

5.5 Solve Linear Trigonometric Equations Lesson with solutions

This document was composed and formatted by Annie Bernatchez.

The course material originated from www.jensenmath.ca

In the previous lesson we have been working with identities. Identities are equations that are true for ANY value of x . In this lesson, we will be working with equations that are not identities. We will have to solve for the value(s) that make the equation true.

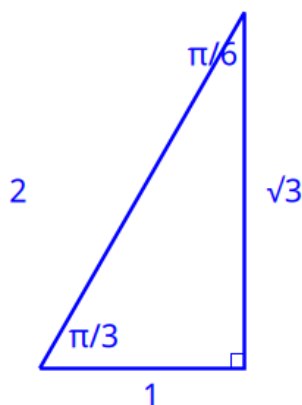
Remember that 2 solutions are possible for an angle between 0 and 2π with a given ratio. Use the reference angle and CAST rule to determine the angles.

When solving a trigonometric equation, consider all 3 tools that can be useful:

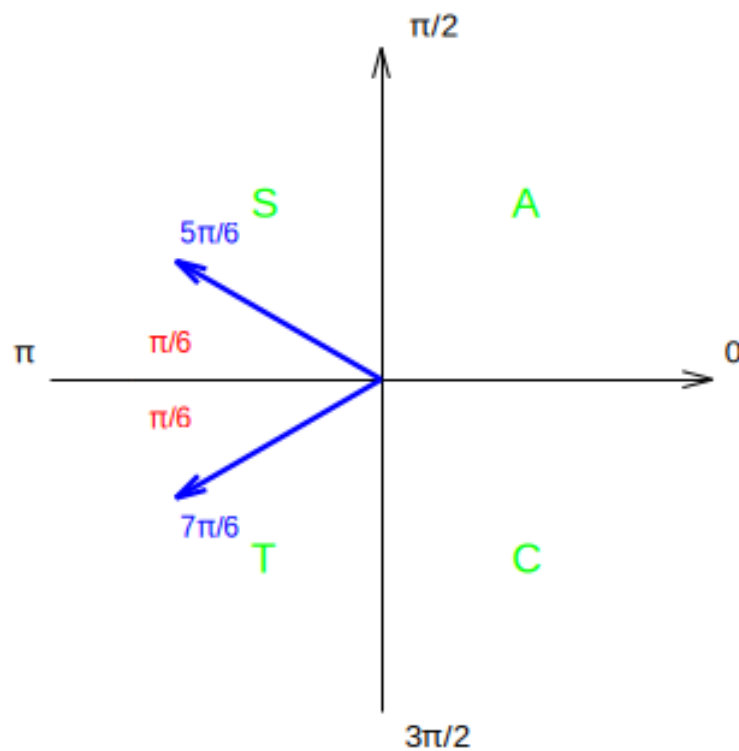
1. Special Triangles
2. Graphs of Trig Functions
3. Calculator

Example 1: Find all solutions for $\cos \theta = -\frac{\sqrt{3}}{2}$ in the interval $0 \leq \theta \leq 2\pi$

$-\frac{\sqrt{3}}{2} \rightarrow$ Graph? No $\rightarrow$ Triangle? Yes



$\cos \frac{\pi}{6} = \frac{\sqrt{3}}{2}$, so the reference angle is $\frac{\pi}{6}$. Put the reference angle into quadrants 2 and 3 to make the ratio negative.



