

5.1 Co-function Identities Worksheet with solutions

This document was composed and formatted by Annie Bernatchez.

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1) Simplify.

a) $\sin x \left(\frac{1}{\cos x} \right)$ $= \frac{\sin x}{\cos x}$ $= \tan x$	b) $(\cos x)(\sec x)$ $= \cos x \left(\frac{1}{\cos x} \right)$ $= 1$
c) $1 - \cos^2 x$ $= \sin^2 x$	d) $1 - \sin^2 x$ $= \cos^2 x$
e) $\frac{\tan x}{\sin x}$ $= \frac{\sin x}{\cos x} \cdot \frac{1}{\sin x}$ $= \frac{1}{\cos x}$ $= \sec x$	f) $(1 - \sin x)(1 + \sin x)$ $= 1 - \sin^2 x$ $= \cos^2 x$
g) $\left(\frac{1}{\tan x} \right) \sin x$ $= \frac{\cos x}{\sin x} (\sin x)$ $= \cos x$	h) $\frac{1 + \tan^2 x}{\tan^2 x}$ $= \frac{\sec^2 x}{\tan^2 x}$ $= \frac{1}{\cos^2 x} \cdot \frac{\cos^2 x}{\sin^2 x}$ $= \frac{1}{\sin^2 x}$ $= \csc^2 x$
i) $\frac{\sin x \cos x}{1 - \sin^2 x}$ $= \frac{\sin x \cos x}{\cos^2 x}$ $= \frac{\sin x}{\cos x}$ $= \tan x$	j) $\frac{1 - \cos^2 x}{\sin x \cos x}$ $= \frac{\sin^2 x}{\sin x \cos x}$ $= \frac{\sin x}{\cos x}$ $= \tan x$

2) Prove the following identities.

a) $\sin^2 x (1 + \cot^2 x) = 1$

LS	RS
$= \sin^2 x (1 + \cot^2 x)$ $= \sin^2 x \left(1 + \frac{\cos^2 x}{\sin^2 x} \right)$ $= \sin^2 x + \cos^2 x$ $= 1$	$= 1$

LS = RS

b) $1 - \cos^2 x = \tan x \cos x \sin x$

LS	RS
$= 1 - \cos^2 x$ $= \sin^2 x$	$= \tan x \cos x \sin x$ $= \left(\frac{\sin x}{\cos x} \right) (\cos x) (\sin x)$ $= \sin^2 x$

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LS = RS

c) $\cos x \tan^3 x = \sin x \tan^2 x$

LS	RS
$\begin{aligned} &= \cos x \tan^3 x \\ &= \cos x \left(\frac{\sin^3 x}{\cos^3 x} \right) \\ &= \frac{\sin^3 x}{\cos^2 x} \end{aligned}$	$\begin{aligned} &= \sin x \tan^2 x \\ &= \sin x \left(\frac{\sin^2 x}{\cos^2 x} \right) \\ &= \frac{\sin^3 x}{\cos^2 x} \end{aligned}$

LS = RS

d) $1 - 2\cos^2 \theta = \sin^4 \theta - \cos^4 \theta$

LS	RS
$= 1 - 2\cos^2 \theta$	$\begin{aligned} &= (\sin^2 \theta)^2 - (\cos^2 \theta)^2 \\ &= (\sin^2 \theta - \cos^2 \theta)(\sin^2 \theta + \cos^2 \theta) \\ &= (\sin^2 \theta - \cos^2 \theta)(1) \\ &= (1 - \cos^2 \theta) - \cos^2 \theta \\ &= 1 - 2\cos^2 \theta \end{aligned}$

LS = RS

e) $\cot x + \frac{\sin x}{1 + \cos x} = \csc x$

LS	RS
$\begin{aligned} &= \frac{\cos x}{\sin x} + \frac{\sin x}{1 + \cos x} \\ &= \frac{\cos x(1 + \cos x) + \sin^2 x}{(1 + \cos x)(\sin x)} \\ &= \frac{\cos x + \cos^2 x + \sin^2 x}{(1 + \cos x)(\sin x)} \\ &= \frac{\cos x + 1}{(1 + \cos x)(\sin x)} \\ &= \frac{1}{\sin x} \end{aligned}$	$\begin{aligned} &= \csc x \\ &= \frac{1}{\sin x} \end{aligned}$

