

5.4 Prouver des identités trigonométriques – Feuille de travail avec solutions

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Le contenu du cours provient de www.jensenmath.ca

Prouver chaque identité en utilisant l'espace des pages suivantes.

a) $\sin(x + y) = \sin x \cos y + \cos x \sin y$

MG	MD
$= \sin(x + y)$ $= \sin[x - (-y)]$ $= \sin x \cos(-y) - \cos x \sin(-y)$ $= \sin x \cos y - \cos x(-\sin y)$ $= \sin x \cos y + \cos x \sin y$	$= \sin x \cos y + \cos x \sin y$

MG = MD

b) $\tan(x - y) = \frac{\tan x - \tan y}{1 + \tan x \tan y}$

MG	MD
$= \tan(x - y)$ $= \tan[x + (-y)]$ $= \frac{\tan x + \tan(-y)}{1 - \tan x \tan(-y)}$ $= \frac{\tan x - \tan y}{1 + \tan x \tan y}$	$= \frac{\tan x - \tan y}{1 + \tan x \tan y}$

MG = MD

c) $\sin(2x) = 2 \sin x \cos x$

MG	MD
$= \sin(2x)$ $= \sin(x + x)$ $= \sin x \cos x + \cos x \sin x$ $= 2 \sin x \cos x$	$= 2 \sin x \cos x$

MG = MD

d) $\cos(2x) = \cos^2 x - \sin^2 x$

MG	MD
$= \cos(2x)$ $= \cos(x + x)$ $= \cos x \cos x - \sin x \sin x$ $= \cos^2 x - \sin^2 x$	$= \cos^2 x - \sin^2 x$

MG = MD

e) $\cot \theta - \tan \theta = 2 \cot(2\theta)$

MG	MD
$= \frac{\cos \theta}{\sin \theta} - \frac{\sin \theta}{\cos \theta}$ $= \frac{\cos^2 \theta - \sin^2 \theta}{\sin \theta \cos \theta}$	$= 2 \cot(2\theta)$ $= \frac{2 \cos(2\theta)}{\sin(2\theta)}$ $= \frac{2 \cos^2 \theta - 2 \sin^2 \theta}{2 \sin \theta \cos \theta}$ $= \frac{\cos^2 \theta - \sin^2 \theta}{\sin \theta \cos \theta}$

MG = MD

f) $\frac{\sin(2\theta)}{1 - \cos(2\theta)} = \cot \theta$

MG	MD
$= \frac{2 \sin \theta \cos \theta}{1 - (1 - 2 \sin^2 \theta)}$ $= \frac{2 \sin \theta \cos \theta}{2 \sin^2 \theta}$ $= \frac{\cos \theta}{\sin \theta}$	$= \cot \theta$ $= \frac{\cos \theta}{\sin \theta}$

MG = MD

g) $\sin x \sec x = \tan x$

MG	MD
$= \sin x \left(\frac{1}{\cos x} \right)$ $= \frac{\sin x}{\cos x}$	$= \tan x$ $= \frac{\sin x}{\cos x}$

MG = MD

h) $\frac{1 - \sin x}{\cos x} = \frac{\cos x}{1 + \sin x}$

MG	MD
$= \frac{1 - \sin x}{\cos x}$	$= \frac{\cos x}{1 + \sin x} \times \frac{1 - \sin x}{1 - \sin x}$ $= \frac{\cos x(1 - \sin x)}{1 - \sin^2 x}$ $= \frac{\cos x(1 - \sin x)}{\cos^2 x}$ $= \frac{1 - \sin x}{\cos x}$

MG = MD

i) $\frac{\sec \theta - 1}{1 - \cos \theta} = \sec \theta$

MG	MD
$= \frac{\frac{1}{\cos \theta} - \frac{\cos \theta}{\cos \theta}}{1 - \cos \theta}$ $= \frac{\left(\frac{1 - \cos \theta}{\cos \theta} \right)}{\left(\frac{1 - \cos \theta}{1} \right)}$ $= \left(\frac{1 - \cos \theta}{\cos \theta} \right) \left(\frac{1}{1 - \cos \theta} \right)$ $= \frac{1}{\cos \theta}$	$= \sec \theta$ $= \frac{1}{\cos \theta}$

MG = MD

$$j) \frac{\sin x - \cos x}{\cos x} + \frac{\sin x + \cos x}{\sin x} = \sec x \csc x$$

MG	MD
$= \frac{\sin x(\sin x - \cos x) + \cos x(\sin x + \cos x)}{\sin x \cos x}$ $= \frac{\sin^2 x - \sin x \cos x + \cos x \sin x + \cos^2 x}{\sin x \cos x}$ $= \frac{\sin^2 x + \cos^2 x}{\sin x \cos x}$ $= \frac{1}{\sin x \cos x}$	$= \sec x \csc x$ $= \left(\frac{1}{\cos x} \right) \left(\frac{1}{\sin x} \right)$ $= \frac{1}{\sin x \cos x}$

MG = MD

$$k) \frac{1 - \sin^2 x \cos^2 x}{\cos^4 x} = \tan^4 x + \tan^2 x + 1$$

MG	MD
$= \frac{1 - \sin^2 x \cos^2 x}{\cos^4 x}$	$= \frac{\sin^4 x}{\cos^4 x} + \frac{\sin^2 x}{\cos^2 x} + 1$ $= \frac{\sin^4 x + \sin^2 x \cos^2 x + \cos^4 x}{\cos^4 x}$ $= \frac{\sin^2 x(1 - \cos^2 x) + \sin^2 x \cos^2 x + \cos^2 x(1 - \sin^2 x)}{\cos^4 x}$ $= \frac{\sin^2 x - \sin^2 x \cos^2 x + \sin^2 x \cos^2 x + \cos^2 x - \sin^2 x \cos^2 x}{\cos^4 x}$ $= \frac{\sin^2 x + \cos^2 x - \sin^2 x \cos^2 x}{\cos^4 x}$ $= \frac{1 - \sin^2 x \cos^2 x}{\cos^4 x}$

