

5.3 Double Angle Formulas Worksheet with solutions

This document was composed and formatted by Annie Bernatchez.

The course material originated from www.jensenmath.ca

1) Express each of the following as a single trig ratio.

a) $2 \sin(5x) \cos(5x)$ $= \sin[2(5x)]$ $= \sin(10x)$	b) $\cos^2 \theta - \sin^2 \theta$ $= \cos(2\theta)$	c) $1 - 2 \sin^2(3x)$ $= \cos[2(3x)]$ $= \cos(6x)$
d) $\frac{2 \tan(4x)}{1 - \tan^2(4x)}$ $= \tan[2(4x)]$ $= \tan(8x)$	e) $4 \sin \theta \cos \theta$ $= 2 \times 2 \sin \theta \cos \theta$ $= 2 \sin(2\theta)$	f) $2 \cos^2 \frac{\theta}{2} - 1$ $= \cos \left[2 \left(\frac{\theta}{2} \right) \right]$ $= \cos \theta$

2) Express each of the following as a single trig ratio and then evaluate

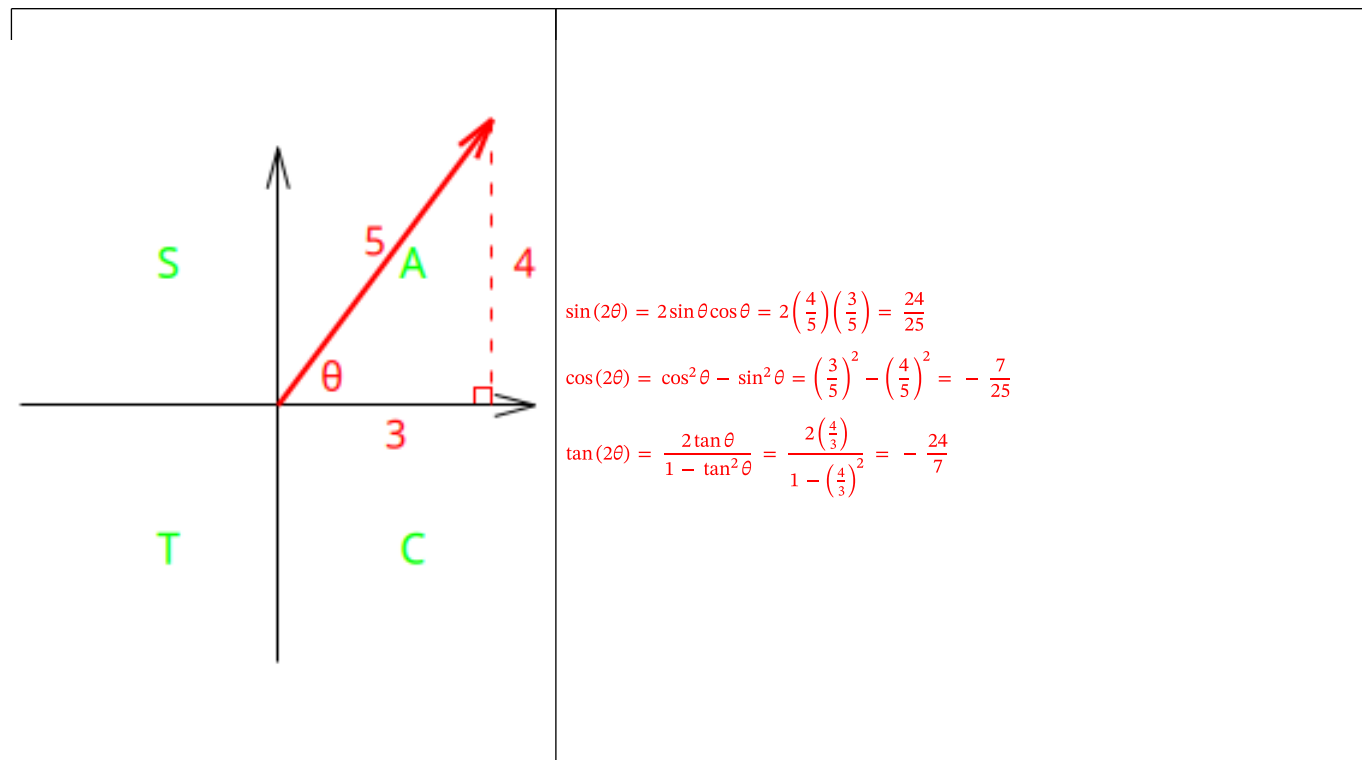
a) $2 \sin 45^\circ \cos 45^\circ$ $= \sin[2(45^\circ)]$ $= \sin(90^\circ)$ $= 1$	b) $\cos^2 30^\circ - \sin^2 30^\circ$ $= \cos[2(30^\circ)]$ $= \cos(60^\circ)$ $= \frac{1}{2}$	c) $2 \sin \frac{\pi}{12} \cos \frac{\pi}{12}$ $= \sin \left[2 \left(\frac{\pi}{12} \right) \right]$ $= \sin \frac{\pi}{6}$ $= \frac{1}{2}$
d) $\cos^2 \frac{\pi}{12} - \sin^2 \frac{\pi}{12}$ $= \cos \left[2 \left(\frac{\pi}{12} \right) \right]$ $= \cos \frac{\pi}{6}$ $= \frac{\sqrt{3}}{2}$	e) $1 - 2 \sin^2 \frac{3\pi}{8}$ $= \cos \left[2 \left(\frac{3\pi}{8} \right) \right]$ $= \cos \frac{3\pi}{4}$ $= -\frac{1}{\sqrt{2}}$	f) $2 \tan 60^\circ \cos^2 60^\circ$ $= 2 \left(\frac{\sin 60^\circ}{\cos 60^\circ} \right) \cos^2 60^\circ$ $= 2 \sin 60^\circ \cos 60^\circ = \sin[2(60^\circ)]$ $= \sin(120^\circ) = \frac{\sqrt{3}}{2}$

