

5.7 Solve Quadratic Trigonometric Equations Lesson with solutions

This document was composed and formatted by Annie Bernatchez.

The course material originated from www.jensenmath.ca

A quadratic trigonometric equation may have multiple solutions in the interval $0 \leq x \leq 2\pi$.

You can often **factor** a quadratic trigonometric equation and then solve the resulting two linear trigonometric equations. In cases where the equation cannot be factored, use the **quadratic formula** and then solve the resulting linear trigonometric equations.

You may need to use a Pythagorean identity, compound angle formula, or double angle formula to create a quadratic equation that contains only a single trigonometric function whose arguments all match.

Remember that when solving a linear trigonometric equation, consider all 3 tools that can be useful:

1. Special Triangles
2. Graphs of Trig Functions
3. Calculator

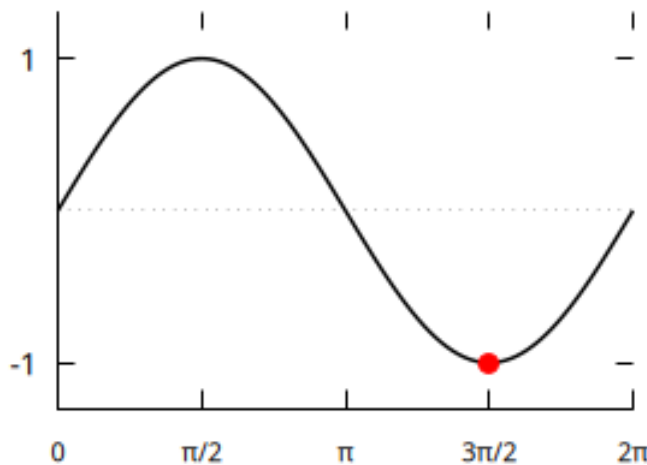
Part 1: Solving Quadratic Trigonometric Equations

Example 1: Solve each of the following equations for $0 \leq x \leq 2\pi$

a) $(\sin x + 1)\left(\sin x - \frac{1}{2}\right) = 0$

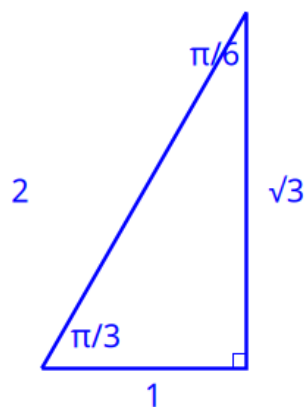
Set both factors equal to zero and solve.

$\sin x + 1 = 0 \rightarrow \sin x = -1 \rightarrow$ Graph? Yes!

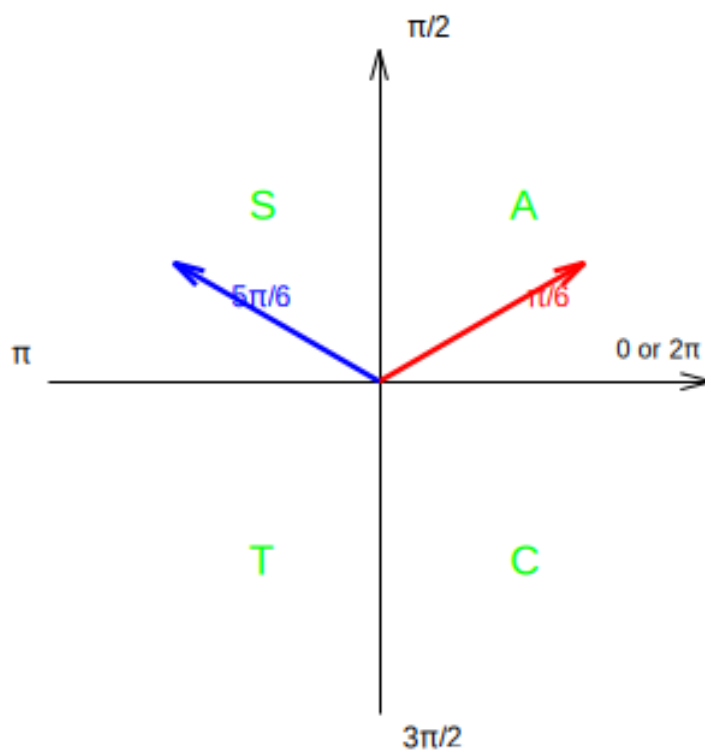


$x_1 = \frac{3\pi}{2}$

$\sin x - \frac{1}{2} = 0 \rightarrow \sin x = \frac{1}{2} \rightarrow$ Graph? No $\rightarrow$ Triangle? Yes



