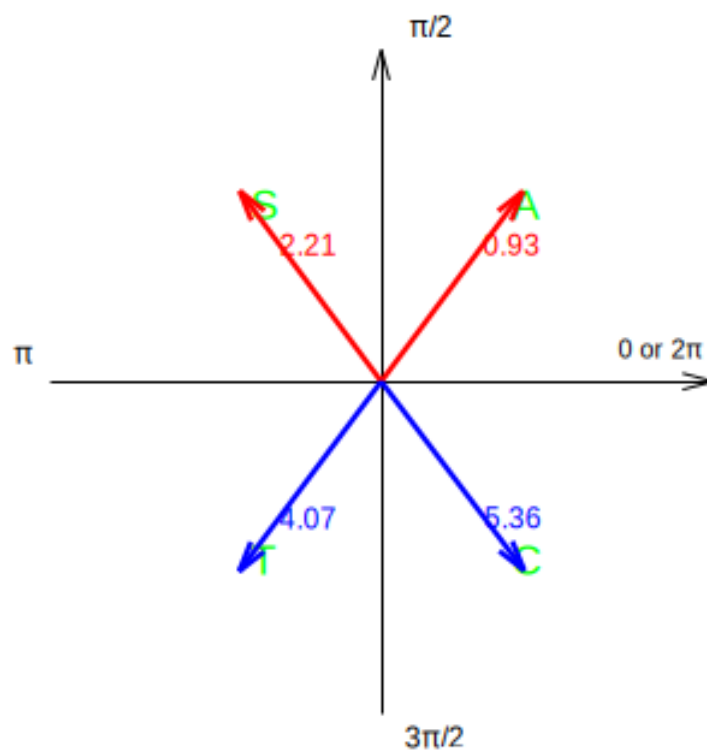
$$x_2 = 2\pi - \frac{\pi}{4} = \frac{7\pi}{4}$$

3) Determine approximate solutions for each equation in the interval $0 \leq x \leq 2\pi$, to the nearest hundredth of a radian.

a) $\sin^2 x - 0.64 = 0$

$$\sin^2 x = 0.64$$

$$\sin x = \pm \sqrt{0.64} = \pm 0.8$$



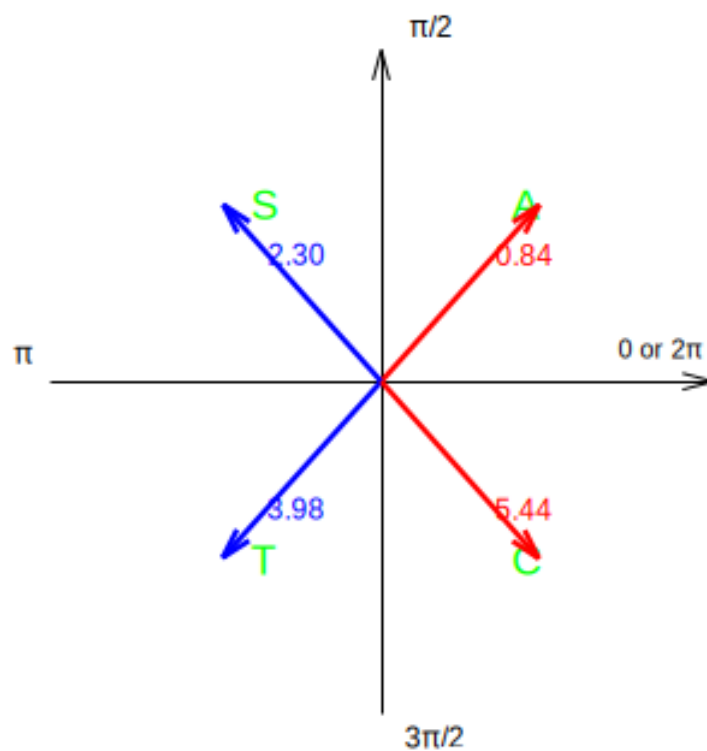
$$\sin x = 0.8 \rightarrow x_1 = \sin^{-1}(0.8) \approx 0.93, x_2 = \pi - 0.93 \approx 2.21$$

