

5.3 Transformations of Trig Functions Lesson with solutions

This document was composed and formatted by Annie Bernatchez.

The course material originated from www.jensenmath.ca

Part 1: Transformation Properties

For a transformed sine or cosine function of the form $y = a \sin[k(x - d)] + c$ or $y = a \cos[k(x - d)] + c$, each parameter affects the graph in the following way:

a	k	d	c
amplitude = $ a $; if $a < 0$, the graph is reflected in the x -axis	period = $\frac{2\pi}{ k }$; if $k < 0$, the graph is reflected in the y -axis	phase shift; the graph shifts d units to the right (if $d > 0$) or left (if $d < 0$)	vertical shift; the graph shifts c units up (if $c > 0$) or down (if $c < 0$)

Example 1: For the function $y = 3 \sin \left[\frac{1}{2} \left(\theta + \frac{\pi}{3} \right) \right] - 1$, determine the amplitude, period, phase shift, vertical shift, maximum, and minimum.

Amplitude: $ a = 3$	Period: $\frac{2\pi}{ k } = \frac{2\pi}{\frac{1}{2}} = 4\pi$
Phase shift: $\frac{\pi}{3}$ to the left	Vertical shift: 1 down
Max: $c + a = -1 + 3 = 2$	Min: $c - a = -1 - 3 = -4$

Part 2: Given the Equation, Graph the Function

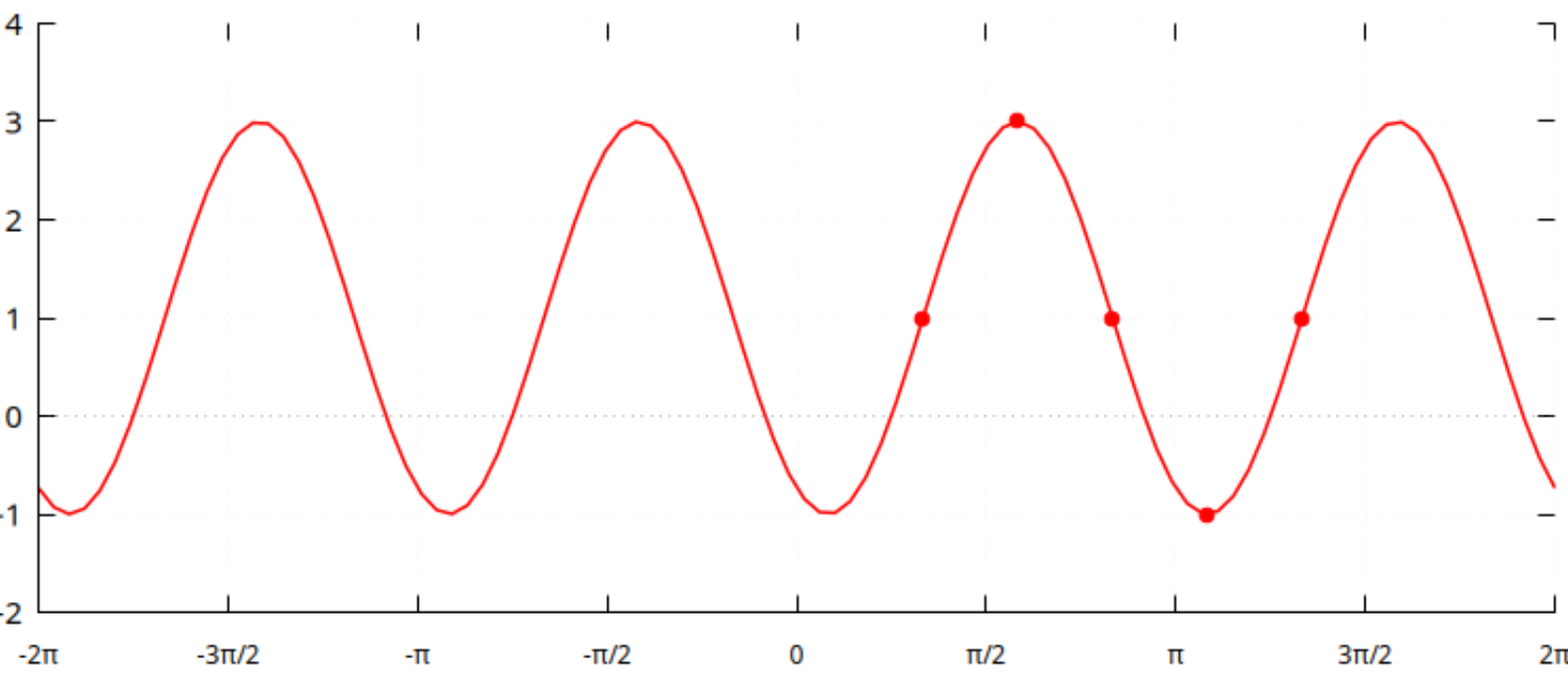
To graph a transformed sine or cosine function, apply the transformations to key points of the parent function $y = \sin x$ or $y = \cos x$.