$\sin \frac{\pi}{6} = \frac{1}{2}$. Put in Q1 and Q2.



$$x_2 = \frac{\pi}{6}$$

$$x_3 = \pi - \frac{\pi}{6} = \frac{5\pi}{6}$$

Solutions are $x = \frac{3\pi}{2}$, $\frac{\pi}{6}$, or $\frac{5\pi}{6}$ radians

b) $\sin^2 x - \sin x = 2$

$$\sin^2 x - \sin x - 2 = 0$$

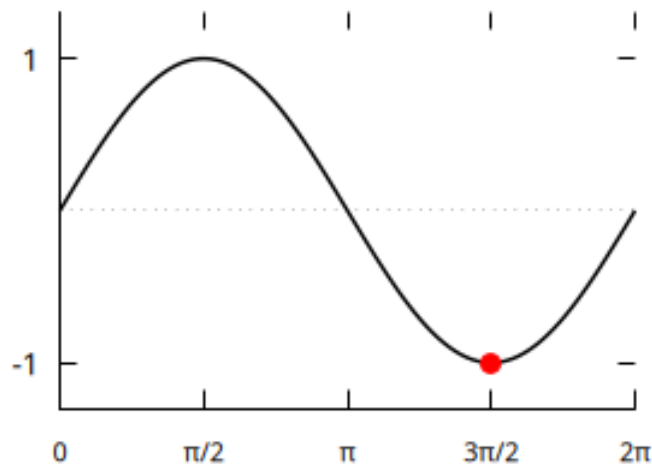
Let $u = \sin x$, $u^2 - u - 2 = 0$

$$(u - 2)(u + 1) = 0$$

$$(\sin x - 2)(\sin x + 1) = 0$$

$$\sin x - 2 = 0 \rightarrow \sin x = 2 \rightarrow \text{No solutions}$$

$$\sin x + 1 = 0 \rightarrow \sin x = -1 \rightarrow \text{Graph? Yes}$$



$$x = \frac{3\pi}{2}$$

The only solution is $x = \frac{3\pi}{2}$

$$\text{c) } 2\sin^2 x - 3\sin x + 1 = 0$$

$$\text{Let } x = \sin x, \quad 2x^2 - 3x + 1 = 0$$

$$2x^2 - 2x - 1x + 1 = 0$$

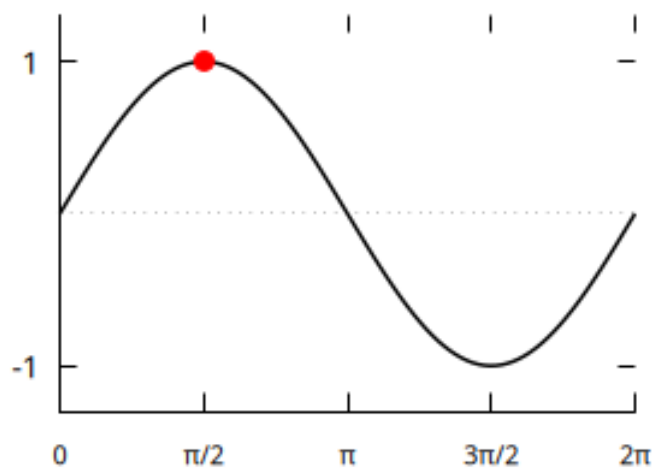
$$(2x^2 - 2x) + (-1x + 1) = 0$$

$$2x(x - 1) - 1(x - 1) = 0$$

$$(x - 1)(2x - 1) = 0$$

$$(\sin x - 1)(2\sin x - 1) = 0$$

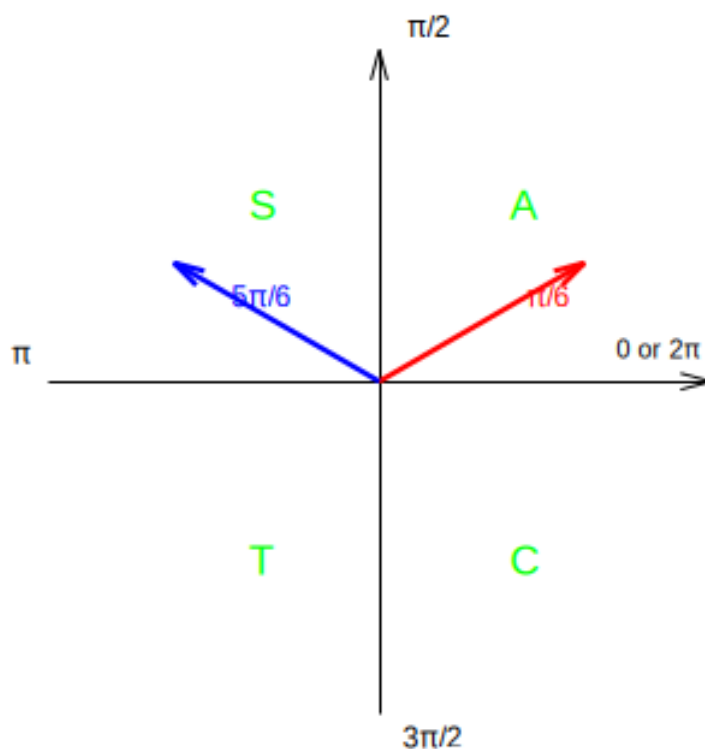
$$\sin x - 1 = 0 \rightarrow \sin x = 1 \rightarrow \text{Graph}$$



