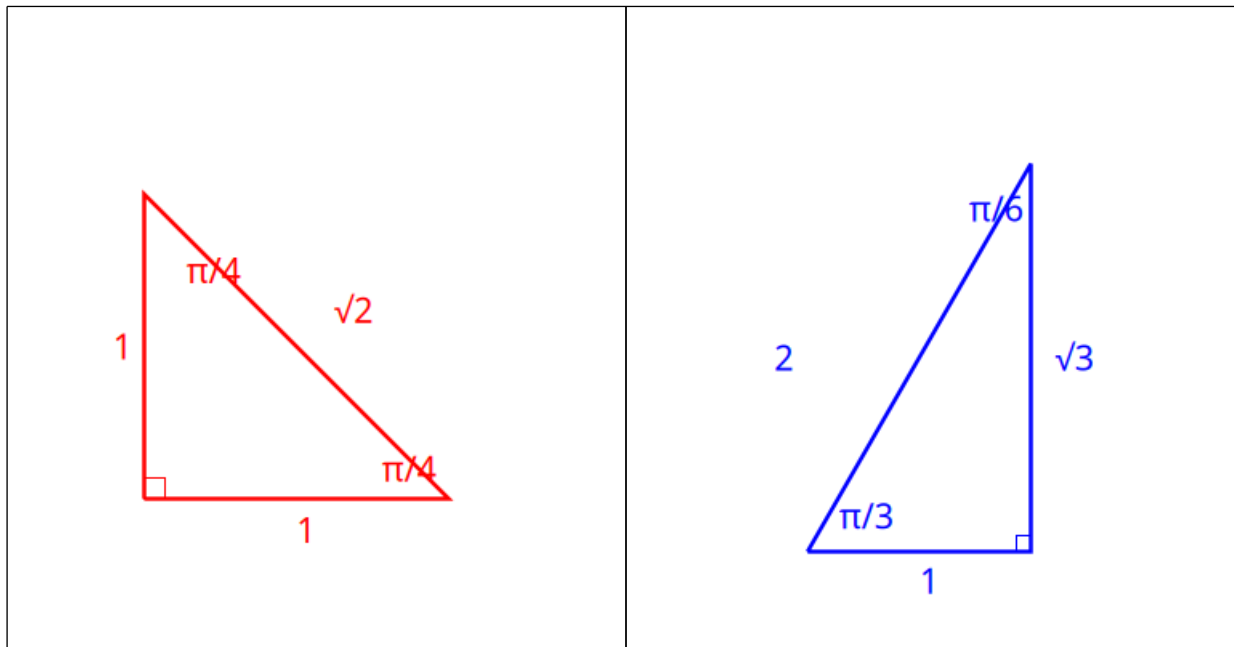


4.2 Trig Ratios and Special Angles Worksheet with solutions

This document was composed and formatted by Annie Bernatchez.

The course material originated from www.jensenmath.ca

1) Draw both special triangles using radian measures.



2) Use a calculator to evaluate each trigonometric ratio, to four decimal places.

a) $\cos 3.43$

$$\cong -0.9587$$

b) $\sin 2.92$

$$\cong 0.2198$$

c) $\tan 5.61$

$$\cong -0.7975$$

d) $\csc 1.27$

$$= \frac{1}{\sin 1.27} \cong 1.0470$$

e) $\cot 4.53$

$$= \frac{1}{\tan 4.53} \cong 0.1844$$

f) $\sec 0.98$

$$= \frac{1}{\cos 0.98} \cong 1.7953$$

3) Use a calculator to evaluate each trigonometric ratio, to four decimal places.