LS = RS

f) $\frac{\sec x}{\sin x} + \frac{\csc x}{\cos x} = \frac{2}{\sin x \cos x}$

LS	RS
$\begin{aligned} &= \frac{\left(\frac{1}{\cos x}\right)}{\sin x} + \frac{\left(\frac{1}{\sin x}\right)}{\cos x} \\ &= \frac{1}{\sin x \cos x} + \frac{1}{\sin x \cos x} \\ &= \frac{2}{\sin x \cos x} \end{aligned}$	$= \frac{2}{\sin x \cos x}$

LS = RS

g) $\frac{\cos^2 x - \sin^2 x}{\cos^2 x + \sin x \cos x} = 1 - \tan x$

LS	RS
$\begin{aligned} &= \frac{(\cos x - \sin x)(\cos x + \sin x)}{\cos x(\cos x + \sin x)} \\ &= \frac{\cos x - \sin x}{\cos x} \end{aligned}$	$\begin{aligned} &= 1 - \tan x \\ &= \frac{\cos x}{\cos x} - \frac{\sin x}{\cos x} \\ &= \frac{\cos x - \sin x}{\cos x} \end{aligned}$

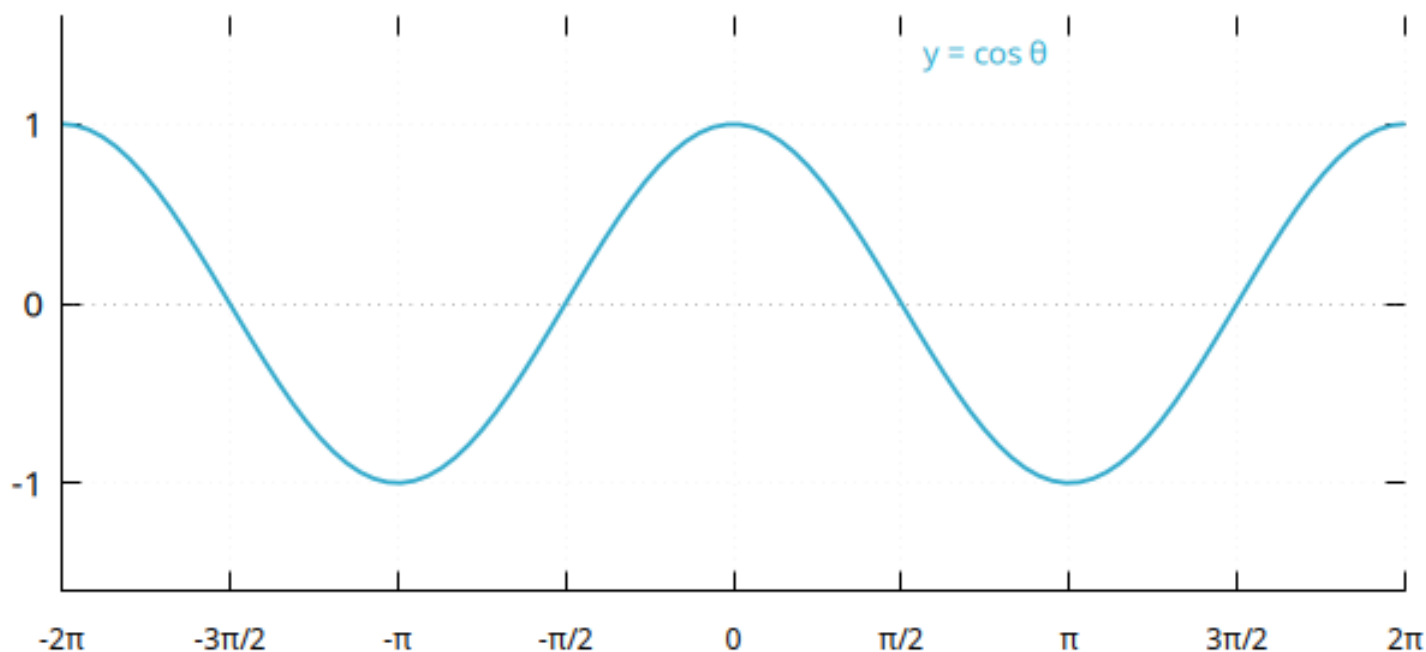
LS = RS

h) $\frac{1}{1 + \cos x} + \frac{1}{1 - \cos x} = 2 \csc^2 x$

LS	RS
$= \frac{1 - \cos x}{(1 + \cos x)(1 - \cos x)} + \frac{1 + \cos x}{(1 - \cos x)(1 + \cos x)}$ $= \frac{1 - \cos x + 1 + \cos x}{(1 + \cos x)(1 - \cos x)}$ $= \frac{2}{1 - \cos^2 x}$ $= \frac{2}{\sin^2 x}$	$= 2 \csc^2 x$ $= 2 \left(\frac{1}{\sin^2 x} \right)$ $= \frac{2}{\sin^2 x}$

LS = RS

- 3)
- a) Use transformations and the cosine function to write three equivalent expressions for the following graph:



1. $y = \cos \theta$
 2. $y = \cos(\theta - 2\pi)$
 3. $y = \cos(\theta + 2\pi)$
- b) Transform your 3 equations from part a) to write the equation of 3 sine functions that represent the graph.

1. $y = \sin\left(\theta + \frac{\pi}{2}\right)$
2. $y = \sin\left(\theta - 2\pi + \frac{\pi}{2}\right) = \sin\left(\theta - \frac{3\pi}{2}\right)$