$$\sin x = -0.8 \rightarrow x_3 = \sin^{-1}(-0.8) = -0.93 + 2\pi \approx 5.36, x_4 = \pi + 0.93 \approx 4.07$$

b) $\cos^2 x - \frac{4}{9} = 0$

$$\cos^2 x = \frac{4}{9}$$

$$\cos x = \pm \sqrt{\frac{4}{9}} = \pm \frac{2}{3}$$



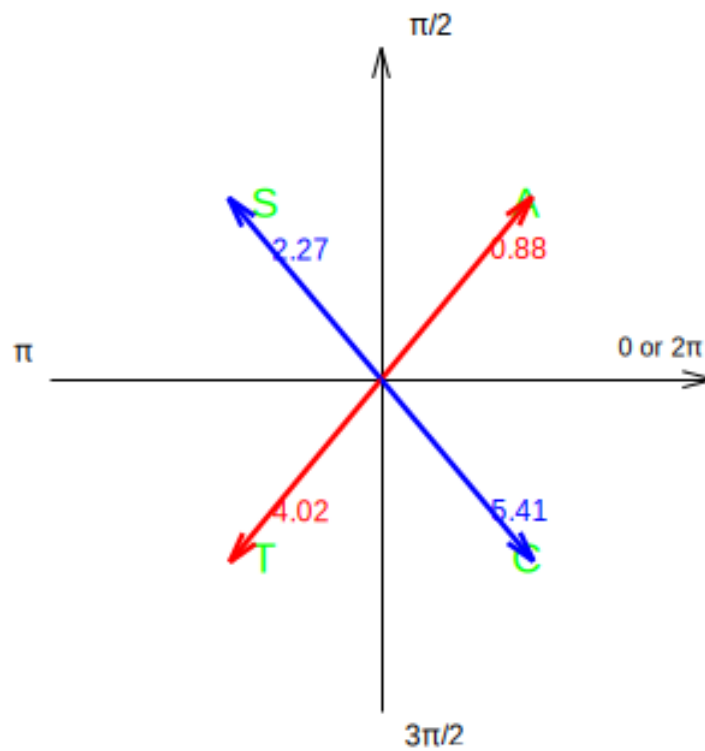
$$\cos x = \frac{2}{3} \rightarrow x_1 = \cos^{-1}\left(\frac{2}{3}\right) \approx 0.84, x_2 = 2\pi - 0.84 \approx 5.44$$

$$\cos x = -\frac{2}{3} \rightarrow x_3 = \cos^{-1}\left(-\frac{2}{3}\right) \approx 2.30, x_4 = \pi + 0.84 \approx 3.98$$

c) $\tan^2 x - 1.44 = 0$

$$\tan^2 x = 1.44$$

$$\tan x = \pm \sqrt{1.44} = \pm 1.2$$



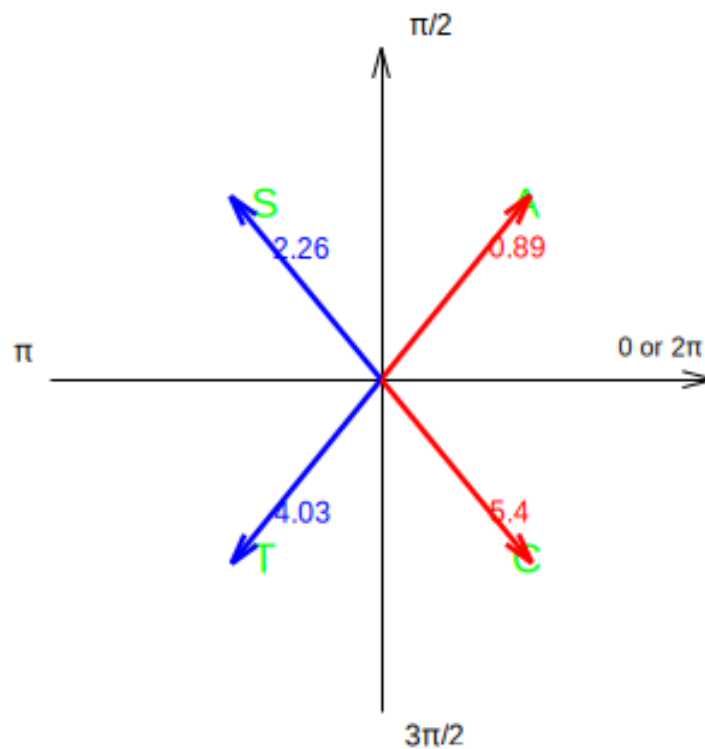
$$\tan x = 1.2 \rightarrow x_1 = \tan^{-1}(1.2) \approx 0.88, x_2 = \pi + 0.88 \approx 4.02$$