MG = MD

$$l) \frac{\cos(2x) + 1}{\sin(2x)} = \cot x$$

MG	MD
$= \frac{2 \cos^2 x - 1 + 1}{2 \sin x \cos x}$ $= \frac{2 \cos^2 x}{2 \sin x \cos x}$ $= \frac{\cos x}{\sin x}$	$= \cot x$ $= \frac{\cos x}{\sin x}$

MG = MD

$$m) \cot \theta - \tan \theta = 2 \cot(2\theta)$$

MG	MD
$= \frac{\cos \theta}{\sin \theta} - \frac{\sin \theta}{\cos \theta}$ $= \frac{\cos^2 \theta - \sin^2 \theta}{\sin \theta \cos \theta}$	$= 2 \cot(2\theta)$ $= \frac{2 \cos(2\theta)}{\sin(2\theta)}$ $= \frac{\cos^2 \theta - \sin^2 \theta}{\sin \theta \cos \theta}$

MG = MD

n) $(\sin x + \cos x)^2 = 1 + \sin(2x)$

MG	MD
$= (\sin x + \cos x)(\sin x + \cos x)$ $= \sin^2 x + \sin x \cos x + \cos x \sin x + \cos^2 x$ $= \sin^2 x + \cos^2 x + 2 \sin x \cos x$ $= 1 + 2 \sin x \cos x$	$= 1 + \sin(2x)$ $= 1 + 2 \sin x \cos x$

MG = MD

o) $\frac{2 \tan x}{1 + \tan^2 x} = \sin(2x)$

MG	MD
$= \frac{2 \tan x}{\sec^2 x}$ $= 2 \left(\frac{\sin x}{\cos x} \right) \cos^2 x$ $= 2 \sin x \cos x$	$= \sin(2x)$ $= 2 \sin x \cos x$

MG = MD

p) $\sin\left(\frac{\pi}{4} + x\right) + \sin\left(\frac{\pi}{4} - x\right) = \sqrt{2} \cos x$

MG	MD
$= \sin \frac{\pi}{4} \cos x + \cos \frac{\pi}{4} \sin x + \sin \frac{\pi}{4} \cos x - \cos \frac{\pi}{4} \sin x$ $= 2 \sin \frac{\pi}{4} \cos x$ $= 2 \left(\frac{\sqrt{2}}{2} \right) \cos x$ $= \sqrt{2} \cos x$	$= \sqrt{2} \cos x$

MG = MD

q) $\cos^4 x - \sin^4 x = \cos(2x)$

MG	MD
$= (\cos^2 x)^2 - (\sin^2 x)^2$ $= (\cos^2 x - \sin^2 x)(\cos^2 x + \sin^2 x)$ $= (\cos^2 x - \sin^2 x)(1)$ $= \cos^2 x - \sin^2 x$	$= \cos(2x)$ $= \cos^2 x - \sin^2 x$

MG = MD

r) $\csc(2x) + \cot(2x) = \cot x$

MG	MD
$= \frac{1}{\sin(2x)} + \frac{\cos(2x)}{\sin(2x)}$ $= \frac{1 + \cos(2x)}{\sin(2x)}$ $= \frac{1 + 2 \cos^2 x - 1}{2 \sin x \cos x}$ $= \frac{2 \cos^2 x}{2 \sin x \cos x}$ $= \frac{\cos x}{\sin x}$	$= \cot x$ $= \frac{\cos x}{\sin x}$

MG = MD

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s) $\cos(2x) = 2\cos^2 x - 1$

MG	MD
$= \cos(x + x)$ $= \cos x \cos x - \sin x \sin x$ $= \cos^2 x - \sin^2 x$ $= \cos^2 x - (1 - \cos^2 x)$ $= 2\cos^2 x - 1$	$= 2\cos^2 x - 1$

MG = MD

t) $\sin\left(\frac{3\pi}{2} - x\right) = -\cos x$

MG	MD
$= \sin \frac{3\pi}{2} \cos x - \cos \frac{3\pi}{2} \sin x$ $= -1 \cos x - 0 \sin x$ $= -\cos x$	$= -\cos x$

MG = MD

u) $\frac{\cos(2x) + 1}{\sin(2x)} = \cot x$

MG	MD
$= \frac{2\cos^2 x - 1 + 1}{2\sin x \cos x}$ $= \frac{2\cos^2 x}{2\sin x \cos x}$ $= \frac{\cos x}{\sin x}$	$= \cot x$ $= \frac{\cos x}{\sin x}$

MG = MD

v) $\cot x + \tan x = 2\csc(2x)$

MG	MD
$= \frac{\cos x}{\sin x} + \frac{\sin x}{\cos x}$ $= \frac{\cos^2 x + \sin^2 x}{\sin x \cos x}$ $= \frac{1}{\sin x \cos x}$	$= 2\csc(2x)$ $= \frac{2}{\sin(2x)}$ $= \frac{2}{2\sin x \cos x}$ $= \frac{1}{\sin x \cos x}$

MG = MD