$$\theta_1 = \pi - \frac{\pi}{6} = \frac{5\pi}{6}$$

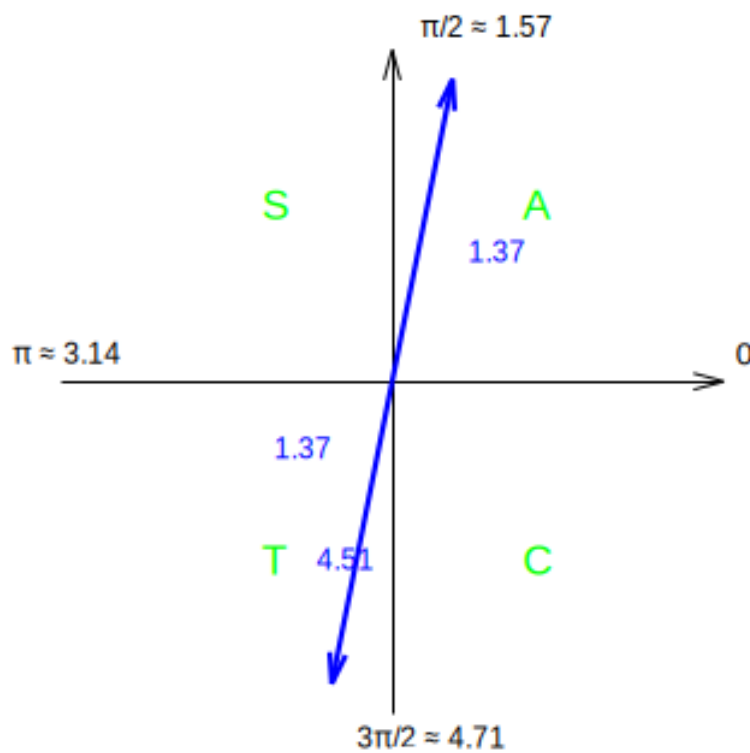
$$\theta_2 = \pi + \frac{\pi}{6} = \frac{7\pi}{6}$$

$$\cos \frac{5\pi}{6} = \cos \frac{7\pi}{6} = -\frac{\sqrt{3}}{2}$$

Example 2: Find all solutions for $\tan \theta = 5$ in the interval $0 \leq \theta \leq 2\pi$

$\tan \theta = 5 \rightarrow$ Graph? No $\rightarrow$ Triangle? No $\rightarrow$ Calculator

Note: Tan is positive in Q1 and Q3.



$$\tan \theta = 5$$

$$\theta_1 = \tan^{-1}(5)$$

$$\theta_1 \approx 1.37 \text{ radians}$$

$$\theta_2 = \pi + 1.37$$

$$\theta_2 \approx 4.51 \text{ radians}$$

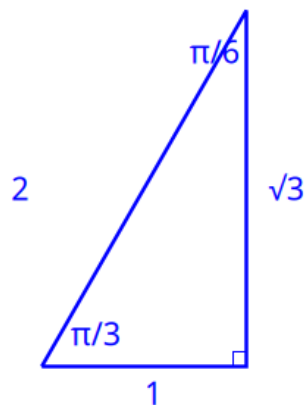
$$\tan(1.37) = \tan(4.51) = 5$$

Example 3: Find all solutions for $2\sin x + 1 = 0$ in the interval $0 \leq x \leq 2\pi$