$$\tan x = -1.2 \rightarrow x_3 = \pi - 0.88 \approx 2.27, x_4 = 2\pi - 0.88 \approx 5.41$$

d) $\sec^2 x - 2.5 = 0$

$$\sec^2 x = 2.5 \rightarrow \cos^2 x = \frac{1}{2.5}$$

$$\cos x = \pm \sqrt{\frac{1}{2.5}} \approx \pm 0.63$$



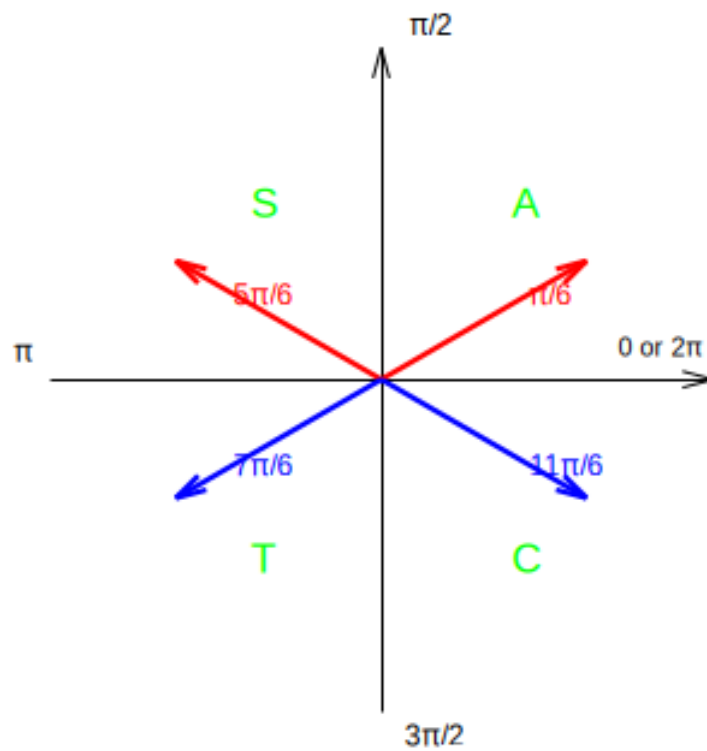
$$\cos x = 0.63 \rightarrow x_1 = \cos^{-1}(0.63) \approx 0.89, x_2 = 2\pi - 0.89 \approx 5.4$$

$$\cos x = -0.63 \rightarrow x_3 = \cos^{-1}(-0.63) \approx 2.26, x_4 = \pi + 0.89 \approx 4.03$$

4) Determine exact solutions for each equation in the interval $0 \leq x \leq 2\pi$.

