

5.2 Compound Angle Formulas Worksheet with solutions

This document was composed and formatted by Annie Bernatchez.

The course material originated from www.jensenmath.ca

1) Use an appropriate compound angle formula to express as a single trig function, and then determine an exact value for each

a) $\sin \frac{\pi}{4} \cos \frac{\pi}{12} + \cos \frac{\pi}{4} \sin \frac{\pi}{12}$ $= \sin \left(\frac{\pi}{4} + \frac{\pi}{12} \right)$ $= \sin \left(\frac{3\pi}{12} + \frac{\pi}{12} \right)$ $= \sin \frac{4\pi}{12} = \sin \frac{\pi}{3}$ $= \frac{\sqrt{3}}{2}$	b) $\sin \frac{\pi}{4} \cos \frac{\pi}{12} - \cos \frac{\pi}{4} \sin \frac{\pi}{12}$ $= \sin \left(\frac{\pi}{4} - \frac{\pi}{12} \right)$ $= \sin \left(\frac{3\pi}{12} - \frac{\pi}{12} \right)$ $= \sin \frac{2\pi}{12} = \sin \frac{\pi}{6}$ $= \frac{1}{2}$
c) $\cos \frac{\pi}{4} \cos \frac{\pi}{12} - \sin \frac{\pi}{4} \sin \frac{\pi}{12}$ $= \cos \left(\frac{\pi}{4} + \frac{\pi}{12} \right)$ $= \cos \left(\frac{3\pi}{12} + \frac{\pi}{12} \right)$ $= \cos \frac{4\pi}{12} = \cos \frac{\pi}{3}$ $= \frac{1}{2}$	d) $\cos \frac{\pi}{4} \cos \frac{\pi}{12} + \sin \frac{\pi}{4} \sin \frac{\pi}{12}$ $= \cos \left(\frac{\pi}{4} - \frac{\pi}{12} \right)$ $= \cos \left(\frac{3\pi}{12} - \frac{\pi}{12} \right)$ $= \cos \frac{2\pi}{12} = \cos \frac{\pi}{6}$ $= \frac{\sqrt{3}}{2}$
e) $\cos \frac{2\pi}{9} \cos \frac{5\pi}{18} - \sin \frac{2\pi}{9} \sin \frac{5\pi}{18}$ $= \cos \left(\frac{2\pi}{9} + \frac{5\pi}{18} \right)$ $= \cos \left(\frac{4\pi}{18} + \frac{5\pi}{18} \right)$ $= \cos \frac{9\pi}{18} = \cos \frac{\pi}{2}$ $= 0$	f) $\cos \frac{10\pi}{9} \cos \frac{5\pi}{18} + \sin \frac{10\pi}{9} \sin \frac{5\pi}{18}$ $= \cos \left(\frac{10\pi}{9} - \frac{5\pi}{18} \right)$ $= \cos \left(\frac{20\pi}{18} - \frac{5\pi}{18} \right)$ $= \cos \frac{15\pi}{18} = \cos \frac{5\pi}{6}$ $= -\frac{\sqrt{3}}{2}$

3) Apply a compound angle formula, and then determine an exact value for each.

a) $\sin \left(\frac{\pi}{3} + \frac{\pi}{4} \right)$ $= \sin \frac{\pi}{3} \cos \frac{\pi}{4} + \cos \frac{\pi}{3} \sin \frac{\pi}{4}$ $= \left(\frac{\sqrt{3}}{2} \right) \left(\frac{\sqrt{2}}{2} \right) + \left(\frac{1}{2} \right) \left(\frac{\sqrt{2}}{2} \right)$ $= \frac{\sqrt{6}}{4} + \frac{\sqrt{2}}{4}$ $= \frac{\sqrt{6} + \sqrt{2}}{4}$	b) $\cos \left(\frac{\pi}{3} + \frac{\pi}{4} \right)$ $= \cos \frac{\pi}{3} \cos \frac{\pi}{4} - \sin \frac{\pi}{3} \sin \frac{\pi}{4}$ $= \left(\frac{1}{2} \right) \left(\frac{\sqrt{2}}{2} \right) - \left(\frac{\sqrt{3}}{2} \right) \left(\frac{\sqrt{2}}{2} \right)$ $= \frac{\sqrt{2}}{4} - \frac{\sqrt{6}}{4}$ $= \frac{\sqrt{2} - \sqrt{6}}{4}$
c) $\cos \left(\frac{2\pi}{3} - \frac{\pi}{4} \right)$ $= \cos \frac{2\pi}{3} \cos \frac{\pi}{4} + \sin \frac{2\pi}{3} \sin \frac{\pi}{4}$ $= \left(-\frac{1}{2} \right) \left(\frac{\sqrt{2}}{2} \right) + \left(\frac{\sqrt{3}}{2} \right) \left(\frac{\sqrt{2}}{2} \right)$ $= -\frac{\sqrt{2}}{4} + \frac{\sqrt{6}}{4}$ $= \frac{\sqrt{6} - \sqrt{2}}{4}$	d) $\sin \left(\frac{2\pi}{3} - \frac{\pi}{4} \right)$ $= \sin \frac{2\pi}{3} \cos \frac{\pi}{4} - \cos \frac{2\pi}{3} \sin \frac{\pi}{4}$ $= \left(\frac{\sqrt{3}}{2} \right) \left(\frac{\sqrt{2}}{2} \right) - \left(-\frac{1}{2} \right) \left(\frac{\sqrt{2}}{2} \right)$ $= \frac{\sqrt{6}}{4} + \frac{\sqrt{2}}{4}$ $= \frac{\sqrt{6} + \sqrt{2}}{4}$

