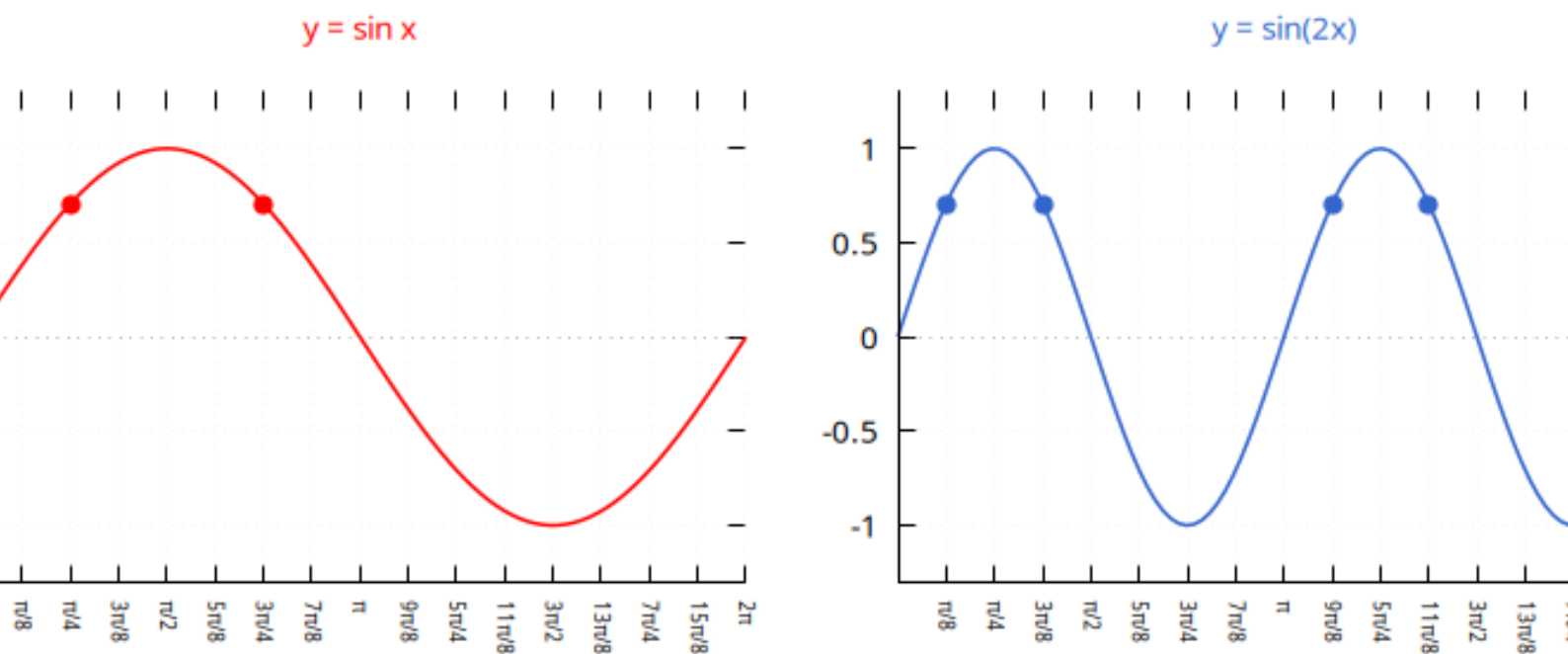


5.6 Solve Double Angle Trigonometric Equations Lesson with solutions

This document was composed and formatted by Annie Bernatchez.

The course material originated from www.jensenmath.ca

Part 1: Investigation



a) What is the period of both of the functions above? How many cycles between 0 and 2π radians?

For $y = \sin x \rightarrow \text{period} = 2\pi$

For $y = \sin(2x) \rightarrow \text{period} = \frac{2\pi}{2} = \pi$

b) Looking at the graph of $y = \sin x$, how many solutions are there for $\sin x = \frac{1}{\sqrt{2}} \approx 0.71$?