a) $\sin^2 x - \frac{1}{4} = 0$

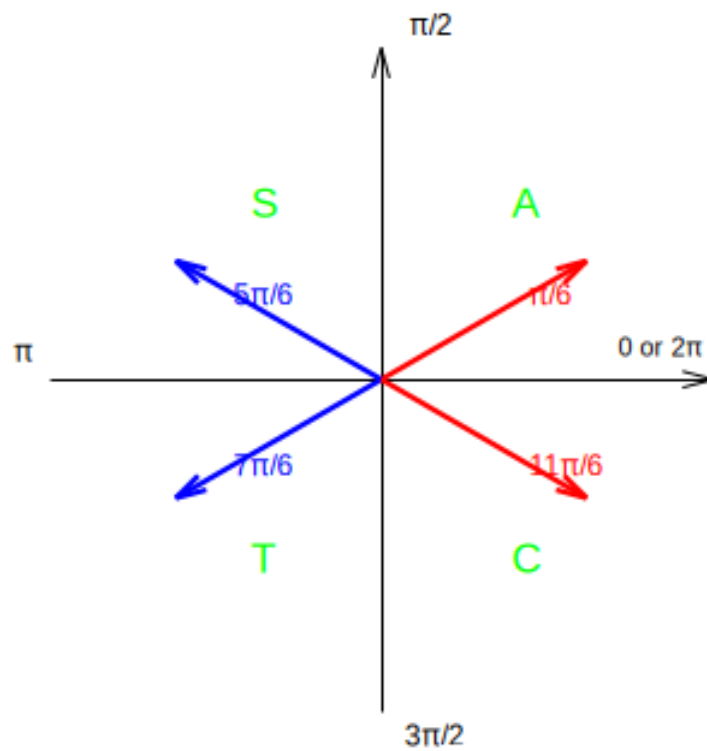
$\sin^2 x = \frac{1}{4} \rightarrow \sin x = \pm \frac{1}{2}$. From special triangle: $\sin \frac{\pi}{6} = \frac{1}{2}$. Place $\frac{\pi}{6}$ in Q1 and Q2 for $\sin x = \frac{1}{2}$, and in Q3 and Q4 for $\sin x = -\frac{1}{2}$.



$$x_1 = \frac{\pi}{6}, x_2 = \frac{5\pi}{6}, x_3 = \pi + \frac{\pi}{6} = \frac{7\pi}{6}, x_4 = 2\pi - \frac{\pi}{6} = \frac{11\pi}{6}$$

b) $\cos^2 x - \frac{3}{4} = 0$

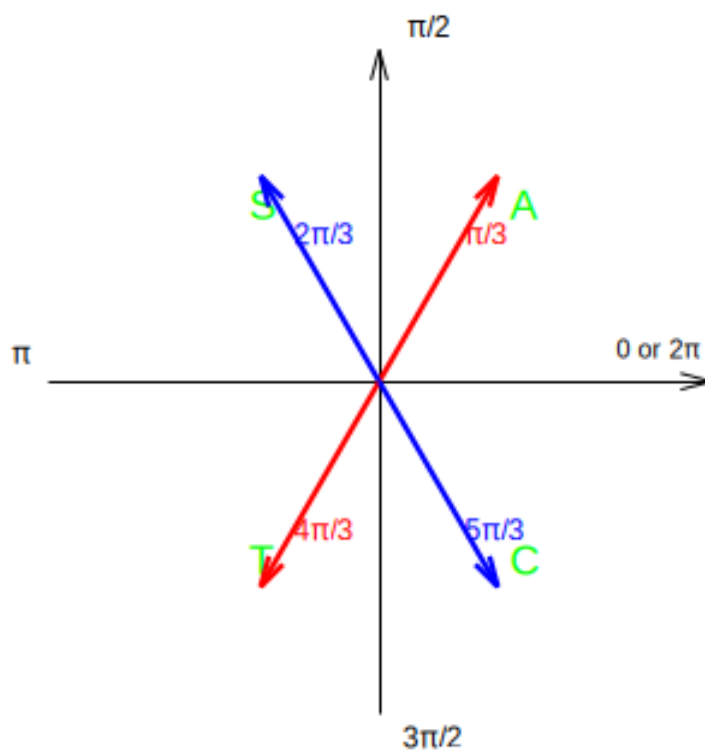
$\cos^2 x = \frac{3}{4} \rightarrow \cos x = \pm \frac{\sqrt{3}}{2}$. From special triangle: $\cos \frac{\pi}{6} = \frac{\sqrt{3}}{2}$. Place $\frac{\pi}{6}$ in Q1 and Q4 for $\cos x = \frac{\sqrt{3}}{2}$, and in Q2 and Q3 for $\cos x = -\frac{\sqrt{3}}{2}$.