3. $y = \sin\left(\theta + 2\pi + \frac{\pi}{2}\right) = \sin\left(\theta + \frac{5\pi}{2}\right)$

- 4) Use the co-function identities to write an expression that is equivalent to each of the following expressions.

4) Use the co-function identities to write an expression that is equivalent to each of the following expressions.

a) $\sin \frac{\pi}{6}$

$$= \cos \left(\frac{\pi}{2} - \frac{\pi}{6} \right)$$

$$= \cos \left(\frac{3\pi}{6} - \frac{\pi}{6} \right)$$

$$= \cos \left(\frac{2\pi}{6} \right)$$

$$= \cos \left(\frac{\pi}{3} \right)$$

b) $\cos \frac{5\pi}{12}$

$$= \sin \left(\frac{\pi}{2} - \frac{5\pi}{12} \right)$$

$$= \sin \left(\frac{6\pi}{12} - \frac{5\pi}{12} \right)$$

$$= \sin \left(\frac{\pi}{12} \right)$$

c) $\cos \frac{5\pi}{16}$

$$= \sin \left(\frac{\pi}{2} - \frac{5\pi}{16} \right)$$

$$= \sin \left(\frac{8\pi}{16} - \frac{5\pi}{16} \right)$$

$$= \sin \left(\frac{3\pi}{16} \right)$$

5) Write an expression that is equivalent to each of the following expressions, using the related acute angle.

a) $\sin \frac{7\pi}{8}$

$\frac{7\pi}{8}$ is in quadrant II, where sine is positive, with related (reference) angle $\frac{\pi}{8}$.

$$\sin \frac{7\pi}{8} = \sin \frac{\pi}{8}$$

b) $\cos \frac{13\pi}{12}$

$\frac{13\pi}{12}$ is in quadrant III, where cosine is negative, with related (reference) angle $\frac{\pi}{12}$.

$$\cos \frac{13\pi}{12} = -\cos \frac{\pi}{12}$$

c) $\cos \frac{11\pi}{6}$

$\frac{11\pi}{6}$ is in quadrant IV, where cosine is positive, with related (reference) angle $\frac{\pi}{6}$.