2 solutions

$$\sin \frac{\pi}{4} = \sin \frac{3\pi}{4} = \frac{1}{\sqrt{2}}$$

c) Looking at the graph of $y = \sin(2x)$, how many solutions are there for $\sin(2x) = \frac{1}{\sqrt{2}} \approx 0.71$?

4 solutions

$$\sin \left[2 \left(\frac{\pi}{8} \right) \right] = \sin \left[2 \left(\frac{3\pi}{8} \right) \right] = \sin \left[2 \left(\frac{9\pi}{8} \right) \right] = \sin \left[2 \left(\frac{11\pi}{8} \right) \right] = \frac{1}{\sqrt{2}}$$

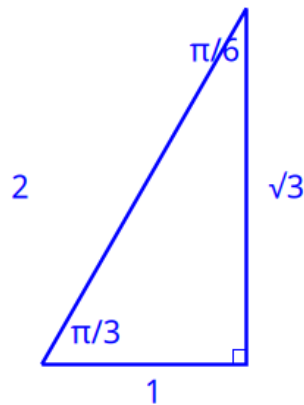
d) When the period of a function is cut in half, what does that do to the number of solutions between 0 and 2π radians?

Doubles the number of solutions

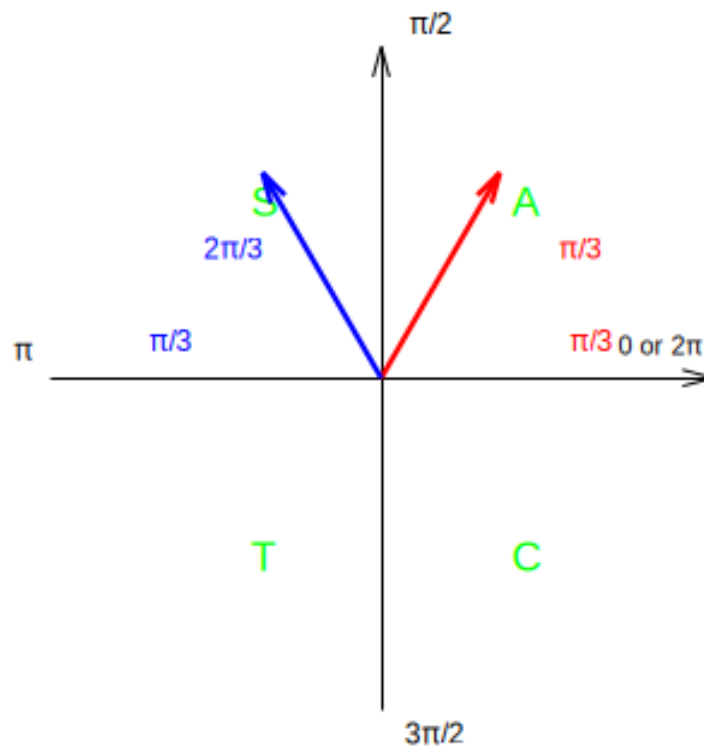
Part 2: Solve Linear Trigonometric Equations that Involve Double Angles

Example 1: $\sin(2\theta) = \frac{\sqrt{3}}{2}$ where $0 \leq \theta \leq 2\pi$

Let $2\theta = x$. $\sin x = \frac{\sqrt{3}}{2} \rightarrow$ Graph? No $\rightarrow$ Triangle? Yes



$\sin \frac{\pi}{3} = \frac{\sqrt{3}}{2}$. Put the reference angle in Q1 and Q2, where sine is positive.



$$x_1 = \frac{\pi}{3}$$

$$x_2 = \pi - \frac{\pi}{3} = \frac{2\pi}{3}$$

$$2\theta = x \rightarrow 2\theta = \frac{\pi}{3} \rightarrow \theta_1 = \frac{\pi}{6}, \text{ and } 2\theta = \frac{2\pi}{3} \rightarrow \theta_2 = \frac{\pi}{3}$$

$y = \sin(2\theta)$ has a period of π ; add π to θ_1 and θ_2 to find other angles $0 \leq \theta \leq 2\pi$ that have equivalent ratios.