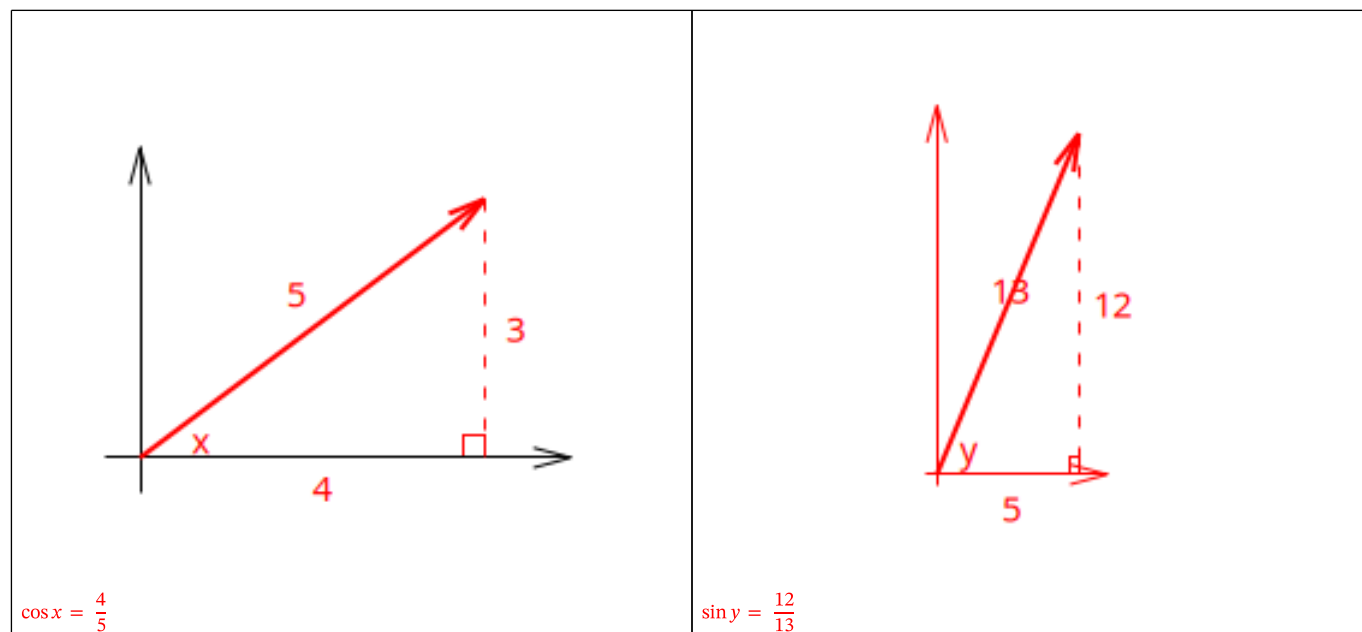
e) $\tan\left(\frac{\pi}{4} + \pi\right)$ $= \frac{\tan \frac{\pi}{4} + \tan \pi}{1 - \tan \frac{\pi}{4} \tan \pi}$ $= \frac{1 + 0}{1 - 1(0)}$ $= 1$	f) $\tan\left(\frac{\pi}{3} - \frac{\pi}{6}\right)$ $= \frac{\tan \frac{\pi}{3} - \tan \frac{\pi}{6}}{1 + \tan \frac{\pi}{3} \tan \frac{\pi}{6}}$ $= \frac{\sqrt{3} - \frac{1}{\sqrt{3}}}{1 + \sqrt{3}\left(\frac{1}{\sqrt{3}}\right)}$ $= \frac{\frac{3}{\sqrt{3}} - \frac{1}{\sqrt{3}}}{2}$ $= \frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{3}$
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4) Use an appropriate compound angle formula to determine an exact value for each.