$$x_1 = \frac{\pi}{6}, x_2 = \pi - \frac{\pi}{6} = \frac{5\pi}{6}, x_3 = \pi + \frac{\pi}{6} = \frac{7\pi}{6}, x_4 = 2\pi - \frac{\pi}{6} = \frac{11\pi}{6}$$

c) $\tan^2 x - 3 = 0$

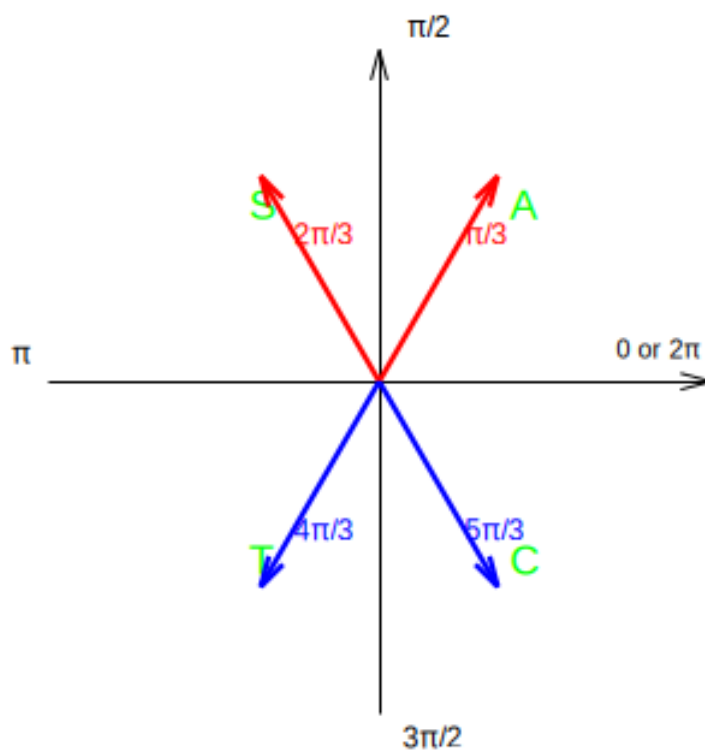
$\tan^2 x = 3 \rightarrow \tan x = \pm \sqrt{3}$. From special triangle: $\tan \frac{\pi}{3} = \sqrt{3}$. Place $\frac{\pi}{3}$ in Q1 and Q3 for $\tan x = \sqrt{3}$, and in Q2 and Q4 for $\tan x = -\sqrt{3}$.



$$x_1 = \frac{\pi}{3}, x_2 = \pi - \frac{\pi}{3} = \frac{2\pi}{3}, x_3 = \pi + \frac{\pi}{3} = \frac{4\pi}{3}, x_4 = 2\pi - \frac{\pi}{3} = \frac{5\pi}{3}$$

d) $3\csc^2 x - 4 = 0$

$\csc^2 x = \frac{4}{3} \rightarrow \sin^2 x = \frac{3}{4} \rightarrow \sin x = \pm \frac{\sqrt{3}}{2}$. From special triangle: $\sin \frac{\pi}{3} = \frac{\sqrt{3}}{2}$. Place $\frac{\pi}{3}$ in Q1 and Q2 for $\sin x = \frac{\sqrt{3}}{2}$, and in Q3 and Q4 for $\sin x = -\frac{\sqrt{3}}{2}$.