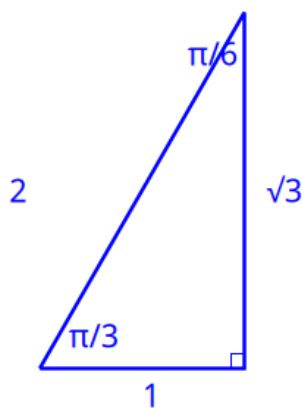
$$\theta_3 = \theta_1 + \pi = \frac{\pi}{6} + \pi = \frac{7\pi}{6}$$

$$\theta_4 = \theta_2 + \pi = \frac{\pi}{3} + \pi = \frac{4\pi}{3}$$

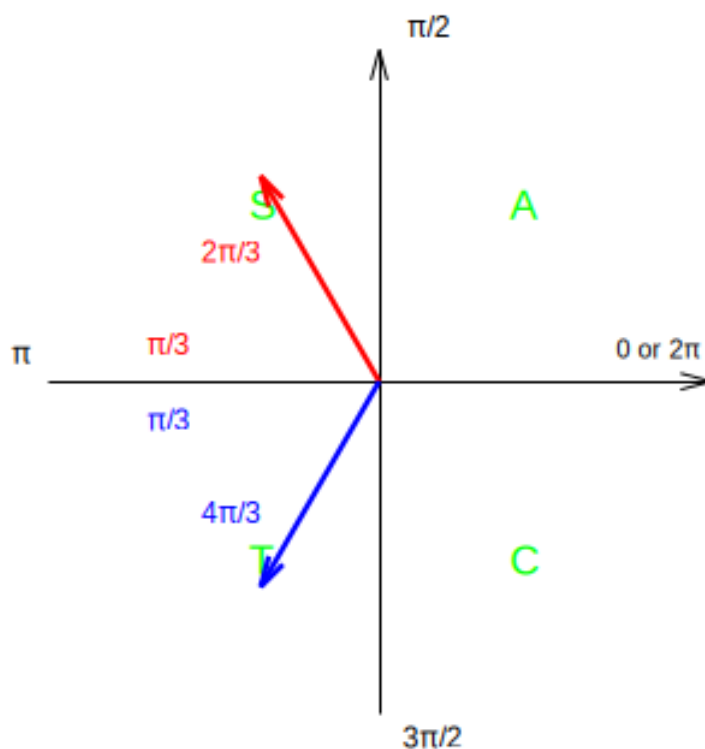
$$\sin\left[2\left(\frac{\pi}{6}\right)\right] = \sin\left[2\left(\frac{\pi}{3}\right)\right] = \sin\left[2\left(\frac{7\pi}{6}\right)\right] = \sin\left[2\left(\frac{4\pi}{3}\right)\right] = \frac{\sqrt{3}}{2}$$

Example 2: $\cos(2\theta) = -\frac{1}{2}$ where $0 \leq \theta \leq 2\pi$

Let $2\theta = x$. $\cos x = -\frac{1}{2} \rightarrow$ Graph? No $\rightarrow$ Triangle? Yes



$\cos \frac{\pi}{3} = \frac{1}{2}$. Put the reference angle in Q2 and Q3, where cosine is negative.



$$x_1 = \pi - \frac{\pi}{3} = \frac{2\pi}{3}$$

$$x_2 = \pi + \frac{\pi}{3} = \frac{4\pi}{3}$$

$$2\theta = x \rightarrow 2\theta = \frac{2\pi}{3} \rightarrow \theta_1 = \frac{\pi}{3}, \text{ and } 2\theta = \frac{4\pi}{3} \rightarrow \theta_2 = \frac{2\pi}{3}$$

Remember that $\cos(2\theta)$ has a period of π ; add π to θ_1 and θ_2 to find other solutions $0 \leq \theta \leq 2\pi$.