$$x_1 = \frac{\pi}{2}$$

$$2\sin x - 1 = 0 \rightarrow \sin x = \frac{1}{2} \rightarrow \text{Triangle}$$

$$\sin \frac{\pi}{6} = \frac{1}{2}. \text{ Put in Q1 and Q2.}$$



$$x_2 = \frac{\pi}{6}$$

$$x_3 = \pi - \frac{\pi}{6} = \frac{5\pi}{6}$$

The solutions are $x = \frac{\pi}{2}, \frac{\pi}{6}, \text{ or } \frac{5\pi}{6}$

Part 2: Use Identities to Help Solve Quadratic Trigonometric Equations

Example 2: Solve each of the following equations for $0 \leq x \leq 2\pi$

a) $2\sec^2 x - 3 + \tan x = 0$

Pythagorean Identity: $\sec^2 x = \tan^2 x + 1$

$$2(\tan^2 x + 1) - 3 + \tan x = 0$$

$$2\tan^2 x + 2 - 3 + \tan x = 0$$

$$2\tan^2 x + \tan x - 1 = 0$$

Let $x = \tan x$, $2x^2 + x - 1 = 0$

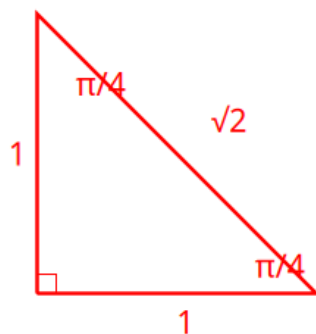
$$2x^2 + 2x - 1x - 1 = 0$$

$$2x(x + 1) - 1(x + 1) = 0$$

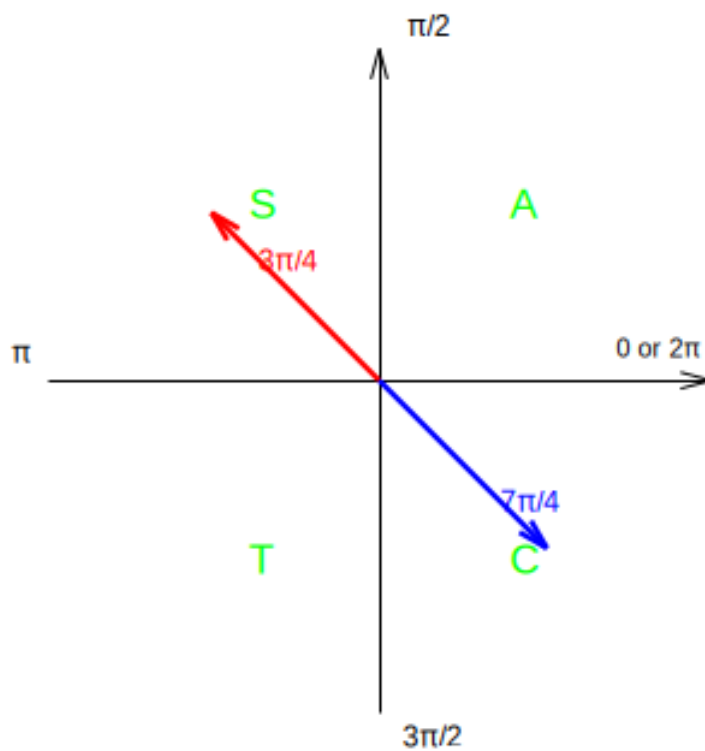
$$(x + 1)(2x - 1) = 0$$

$$(\tan x + 1)(2\tan x - 1) = 0$$

$$\tan x + 1 = 0 \rightarrow \tan x = -1$$



Put $\frac{\pi}{4}$ in Q2 and Q4.

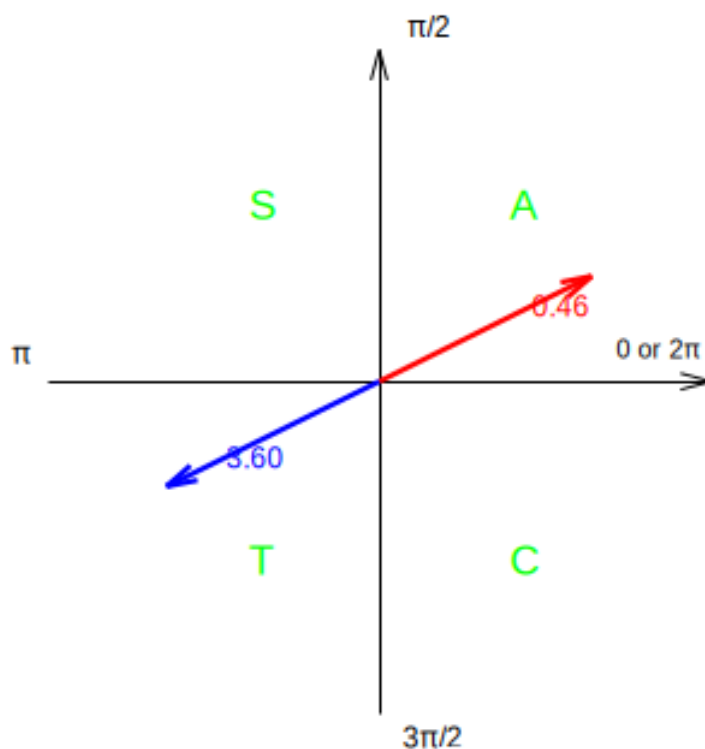


$$x_1 = \pi - \frac{\pi}{4} = \frac{3\pi}{4}$$

$$x_2 = 2\pi - \frac{\pi}{4} = \frac{7\pi}{4}$$

$$2 \tan x - 1 = 0 \rightarrow \tan x = \frac{1}{2}$$

$$x = \tan^{-1}\left(\frac{1}{2}\right) \approx 0.46 \text{ radians. Put in Q1 and Q3.}$$