a) $\cot \frac{3\pi}{7}$

$$= \frac{1}{\tan \left(\frac{3\pi}{7} \right)} \cong 0.2282$$

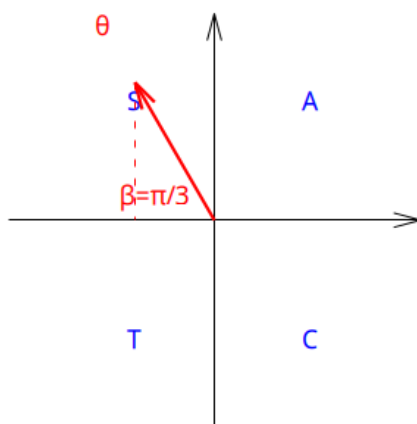
b) $\sec \frac{16\pi}{3}$

$$= \frac{1}{\cos \left(\frac{16\pi}{3} \right)} = -2$$

$$\begin{aligned} \text{c) } & \csc \frac{5\pi}{11} \\ &= \frac{1}{\sin\left(\frac{5\pi}{11}\right)} \cong 1.0103 \end{aligned}$$

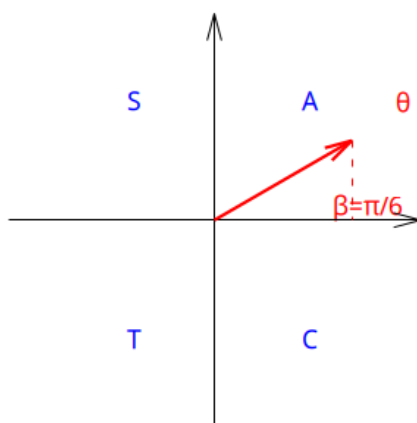
4) Use the unit circle and the cast rule to find exact expressions for each ratio

a) $\sin \frac{2\pi}{3}$



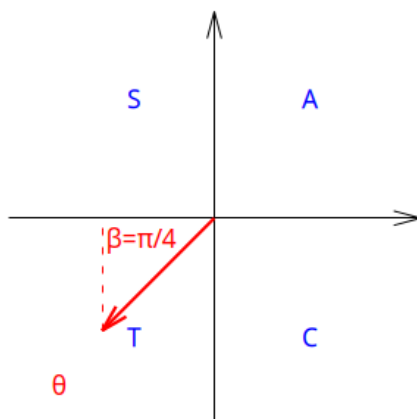
$$\sin\left(\frac{2\pi}{3}\right) = \sin\left(\frac{\pi}{3}\right) = \frac{\sqrt{3}}{2}$$

b) $\tan \frac{\pi}{6}$



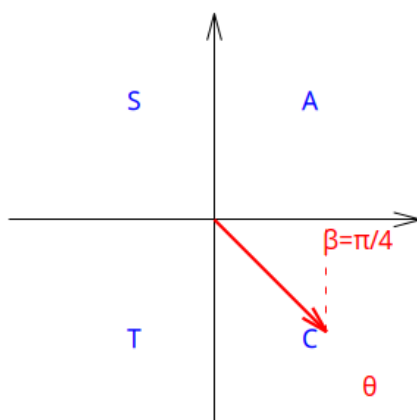
$$\tan \frac{\pi}{6} = \frac{1}{\sqrt{3}}$$

c) $\cos \frac{5\pi}{4}$



$$\cos \left(\frac{5\pi}{4} \right) = - \cos \left(\frac{\pi}{4} \right) = - \frac{1}{\sqrt{2}}$$

d) $\tan \frac{7\pi}{4}$

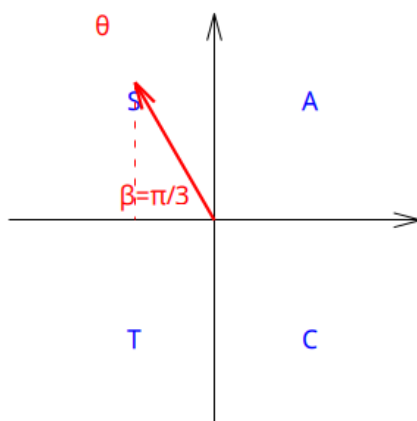


$$\tan \left(\frac{7\pi}{4} \right) = - \tan \left(\frac{\pi}{4} \right) = - 1$$

5) Use the unit circle and cast rule to determine exact values of the primary trig ratios for each angle.