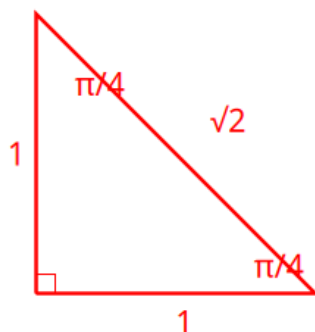
$$\theta_3 = \theta_1 + \pi = \frac{\pi}{3} + \pi = \frac{4\pi}{3}$$

$$\theta_4 = \theta_2 + \pi = \frac{2\pi}{3} + \pi = \frac{5\pi}{3}$$

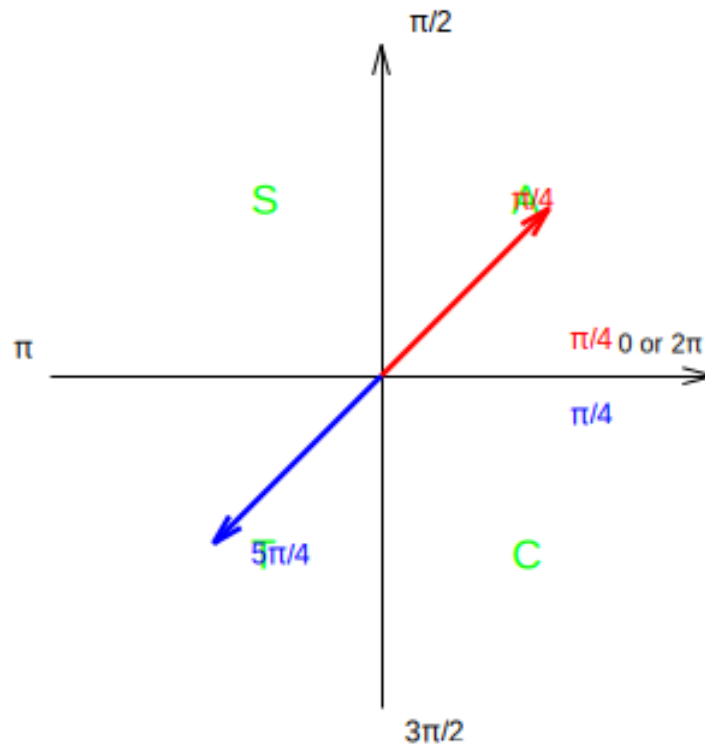
$$\cos\left[2\left(\frac{\pi}{3}\right)\right] = \cos\left[2\left(\frac{2\pi}{3}\right)\right] = \cos\left[2\left(\frac{4\pi}{3}\right)\right] = \cos\left[2\left(\frac{5\pi}{3}\right)\right] = -\frac{1}{2}$$

Example 3: $\tan(2\theta) = 1$ where $0 \leq \theta \leq 2\pi$

Let $2\theta = x$. $\tan x = 1 \rightarrow$ Graph? No $\rightarrow$ Triangle? Yes



$\tan \frac{\pi}{4} = 1$. Put the reference angle in Q1 and Q3, where tangent is positive.



$$x_1 = \frac{\pi}{4}$$

$$x_2 = \pi + \frac{\pi}{4} = \frac{5\pi}{4}$$

$$2\theta = x \rightarrow 2\theta = \frac{\pi}{4} \rightarrow \theta_1 = \frac{\pi}{8}, \text{ and } 2\theta = \frac{5\pi}{4} \rightarrow \theta_2 = \frac{5\pi}{8}$$

Remember that $\tan(2\theta)$ has a period of $\frac{\pi}{2}$; add $\frac{\pi}{2}$ to θ_1 and θ_2 to find other solutions $0 \leq \theta \leq 2\pi$.

$$\theta_3 = \theta_2 + \frac{\pi}{2} = \frac{5\pi}{8} + \frac{4\pi}{8} = \frac{9\pi}{8}$$

$$\theta_4 = \theta_3 + \frac{\pi}{2} = \frac{9\pi}{8} + \frac{4\pi}{8} = \frac{13\pi}{8}$$

$$\tan \left[2 \left(\frac{\pi}{8} \right) \right] = \tan \left[2 \left(\frac{5\pi}{8} \right) \right] = \tan \left[2 \left(\frac{9\pi}{8} \right) \right] = \tan \left[2 \left(\frac{13\pi}{8} \right) \right] = 1$$