Example 2: Graph $y = 2 \sin[2(x - \frac{\pi}{3})] + 1$ using transformations.

x	$y = \sin x$
0	0
$\frac{\pi}{2}$	1
π	0
$\frac{3\pi}{2}$	-1
2π	0

$\frac{x}{2} + \frac{\pi}{3}$	$2y + 1$
$\frac{\pi}{3}$	1
$\frac{7\pi}{12}$	3
$\frac{5\pi}{6}$	1
$\frac{13\pi}{12}$	-1
$\frac{4\pi}{3}$	1

Amplitude = 2; Period = π radians

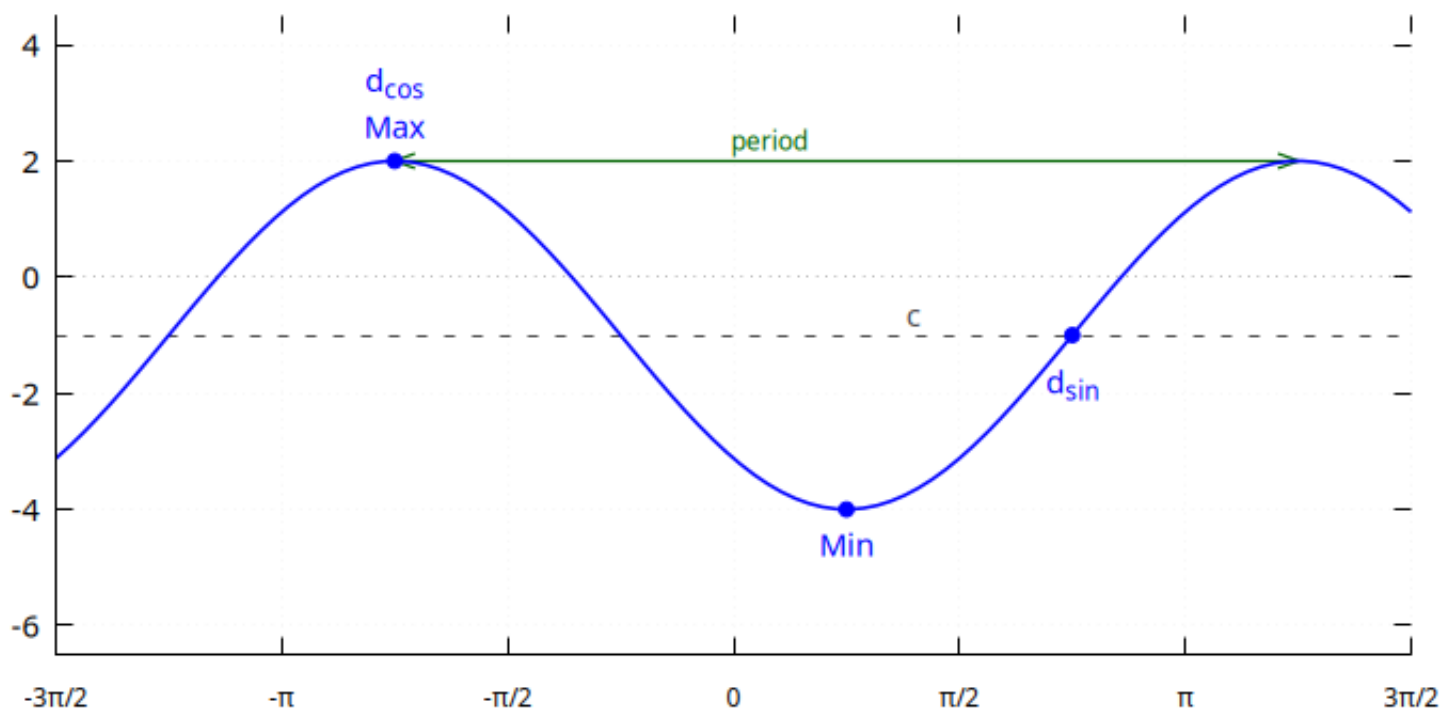


Part 3: Given the Graph, Write the Equation

To find the equation of a transformed sine or cosine function from its graph, use the following relationships:

a	k	d	c
$a = \frac{\text{max} - \text{min}}{2}$	$k = \frac{2\pi}{\text{period}}$, found from the start and end of one cycle	$d_{\sin} = d_{\cos} - \frac{\pi}{2k}$ is the x -coordinate of a point where the graph crosses the midline while increasing; $d_{\cos} = d_{\sin} + \frac{\pi}{2k}$ is the x -coordinate of a maximum	$c = \text{max} - a $, or equivalently $c = \frac{\text{max} + \text{min}}{2}$