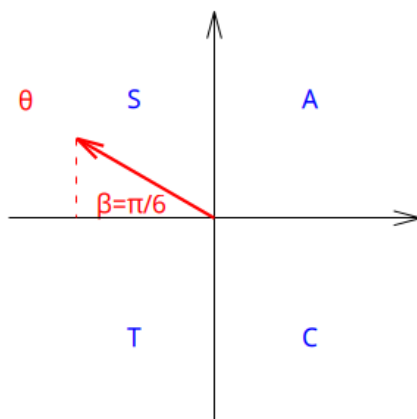
5) Use the unit circle and cast rule to determine exact values of the primary trig ratios for each angle.

a) $\frac{2\pi}{3}$



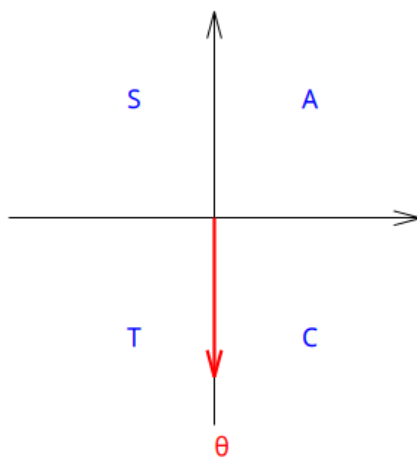
$$\sin \frac{2\pi}{3} = \frac{\sqrt{3}}{2} ; \cos \frac{2\pi}{3} = -\frac{1}{2} ; \tan \frac{2\pi}{3} = -\sqrt{3}$$

b) $\frac{5\pi}{6}$



$$\sin \frac{5\pi}{6} = \frac{1}{2} ; \cos \frac{5\pi}{6} = -\frac{\sqrt{3}}{2} ; \tan \frac{5\pi}{6} = -\frac{1}{\sqrt{3}}$$

c) $\frac{3\pi}{2}$

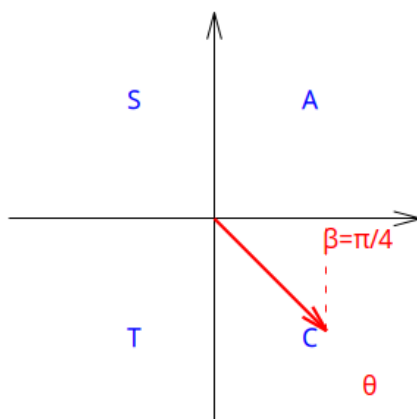


$$\sin \theta = \frac{y}{r} = \frac{-1}{1} = -1$$

$$\cos \theta = \frac{x}{r} = \frac{0}{1} = 0$$

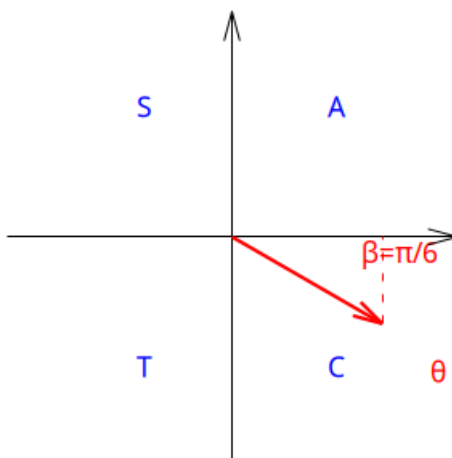
$$\tan \theta = \frac{y}{x} = \frac{-1}{0} = \text{undefined}$$

d) $\frac{7\pi}{4}$



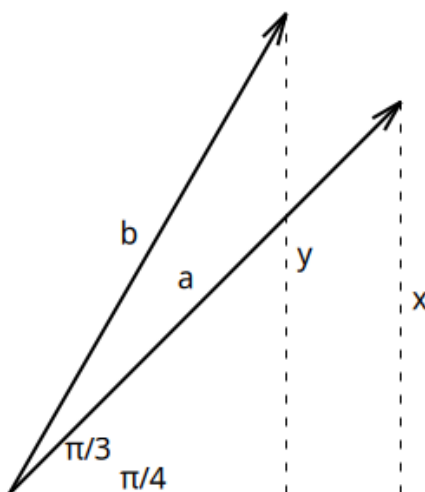
$$\sin \frac{7\pi}{4} = -\frac{1}{\sqrt{2}} ; \cos \frac{7\pi}{4} = \frac{1}{\sqrt{2}} ; \tan \frac{7\pi}{4} = -1$$

6) Use the special triangles determine exact values for the six trigonometric ratios for $\frac{11\pi}{6}$.