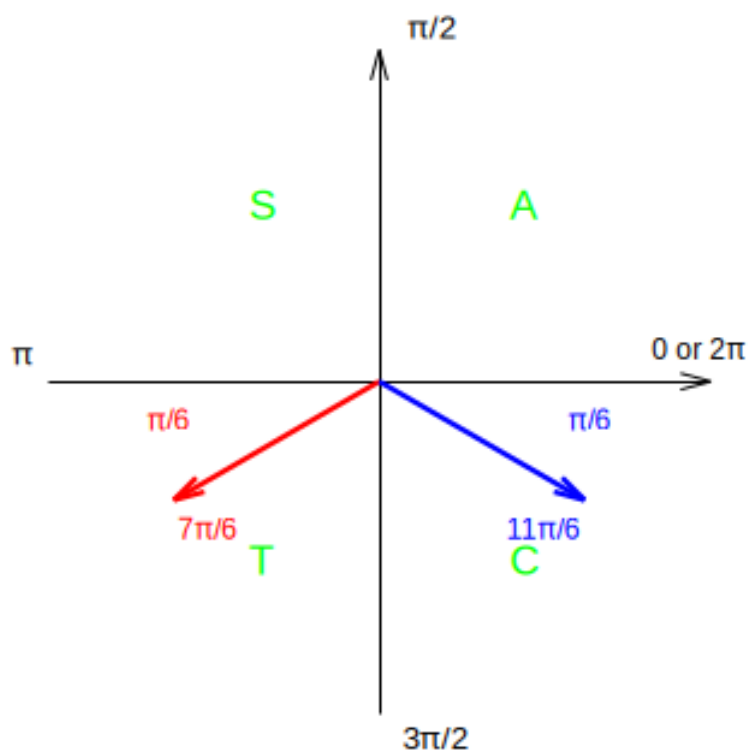
$$2\sin x + 1 = 0$$

$$2\sin x = -1$$

$$\sin x = -\frac{1}{2} \rightarrow \text{Graph? No} \rightarrow \text{Triangle? Yes}$$



$\sin \frac{\pi}{6} = \frac{1}{2}$, use as reference angle in Q3 and Q4 to make the ratio negative.



$$\theta_1 = \pi + \frac{\pi}{6} = \frac{7\pi}{6}$$

$$\theta_2 = 2\pi - \frac{\pi}{6} = \frac{11\pi}{6}$$

$$\sin \frac{7\pi}{6} = \sin \frac{11\pi}{6} = -\frac{1}{2}$$

Example 4: Solve $3(\tan x + 1) = 2$, where $0 \leq x \leq 2\pi$

$$3(\tan x + 1) = 2$$

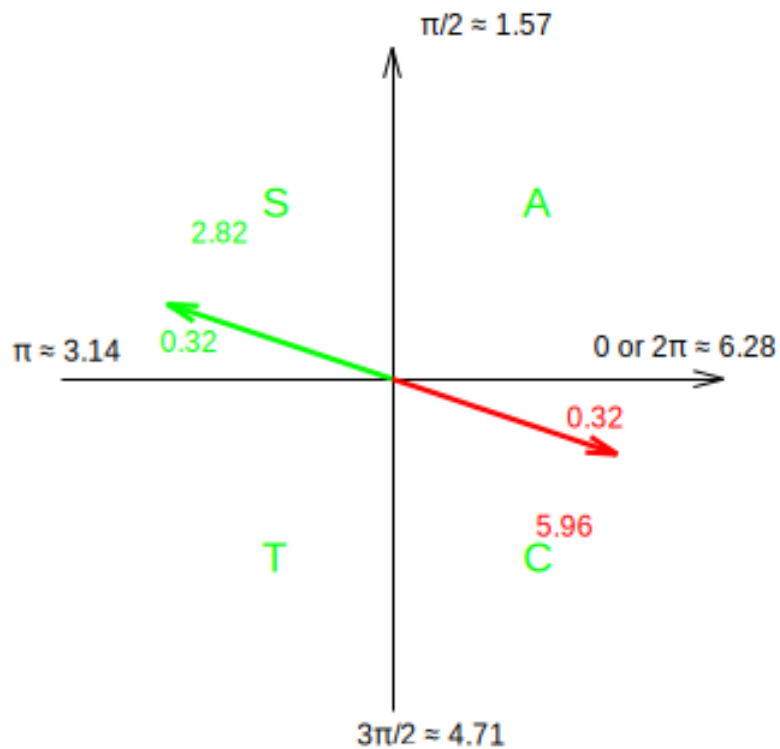
$$\tan x + 1 = \frac{2}{3}$$

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$$\tan x = \frac{2}{3} - 1$$

$$\tan x = -\frac{1}{3} \rightarrow \text{Graph? No} \rightarrow \text{Triangle? No} \rightarrow \text{Calculator}$$

Note: Tan is negative in Q2 and Q4.



$$\tan x = -\frac{1}{3}$$

$$x_1 = \tan^{-1}\left(-\frac{1}{3}\right)$$

$$x_1 = -0.32175$$

$$x_1 = 2\pi - 0.32175$$

$$x_1 \approx 5.96 \text{ radians}$$

$$x_2 = \pi - 0.32$$

$$x_2 \approx 2.82 \text{ radians}$$

$$\tan(5.96) = \tan(2.82) = -\frac{1}{3}$$