Example 3: Determine a sine equation and a cosine equation for the function shown in the graph.



$$a = \frac{\text{max} - \text{min}}{2} = \frac{2 - (-4)}{2} = 3$$

$$k = \frac{2\pi}{\text{period}} = \frac{2\pi}{2\pi} = 1$$

$$c = \text{max} - |a| = 2 - 3 = -1$$

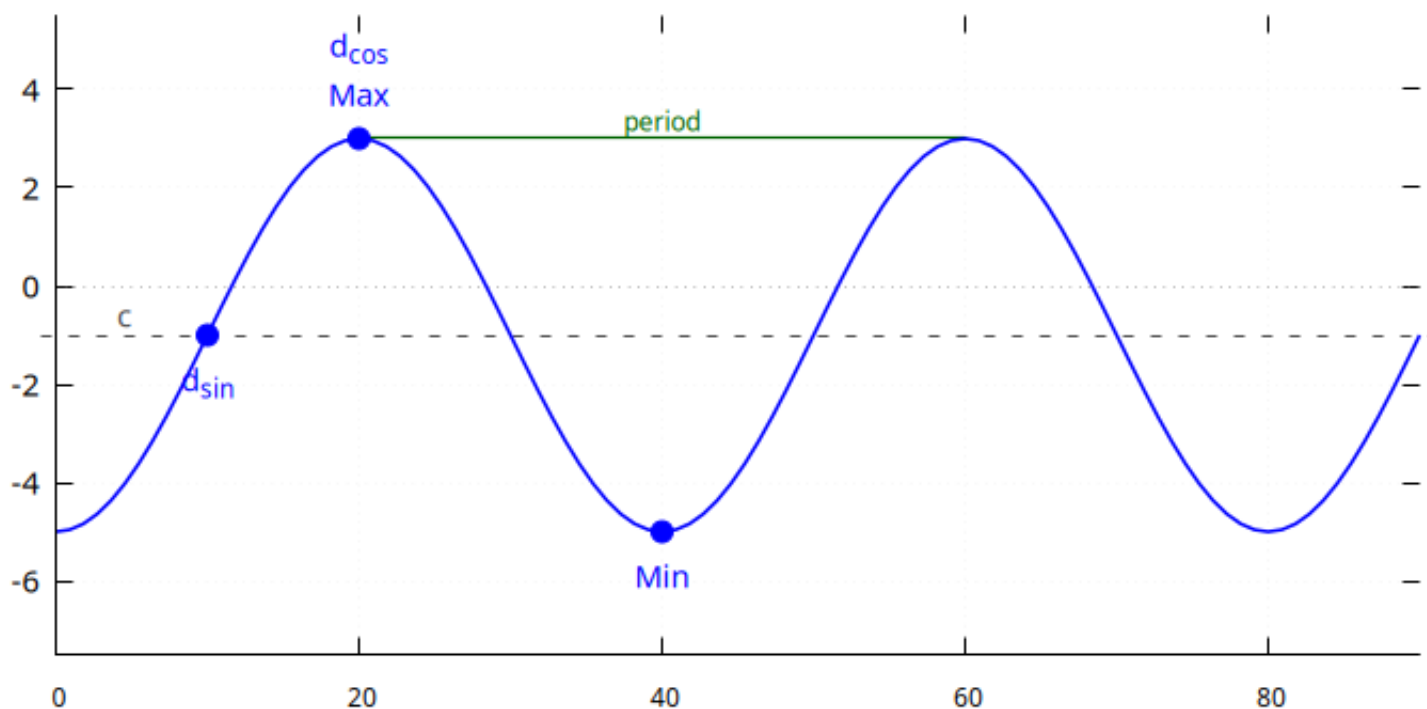
$$d_{\cos} = -\frac{3\pi}{4} \text{ (read from the graph)}$$

$$d_{\sin} = \frac{3\pi}{4} \text{ (read from the graph)}$$

$$y = 3 \sin \left(x - \frac{3\pi}{4} \right) - 1$$

$$y = 3 \cos \left(x + \frac{3\pi}{4} \right) - 1$$

Example 4: Determine a sine equation and a cosine equation for the function shown in the graph.



