

5.4 Prove Trig Identities Lesson with solutions

This document was composed and formatted by Annie Bernatchez.

The course material originated from www.jensenmath.ca

Using your sheet of all identities learned this unit, prove each of the following:

Example 1: Prove $\frac{\sin(2x)}{1 + \cos(2x)} = \tan x$

LS	RS
$= \frac{\sin(2x)}{1 + \cos(2x)}$ $= \frac{\sin(2x)}{1 + 2\cos^2 x - 1}$ $= \frac{2\sin x \cos x}{2\cos^2 x}$ $= \frac{\sin x}{\cos x}$	$= \tan x$ $= \frac{\sin x}{\cos x}$

LS = RS

Example 2: Prove $\cos\left(\frac{\pi}{2} + x\right) = -\sin x$

LS	RS
$= \cos\left(\frac{\pi}{2} + x\right)$ $= \cos \frac{\pi}{2} \cos x - \sin \frac{\pi}{2} \sin x$ $= 0(\cos x) - 1 \sin x$ $= -\sin x$	$= -\sin x$

LS = RS

Example 3: Prove $\csc(2x) = \frac{\csc x}{2\cos x}$

LS	RS
$= \csc(2x)$ $= \frac{1}{\sin(2x)}$ $= \frac{1}{2\sin x \cos x}$	$= \frac{\csc x}{2\cos x}$ $= \csc x \cdot \frac{1}{2\cos x}$ $= \frac{1}{\sin x} \cdot \frac{1}{2\cos x}$ $= \frac{1}{2\sin x \cos x}$

LS = RS

Example 4: Prove $\cos x = \frac{1}{\cos x} - \sin x \tan x$

LS	RS
$= \cos x$	$= \frac{1}{\cos x} - \sin x \tan x$ $= \frac{1}{\cos x} - \sin x \left(\frac{\sin x}{\cos x} \right)$ $= \frac{1}{\cos x} - \frac{\sin^2 x}{\cos x}$ $= \frac{\cos^2 x}{\cos x}$ $= \cos x$

LS = RS

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Example 5: Prove $\tan(2x) - 2 \tan(2x) \sin^2 x = \sin 2x$

LS	RS
$\begin{aligned} &= \tan(2x) - 2 \tan(2x) \sin^2 x \\ &= \tan(2x)[1 - 2 \sin^2 x] \\ &= \tan(2x) \cos(2x) \\ &= \frac{\sin(2x)}{\cos(2x)} \cdot \cos(2x) \\ &= \sin(2x) \end{aligned}$	$= \sin(2x)$

LS = RS

Example 6: Prove $\frac{\cos(x - y)}{\cos(x + y)} = \frac{1 + \tan x \tan y}{1 - \tan x \tan y}$

LS	RS
$\begin{aligned} &= \frac{\cos(x - y)}{\cos(x + y)} \\ &= \frac{\cos x \cos y + \sin x \sin y}{\cos x \cos y - \sin x \sin y} \end{aligned}$	$\begin{aligned} &= \frac{1 + \left(\frac{\sin x}{\cos x}\right)\left(\frac{\sin y}{\cos y}\right)}{1 - \left(\frac{\sin x}{\cos x}\right)\left(\frac{\sin y}{\cos y}\right)} \times \frac{\cos x \cos y}{\cos x \cos y} \\ &= \frac{\cos x \cos y + \sin x \sin y}{\cos x \cos y - \sin x \sin y} \end{aligned}$

LS = RS