a) $\sin \frac{7\pi}{12}$ $= \sin\left(\frac{3\pi}{12} + \frac{4\pi}{12}\right) = \sin\left(\frac{\pi}{4} + \frac{\pi}{3}\right)$ $= \sin \frac{\pi}{4} \cos \frac{\pi}{3} + \cos \frac{\pi}{4} \sin \frac{\pi}{3}$ $= \left(\frac{\sqrt{2}}{2}\right)\left(\frac{1}{2}\right) + \left(\frac{\sqrt{2}}{2}\right)\left(\frac{\sqrt{3}}{2}\right)$ $= \frac{\sqrt{2}}{4} + \frac{\sqrt{6}}{4} = \frac{\sqrt{2} + \sqrt{6}}{4}$	b) $\sin \frac{5\pi}{12}$ $= \sin\left(\frac{2\pi}{12} + \frac{3\pi}{12}\right) = \sin\left(\frac{\pi}{6} + \frac{\pi}{4}\right)$ $= \sin \frac{\pi}{6} \cos \frac{\pi}{4} + \cos \frac{\pi}{6} \sin \frac{\pi}{4}$ $= \left(\frac{1}{2}\right)\left(\frac{\sqrt{2}}{2}\right) + \left(\frac{\sqrt{3}}{2}\right)\left(\frac{\sqrt{2}}{2}\right)$ $= \frac{\sqrt{2}}{4} + \frac{\sqrt{6}}{4} = \frac{\sqrt{2} + \sqrt{6}}{4}$
c) $\cos \frac{11\pi}{12}$ $= \cos\left(\frac{8\pi}{12} + \frac{3\pi}{12}\right) = \cos\left(\frac{2\pi}{3} + \frac{\pi}{4}\right)$ $= \cos \frac{2\pi}{3} \cos \frac{\pi}{4} - \sin \frac{2\pi}{3} \sin \frac{\pi}{4}$ $= \left(-\frac{1}{2}\right)\left(\frac{\sqrt{2}}{2}\right) - \left(\frac{\sqrt{3}}{2}\right)\left(\frac{\sqrt{2}}{2}\right)$ $= -\frac{\sqrt{2}}{4} - \frac{\sqrt{6}}{4} = \frac{-\sqrt{2} - \sqrt{6}}{4}$	d) $\cos \frac{5\pi}{12}$ $= \cos\left(\frac{2\pi}{12} + \frac{3\pi}{12}\right) = \cos\left(\frac{\pi}{6} + \frac{\pi}{4}\right)$ $= \cos \frac{\pi}{6} \cos \frac{\pi}{4} - \sin \frac{\pi}{6} \sin \frac{\pi}{4}$ $= \left(\frac{\sqrt{3}}{2}\right)\left(\frac{\sqrt{2}}{2}\right) - \left(\frac{1}{2}\right)\left(\frac{\sqrt{2}}{2}\right)$ $= \frac{\sqrt{6}}{4} - \frac{\sqrt{2}}{4} = \frac{\sqrt{6} - \sqrt{2}}{4}$
e) $\sin \frac{13\pi}{12}$ $= \sin\left(\frac{9\pi}{12} + \frac{4\pi}{12}\right) = \sin\left(\frac{3\pi}{4} + \frac{\pi}{3}\right)$ $= \sin \frac{3\pi}{4} \cos \frac{\pi}{3} + \cos \frac{3\pi}{4} \sin \frac{\pi}{3}$ $= \left(\frac{\sqrt{2}}{2}\right)\left(\frac{1}{2}\right) + \left(-\frac{\sqrt{2}}{2}\right)\left(\frac{\sqrt{3}}{2}\right)$ $= \frac{\sqrt{2}}{4} - \frac{\sqrt{6}}{4} = \frac{\sqrt{2} - \sqrt{6}}{4}$	f) $\cos \frac{17\pi}{12}$ $= \cos\left(\frac{15\pi}{12} + \frac{2\pi}{12}\right) = \cos\left(\frac{5\pi}{4} + \frac{\pi}{6}\right)$ $= \cos \frac{5\pi}{4} \cos \frac{\pi}{6} - \sin \frac{5\pi}{4} \sin \frac{\pi}{6}$ $= \left(-\frac{\sqrt{2}}{2}\right)\left(\frac{\sqrt{3}}{2}\right) - \left(-\frac{\sqrt{2}}{2}\right)\left(\frac{1}{2}\right)$ $= -\frac{\sqrt{6}}{4} + \frac{\sqrt{2}}{4} = \frac{\sqrt{2} - \sqrt{6}}{4}$
g) $\sin \frac{19\pi}{12}$ $= \sin\left(\frac{15\pi}{12} + \frac{4\pi}{12}\right) = \sin\left(\frac{5\pi}{4} + \frac{\pi}{3}\right)$ $= \sin \frac{5\pi}{4} \cos \frac{\pi}{3} + \cos \frac{5\pi}{4} \sin \frac{\pi}{3}$ $= \left(-\frac{\sqrt{2}}{2}\right)\left(\frac{1}{2}\right) + \left(-\frac{\sqrt{2}}{2}\right)\left(\frac{\sqrt{3}}{2}\right)$ $= -\frac{\sqrt{2}}{4} - \frac{\sqrt{6}}{4} = \frac{-\sqrt{2} - \sqrt{6}}{4}$	h) $\cos \frac{23\pi}{12}$ $= \cos\left(\frac{15\pi}{12} + \frac{8\pi}{12}\right) = \cos\left(\frac{5\pi}{4} + \frac{2\pi}{3}\right)$ $= \cos \frac{5\pi}{4} \cos \frac{2\pi}{3} - \sin \frac{5\pi}{4} \sin \frac{2\pi}{3}$ $= \left(-\frac{\sqrt{2}}{2}\right)\left(-\frac{1}{2}\right) - \left(-\frac{\sqrt{2}}{2}\right)\left(\frac{\sqrt{3}}{2}\right)$ $= \frac{\sqrt{2}}{4} + \frac{\sqrt{6}}{4} = \frac{\sqrt{2} + \sqrt{6}}{4}$