3) Use a double angle formula to rewrite each trig ratio

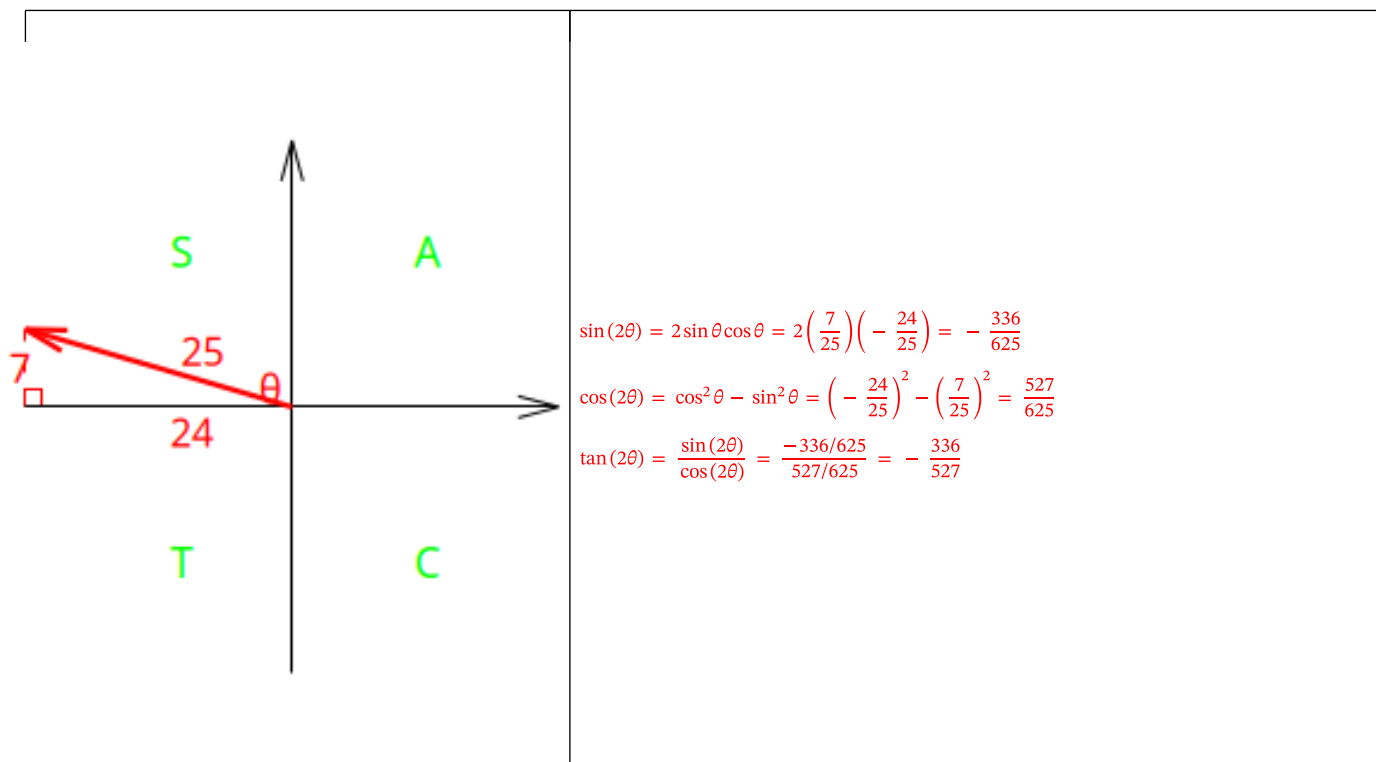
a) $\sin(4\theta)$ $= \sin[2(2\theta)]$ $= 2 \sin(2\theta) \cos(2\theta)$	b) $\cos(3x)$ $= \cos \left[2 \left(\frac{3}{2}x \right) \right]$ $= \cos^2 \left(\frac{3}{2}x \right) - \sin^2 \left(\frac{3}{2}x \right)$	c) $\tan x$ $= \tan \left[2 \left(\frac{1}{2}x \right) \right]$ $= \frac{2 \tan(\frac{1}{2}x)}{1 - \tan^2(\frac{1}{2}x)}$
d) $\cos(6\theta)$ $= \cos[2(3\theta)]$ $= \cos^2(3\theta) - \sin^2(3\theta)$	e) $\sin x$ $= \sin \left[2 \left(\frac{1}{2}x \right) \right]$ $= 2 \sin \left(\frac{1}{2}x \right) \cos \left(\frac{1}{2}x \right)$	f) $\tan(5\theta)$ $= \tan \left[2 \left(\frac{5}{2}\theta \right) \right]$ $= \frac{2 \tan(\frac{5}{2}\theta)}{1 - \tan^2(\frac{5}{2}\theta)}$