$$\cos \frac{11\pi}{6} = \cos \frac{\pi}{6}$$

6) Given that $\sin \frac{\pi}{6} = \frac{1}{2}$, use an equivalent trigonometric expression to show that $\cos \frac{\pi}{3} = \frac{1}{2}$

$$\cos \frac{\pi}{3} = \sin \left(\frac{\pi}{2} - \frac{\pi}{3} \right)$$

$$= \sin \left(\frac{\pi}{6} \right)$$

$$= \frac{1}{2}$$

7) Given that $\sin \frac{\pi}{6} = \frac{1}{2}$, use an equivalent trigonometric expression to show that $\cos \frac{2\pi}{3} = -\frac{1}{2}$

$$\cos \frac{2\pi}{3} = \sin \left(\frac{\pi}{2} - \frac{2\pi}{3} \right)$$

$$= \sin \left(\frac{3\pi}{6} - \frac{4\pi}{6} \right)$$

$$= \sin \left(-\frac{\pi}{6} \right)$$

$$= -\sin \frac{\pi}{6}$$

$$= -\frac{1}{2}$$

8) Given that $\csc \frac{\pi}{4} = \sqrt{2}$, use an equivalent trigonometric expression to show that $\sec \frac{3\pi}{4} = -\sqrt{2}$

$$\begin{aligned}\sec \frac{3\pi}{4} &= \frac{1}{\cos \frac{3\pi}{4}} \\&= \frac{1}{\sin \left(\frac{\pi}{2} - \frac{3\pi}{4} \right)} \\&= \frac{1}{\sin \left(-\frac{\pi}{4} \right)} \\&= \frac{1}{-\sin \frac{\pi}{4}} \\&= -\csc \frac{\pi}{4} \\&= -\sqrt{2}\end{aligned}$$

9) Given that $\cos \frac{3\pi}{11} \approx 0.6549$, use equivalent trigonometric expressions to evaluate the following, to four decimal places.

$$\begin{aligned}\text{a) } \sin \frac{5\pi}{22} &= \cos \left(\frac{\pi}{2} - \frac{5\pi}{22} \right) \\&= \cos \left(\frac{11\pi}{22} - \frac{5\pi}{22} \right) \\&= \cos \left(\frac{6\pi}{22} \right) \\&= \cos \left(\frac{3\pi}{11} \right) \\&\approx 0.6549\end{aligned}$$

$$\begin{aligned}\text{b) } \sin \frac{17\pi}{22} &= \cos \left(\frac{\pi}{2} - \frac{17\pi}{22} \right) \\&= \cos \left(\frac{11\pi}{22} - \frac{17\pi}{22} \right) \\&= \cos \left(-\frac{6\pi}{22} \right) \\&= \cos \left(\frac{3\pi}{11} \right) \\&\approx 0.6549\end{aligned}$$

Answer Key

1) a) $\tan x$ b) 1 c) $\sin^2 x$ d) $\cos^2 x$ e) $\sec x$ f) $\cos^2 x$ g) $\cos x$ h) $\csc^2 x$ i) $\tan x$ j) $\tan x$

3) Answers will vary depending but possible solutions are:

$$\text{a) } y = \cos(\theta + 2\pi), y = \cos(\theta - 2\pi), y = \cos(\theta - 4\pi)$$

$$\text{b) } y = \sin\left(\theta + \frac{5\pi}{2}\right), y = \sin\left(\theta - \frac{3\pi}{2}\right), y = \sin\left(\theta - \frac{7\pi}{2}\right)$$

$$\text{4) a) } \cos \frac{\pi}{3} \text{ b) } \sin \frac{\pi}{12} \text{ c) } \sin \frac{3\pi}{16}$$

$$\text{5) a) } \sin \frac{\pi}{8} \text{ b) } -\cos \frac{\pi}{12} \text{ c) } \cos \frac{\pi}{6}$$

$$\text{9) a) } \sin \frac{5\pi}{22} = \cos \frac{3\pi}{11} \approx 0.6549 \text{ b) } \sin \frac{17\pi}{22} = \cos \left(-\frac{3\pi}{11} \right) = \cos \frac{3\pi}{11} \approx 0.6549$$