$\sin \frac{11\pi}{6} = -\sin \frac{\pi}{6} = -\frac{1}{2}$	$\csc \frac{11\pi}{6} = -2$
$\cos \frac{11\pi}{6} = \cos \frac{\pi}{6} = \frac{\sqrt{3}}{2}$	$\sec \frac{11\pi}{6} = \frac{2}{\sqrt{3}}$
$\tan \frac{11\pi}{6} = -\tan \frac{\pi}{6} = -\frac{1}{\sqrt{3}}$	$\cot \frac{11\pi}{6} = -\sqrt{3}$

7) Lynda is flying her kite at the end of a 40-m string. The string makes an angle of $\frac{\pi}{4}$ with the ground. The wind speed increases, and the kite flies higher until the string makes an angle of $\frac{\pi}{3}$ with the ground.



a) Determine an exact expression for the horizontal distance that the kite moves between the two positions.

$$\begin{aligned}\cos \frac{\pi}{4} &= \frac{x}{40} \\ \frac{1}{\sqrt{2}} &= \frac{x}{40} \\ x &= \frac{40}{\sqrt{2}} \\ x &= 20\sqrt{2}\end{aligned}$$

$$\begin{aligned}\cos \frac{\pi}{3} &= \frac{y}{40} \\ \frac{1}{2} &= \frac{y}{40} \\ y &= 20\end{aligned}$$

$$\text{Horizontal distance} = x - y = 20\sqrt{2} - 20 = 20(\sqrt{2} - 1) \text{ m}$$

b) Determine an exact expression for the vertical distance that the kite moves between the two positions.

$$\begin{aligned}\sin \frac{\pi}{4} &= \frac{a}{40} \\ \frac{1}{\sqrt{2}} &= \frac{a}{40} \\ a &= \frac{40}{\sqrt{2}} \\ a &= 20\sqrt{2} \\ \sin \frac{\pi}{3} &= \frac{b}{40} \\ \frac{\sqrt{3}}{2} &= \frac{b}{40} \\ b &= 20\sqrt{3}\end{aligned}$$

$$\text{Vertical distance} = b - a = 20\sqrt{3} - 20\sqrt{2} = 20(\sqrt{3} - \sqrt{2}) \text{ m}$$

8) Determine an exact value for each expression

a) $\frac{\sin \frac{\pi}{3} \tan \frac{\pi}{6}}{\cos \frac{\pi}{4}}$

$$\begin{aligned}\frac{\sin \frac{\pi}{3} \tan \frac{\pi}{6}}{\cos \frac{\pi}{4}} &= \frac{\left(\frac{\sqrt{3}}{2}\right)\left(\frac{1}{\sqrt{3}}\right)}{\left(\frac{1}{\sqrt{2}}\right)} \\ &= \frac{\left(\frac{1}{2}\right)}{\left(\frac{1}{\sqrt{2}}\right)} \\ &= \frac{\sqrt{2}}{2}\end{aligned}$$

b) $\cot \frac{5\pi}{4} + \tan \frac{11\pi}{6} \tan \frac{5\pi}{3}$

$$\begin{aligned}\cot \frac{5\pi}{4} + \tan \frac{11\pi}{6} \tan \frac{5\pi}{3} &= \cot \frac{\pi}{4} - \tan \frac{\pi}{6} \left(-\tan \frac{\pi}{3}\right) \\ &= 1 - \left(\frac{1}{\sqrt{3}}\right)(-\sqrt{3}) \\ &= 1 - (-1) \\ &= 2\end{aligned}$$