$$x_3 \approx 0.46$$

$$x_4 = \pi + 0.46 \approx 3.60$$

The solutions are $x = \frac{3\pi}{4}, \frac{7\pi}{4}, 0.46$, or 3.60 radians

b) $3 \sin x + 3 \cos(2x) = 2$

$$3 \sin x + 3(1 - 2 \sin^2 x) = 2$$

$$3 \sin x + 3 - 6 \sin^2 x - 2 = 0$$

$$-6 \sin^2 x + 3 \sin x + 1 = 0$$

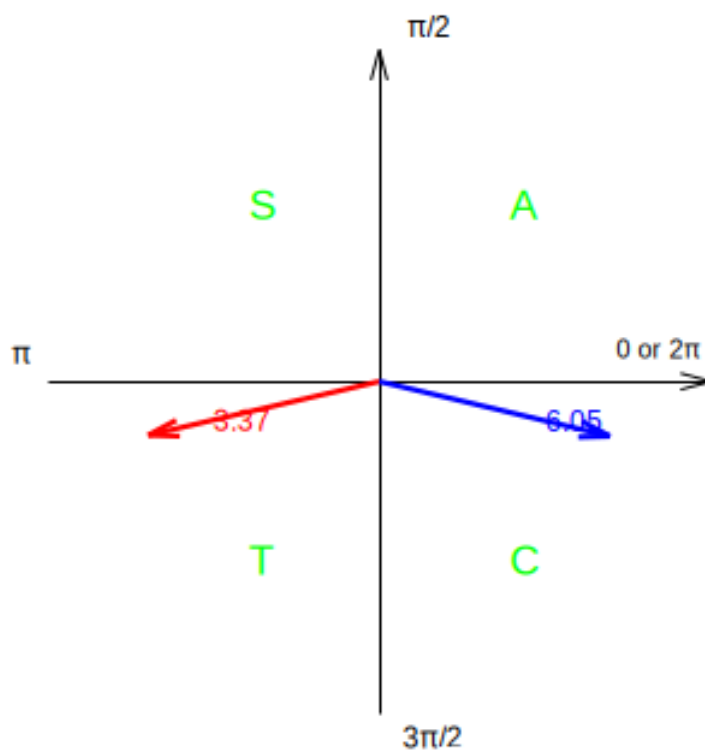
Let $x = \sin x$. $-6x^2 + 3x + 1 = 0$. Not factorable, use the quadratic formula.

$$x = \frac{-3 \pm \sqrt{(3)^2 - 4(-6)(1)}}{2(-6)}$$

$$x = \frac{-3 \pm \sqrt{33}}{-12}$$

$$x \approx -0.23 \text{ or } x \approx 0.73$$

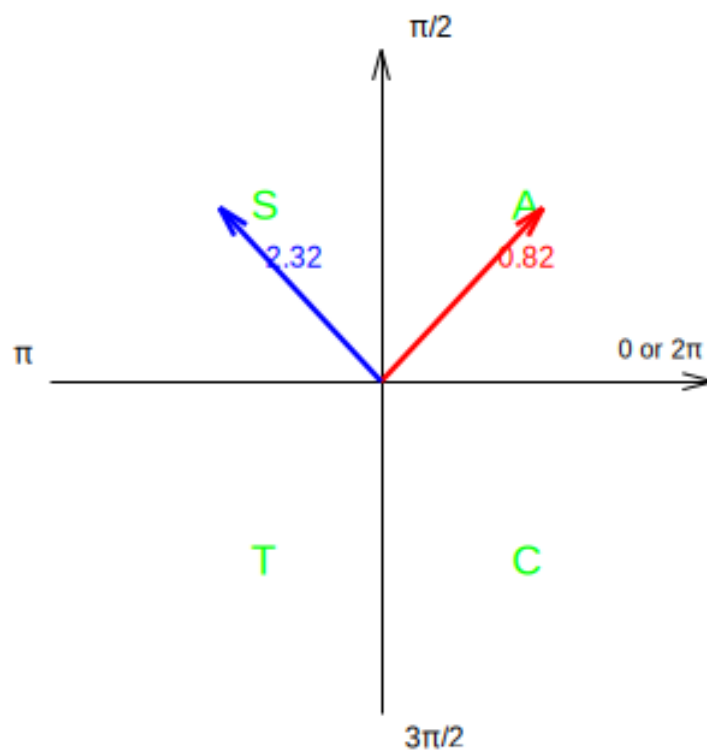
$\sin x = -0.23 \rightarrow x = \sin^{-1}(-0.23)$. Put in Q3 and Q4.



$$x_1 = 2\pi - 0.23 \approx 6.05$$

$$x_2 = \pi + 0.23 \approx 3.37$$

$\sin x = 0.73 \rightarrow x = \sin^{-1}(0.73)$. Put in Q1 and Q2.



$$x_3 \approx 0.82$$

$$x_4 = \pi - 0.82 \approx 2.32$$

The solutions are $x = 0.82, 2.32, 3.37, \text{ or } 6.05$ radians