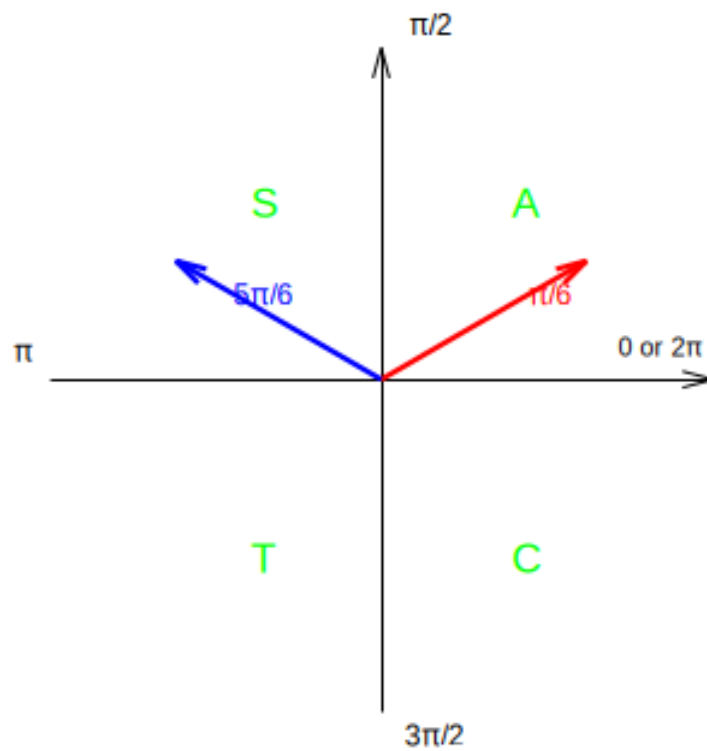


$$x_1 = \frac{\pi}{3}, x_2 = \pi - \frac{\pi}{3} = \frac{2\pi}{3}, x_3 = \pi + \frac{\pi}{3} = \frac{4\pi}{3}, x_4 = 2\pi - \frac{\pi}{3} = \frac{5\pi}{3}$$

5) Determine solutions for each equation in the interval $0 \leq x \leq 2\pi$.

a) $3 \sin x = \sin x + 1$

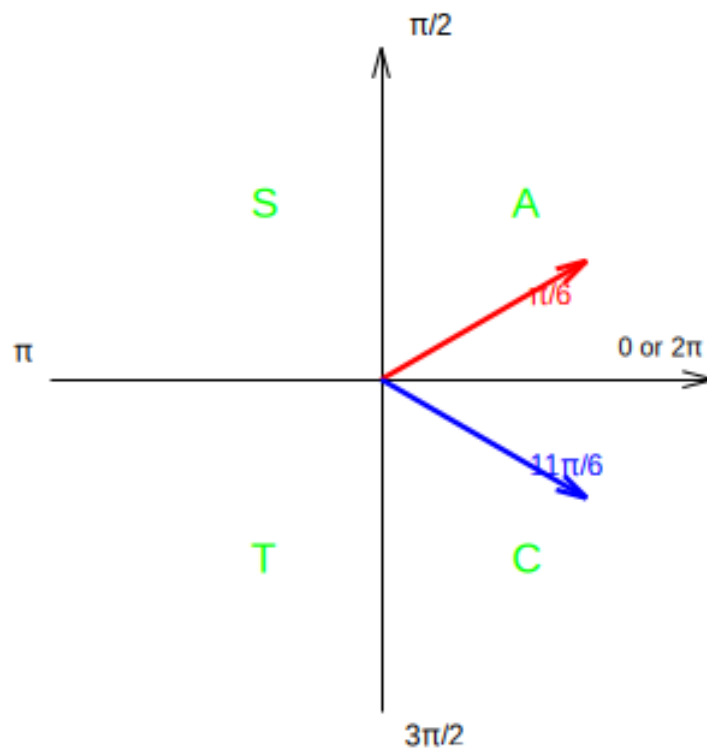
$2 \sin x = 1 \rightarrow \sin x = \frac{1}{2}$. From special triangle: $\sin \frac{\pi}{6} = \frac{1}{2}$. Place in Q1 and Q2.



$$x_1 = \frac{\pi}{6}, x_2 = \pi - \frac{\pi}{6} = \frac{5\pi}{6}$$

b) $5 \cos x - \sqrt{3} = 3 \cos x$

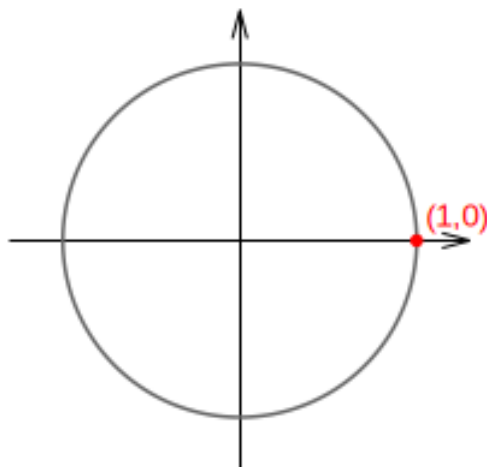
$$2 \cos x = \sqrt{3} \rightarrow \cos x = \frac{\sqrt{3}}{2}. \text{ From special triangle: } \cos \frac{\pi}{6} = \frac{\sqrt{3}}{2}. \text{ Place in Q1 and Q4.}$$



