

5.2 Compound Angle Formulas Lesson

This document was composed and formatted by Annie Bernatchez.

The course material originated from www.jensenmath.ca

Compound angle: an angle that is created by adding or subtracting two or more angles.

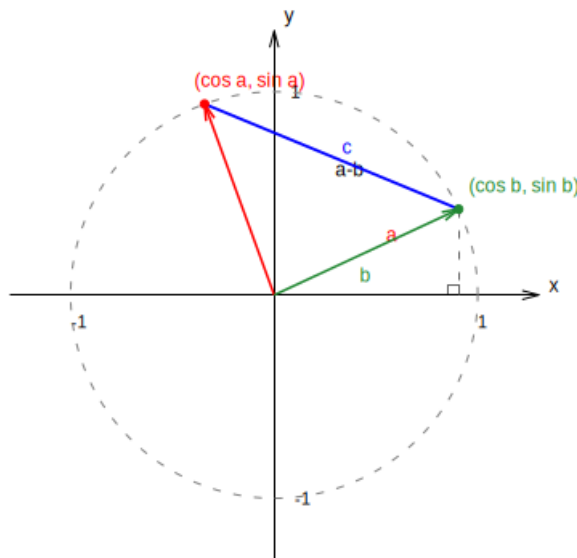
Part 1: Proof of $\cos(x - y)$

Normal algebra rules do not apply:

$$\cos(x - y) \neq \cos x - \cos y$$

So what does $\cos(x - y) = ?$

Consider the diagram to the right...



By the cosine law:

$$c^2 = 1^2 + 1^2 - 2(1)(1)\cos(a - b)$$

$$c^2 = 2 - 2\cos(a - b) \text{ THIS IS EQUATION 1}$$

But notice that c has endpoints of $(\cos a, \sin a)$ and $(\cos b, \sin b)$

Using the distance formula $\text{distance} = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$$c = \sqrt{(\cos a - \cos b)^2 + (\sin a - \sin b)^2}$$

$$c^2 = (\cos a - \cos b)^2 + (\sin a - \sin b)^2$$

$$c^2 = \cos^2 a - 2\cos a \cos b + \cos^2 b + \sin^2 a - 2\sin a \sin b + \sin^2 b$$

$$c^2 = 1 - 2\cos a \cos b - 2\sin a \sin b + 1$$

$$c^2 = 2 - 2\cos a \cos b - 2\sin a \sin b \text{ THIS IS EQUATION 2}$$

Set equations 1 and 2 equal

$$2 - 2\cos(a - b) = 2 - 2\cos a \cos b - 2\sin a \sin b$$

$$-2\cos(a - b) = -2\cos a \cos b - 2\sin a \sin b$$

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Part 2: Proofs of other compound angle formulas

Table 1. Even/Odd Properties

$\cos(-x) = \cos x$	$\sin(-x) = -\sin x$
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Example 1: Prove $\cos(x - y) = \cos x \cos y + \sin x \sin y$

Example 2: Prove addition and subtraction formulas for sine using co-function identities and the subtraction formula for cosine.

Table 2. Co-Function Identities

$\cos\left(\frac{\pi}{2} - x\right) = \sin x$	$\sin\left(\frac{\pi}{2} - x\right) = \cos x$
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a) Prove $\sin(x + y) = \sin x \cos y + \cos x \sin y$

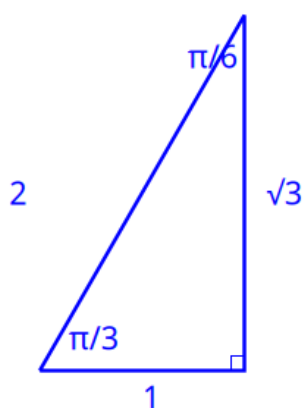
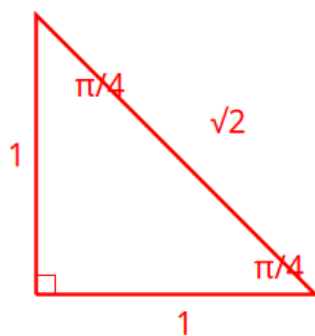
b) Prove $\sin(x - y) = \sin x \cos y - \cos x \sin y$

Table 3. Compound Angle Formulas

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Part 3: Determine Exact Trig Ratios for Angles other than Special Angles

By expressing an angle as a sum or difference of angles in the special triangles, exact values of other angles can be determined.



Example 3: Use compound angle formulas to determine exact values for

a) $\sin \frac{\pi}{12}$

b) $\tan\left(-\frac{5\pi}{12}\right)$

Part 4: Use Compound Angle Formulas to Simplify Trig Expressions

Example 4: Simplify the following expression

$$\cos \frac{7\pi}{12} \cos \frac{5\pi}{12} + \sin \frac{7\pi}{12} \sin \frac{5\pi}{12}$$

Part 5: Application

Example 5: Evaluate $\sin(a + b)$, where a and b are both angles in the second quadrant; given $\sin a = \frac{3}{5}$ and $\sin b = \frac{5}{13}$. Start by drawing both terminal arms in the second quadrant and solving for the third side.