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4) Determine the values of $\sin 2\theta$, $\cos 2\theta$, and $\tan 2\theta$, given $\cos \theta = \frac{3}{5}$ and $0 \leq \theta \leq \frac{\pi}{2}$



5) Determine the values of $\sin 2\theta$, $\cos 2\theta$, and $\tan 2\theta$, given $\tan \theta = -\frac{7}{24}$ and $\frac{\pi}{2} \leq \theta \leq \pi$



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6) Determine the values of $\sin 2\theta$, $\cos 2\theta$, and $\tan 2\theta$, given $\sin \theta = -\frac{12}{13}$ and $\frac{3\pi}{2} \leq \theta \leq 2\pi$

	$\sin(2\theta) = 2\sin\theta\cos\theta = 2\left(-\frac{12}{13}\right)\left(\frac{5}{13}\right) = -\frac{120}{169}$ $\cos(2\theta) = \cos^2\theta - \sin^2\theta = \left(\frac{5}{13}\right)^2 - \left(-\frac{12}{13}\right)^2 = -\frac{119}{169}$ $\tan(2\theta) = \frac{\sin(2\theta)}{\cos(2\theta)} = \frac{-120/169}{-119/169} = \frac{120}{119}$
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7) Determine the values of $\sin 2\theta$, $\cos 2\theta$, and $\tan 2\theta$, given $\cos \theta = -\frac{4}{5}$ and $\frac{\pi}{2} \leq \theta \leq \pi$

	$\sin(2\theta) = 2\sin\theta\cos\theta = 2\left(\frac{3}{5}\right)\left(-\frac{4}{5}\right) = -\frac{24}{25}$ $\cos(2\theta) = \cos^2\theta - \sin^2\theta = \left(-\frac{4}{5}\right)^2 - \left(\frac{3}{5}\right)^2 = \frac{7}{25}$ $\tan(2\theta) = \frac{\sin(2\theta)}{\cos(2\theta)} = \frac{-24/25}{7/25} = -\frac{24}{7}$
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8) Determine the value of a in the equation $2\tan x - \tan(2x) + 2a = 1 - \tan(2x)\tan^2 x$

$$2\tan x + 2a - 1 = \tan(2x) - \tan(2x)\tan^2 x$$

$$2\tan x + 2a - 1 = \tan(2x)(1 - \tan^2 x)$$

$$2\tan x + 2a - 1 = \frac{2\tan x}{1 - \tan^2 x}(1 - \tan^2 x)$$

$$2\tan x + 2a - 1 = 2\tan x$$

$$2a - 1 = 0$$

$$a = \frac{1}{2}$$

Answer Key

1) a) $\sin(10x)$ b) $\cos(2\theta)$ c) $\cos(6x)$ d) $\tan(8x)$ e) $2\sin(2\theta)$ f) $\cos\theta$

2) a) $\sin 90^\circ; 1$ b) $\cos 60^\circ; \frac{1}{2}$ c) $\sin \frac{\pi}{6}; \frac{1}{2}$ d) $\cos \frac{\pi}{6}; \frac{\sqrt{3}}{2}$ e) $\cos \frac{3\pi}{4}; -\frac{1}{\sqrt{2}}$ f) $\sin 120^\circ; \frac{\sqrt{3}}{2}$

3) a) $2\sin(2\theta)\cos(2\theta)$ b) $2\cos^2(1.5x) - 1$ c) $\frac{2\tan(0.5x)}{1 - \tan^2(0.5x)}$ d) $\cos^2(3\theta) - \sin^2(3\theta)$ e) $2\sin(0.5x)\cos(0.5x)$ f) $\frac{2\tan(2.5\theta)}{1 - \tan^2(2.5\theta)}$

4) $\sin(2\theta) = \frac{24}{25}$, $\cos(2\theta) = -\frac{7}{25}$, $\tan(2\theta) = -\frac{24}{7}$

5) $\sin(2\theta) = -\frac{336}{625}$, $\cos(2\theta) = \frac{527}{625}$, $\tan(2\theta) = -\frac{336}{527}$

6) $\sin(2\theta) = -\frac{120}{169}$, $\cos(2\theta) = -\frac{119}{169}$, $\tan(2\theta) = \frac{120}{119}$

7) $\sin(2\theta) = -\frac{24}{25}$, $\cos(2\theta) = \frac{7}{25}$, $\tan(2\theta) = -\frac{24}{7}$

8) $a = \frac{1}{2}$