$$x_1 = \frac{\pi}{6}, x_2 = 2\pi - \frac{\pi}{6} = \frac{11\pi}{6}$$

c) $7\sec x = 7$

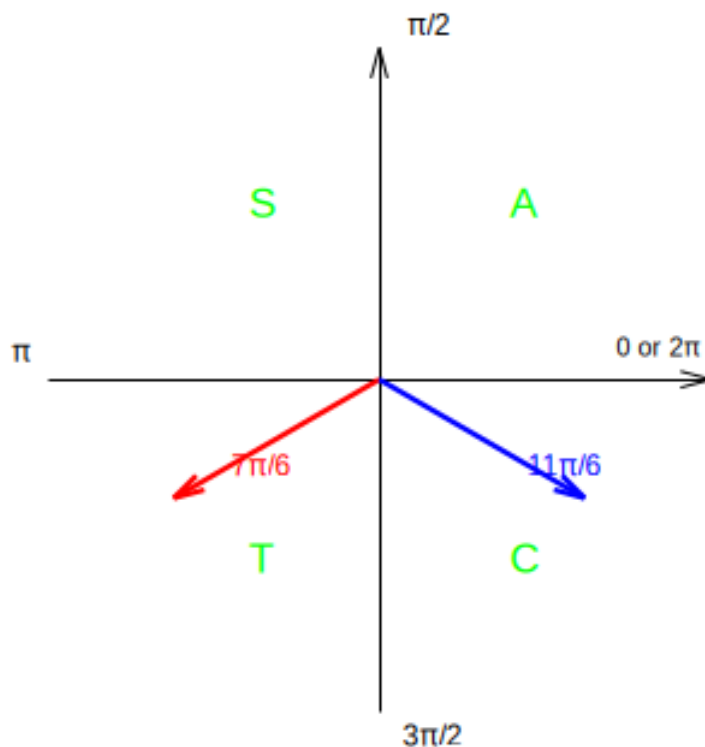
$\sec x = 1 \rightarrow \cos x = 1$. Use the unit circle, where each point is $(\cos x, \sin x)$.



$$x_1 = 0, x_2 = 2\pi$$

d) $2\csc x + 17 = 15 + \csc x$

$\csc x = -2 \rightarrow \sin x = -\frac{1}{2}$. From special triangle: $\sin \frac{\pi}{6} = \frac{1}{2}$. Place $\frac{\pi}{6}$ in Q3 and Q4.



$$x_1 = \pi + \frac{\pi}{6} = \frac{7\pi}{6}, \quad x_2 = 2\pi - \frac{\pi}{6} = \frac{11\pi}{6}$$

Answer Key

1) a) 0.25, 2.89 b) 2.42, 3.86 c) 1.37, 4.51 d) 1.32, 4.97 e) 2.16, 5.3 f) 3.55, 5.87

2) a) $\frac{4\pi}{3}, \frac{5\pi}{3}$ b) $\frac{\pi}{3}, \frac{5\pi}{3}$ c) $\frac{\pi}{4}, \frac{5\pi}{4}$ d) $\frac{3\pi}{4}, \frac{7\pi}{4}$

3) a) 0.93, 2.21, 4.07, 5.36 b) 0.84, 2.3, 3.98, 5.44 c) 0.88, 2.27, 4.02, 5.41 d) 0.89, 2.26, 4.03, 5.4

4) a) $\frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6}$ b) $\frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6}$ c) $\frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{3}$ d) $\frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{3}$

5) a) $\frac{\pi}{6}, \frac{5\pi}{6}$ b) $\frac{\pi}{6}, \frac{11\pi}{6}$ c) 0 or 2π d) 3.67 or 5.76