5.2 Compound Angle Formulas Worksheet with solutions

5) Angles x and y are located in the first quadrant such that $\sin x = \frac{3}{5}$ and $\cos y = \frac{5}{13}$. Determine exact values for $\cos x$ and $\sin y$.



6) Refer to the previous question. Determine an exact value for each of the following.

a) $\sin(x + y)$ $= \sin x \cos y + \cos x \sin y$ $= \left(\frac{3}{5}\right)\left(\frac{5}{13}\right) + \left(\frac{4}{5}\right)\left(\frac{12}{13}\right)$ $= \frac{15}{65} + \frac{48}{65}$ $= \frac{63}{65}$	b) $\sin(x - y)$ $= \sin x \cos y - \cos x \sin y$ $= \left(\frac{3}{5}\right)\left(\frac{5}{13}\right) - \left(\frac{4}{5}\right)\left(\frac{12}{13}\right)$ $= \frac{15}{65} - \frac{48}{65}$ $= -\frac{33}{65}$
c) $\cos(x + y)$ $= \cos x \cos y - \sin x \sin y$ $= \left(\frac{4}{5}\right)\left(\frac{5}{13}\right) - \left(\frac{3}{5}\right)\left(\frac{12}{13}\right)$ $= \frac{20}{65} - \frac{36}{65}$ $= -\frac{16}{65}$	d) $\cos(x - y)$ $= \cos x \cos y + \sin x \sin y$ $= \left(\frac{4}{5}\right)\left(\frac{5}{13}\right) + \left(\frac{3}{5}\right)\left(\frac{12}{13}\right)$ $= \frac{20}{65} + \frac{36}{65}$ $= \frac{56}{65}$

7) Use a compound angle formula to show that $\cos(2x) = \cos^2 x - \sin^2 x$

LS	RS
$= \cos(2x)$ $= \cos(x + x)$ $= \cos x \cos x - \sin x \sin x$ $= \cos^2 x - \sin^2 x$	$= \cos^2 x - \sin^2 x$

LS = RS

Answer Key

1) a) $\frac{\sqrt{3}}{2}$ b) $\frac{1}{2}$ c) $\frac{1}{2}$ d) $\frac{\sqrt{3}}{2}$ e) 0 f) $-\frac{\sqrt{3}}{2}$

3) a) $\frac{\sqrt{6} + \sqrt{2}}{4}$ b) $\frac{\sqrt{2} - \sqrt{6}}{4}$ c) $\frac{\sqrt{6} - \sqrt{2}}{4}$ d) $\frac{\sqrt{6} + \sqrt{2}}{4}$ e) 1 f) $\frac{\sqrt{3}}{3}$

4) a) $\frac{\sqrt{2} + \sqrt{6}}{4}$ b) $\frac{\sqrt{2} + \sqrt{6}}{4}$ c) $\frac{-\sqrt{2} - \sqrt{6}}{4}$ d) $\frac{\sqrt{6} - \sqrt{2}}{4}$ e) $\frac{\sqrt{2} - \sqrt{6}}{4}$ f) $\frac{\sqrt{2} - \sqrt{6}}{4}$ g) $\frac{-\sqrt{2} - \sqrt{6}}{4}$ h) $\frac{\sqrt{2} + \sqrt{6}}{4}$

5) $\cos x = \frac{4}{5}$, $\sin y = \frac{12}{13}$

6) a) $\frac{63}{65}$ b) $-\frac{33}{65}$ c) $-\frac{16}{65}$ d) $\frac{56}{65}$