$$a = \frac{\text{max} - \text{min}}{2} = \frac{3 - (-5)}{2} = 4$$

$$k = \frac{2\pi}{\text{period}} = \frac{2\pi}{40} = \frac{\pi}{20}$$

$$c = \text{max} - |a| = 3 - 4 = -1$$

$$d_{\cos} = 20 \text{ (read from the graph)}$$

$$d_{\sin} = 10 \text{ (read from the graph)}$$

$$y = 4 \cos \left[\frac{\pi}{20}(x - 20) \right] - 1$$

$$y = 4 \sin \left[\frac{\pi}{20}(x - 10) \right] - 1$$

Example 5:

a) Create a sine function with amplitude 7, period π , phase shift $\frac{\pi}{4}$ to the right, and vertical displacement 3 down.

$$a = 7$$

$$k = \frac{2\pi}{\pi} = 2$$

$$c = -3$$

$$d = \frac{\pi}{4}$$

$$y = 7 \sin \left[2 \left(x - \frac{\pi}{4} \right) \right] - 3$$

b) Convert this function to an equivalent cosine equation.

$$d_{\cos} = d_{\sin} + \frac{\pi}{2k} = \frac{\pi}{4} + \frac{\pi}{4} = \frac{\pi}{2}$$

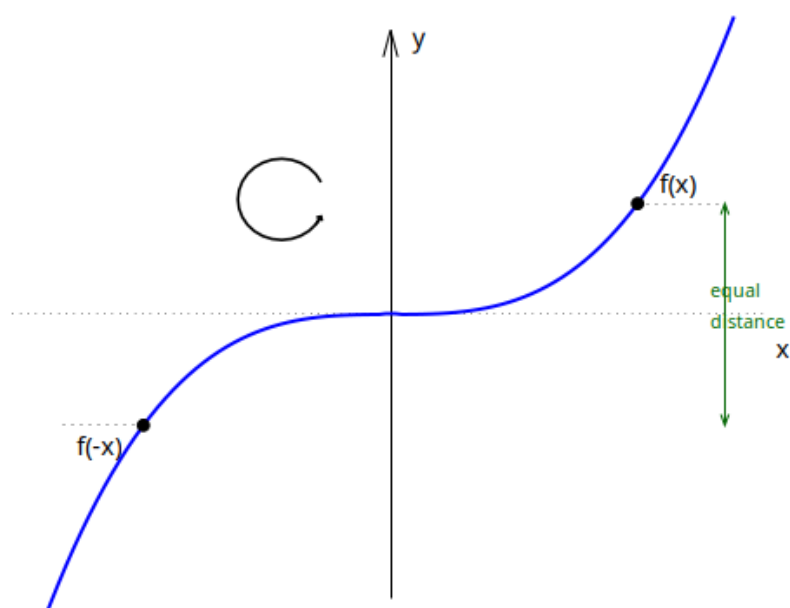
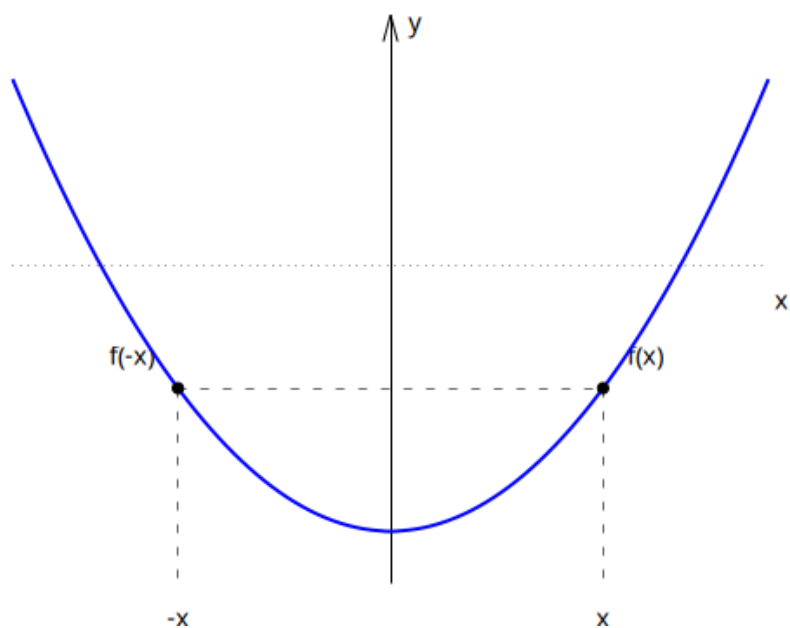
$$y = 7 \cos \left[2 \left(x - \frac{\pi}{2} \right) \right] - 3$$

Part 4: Even and Odd Functions

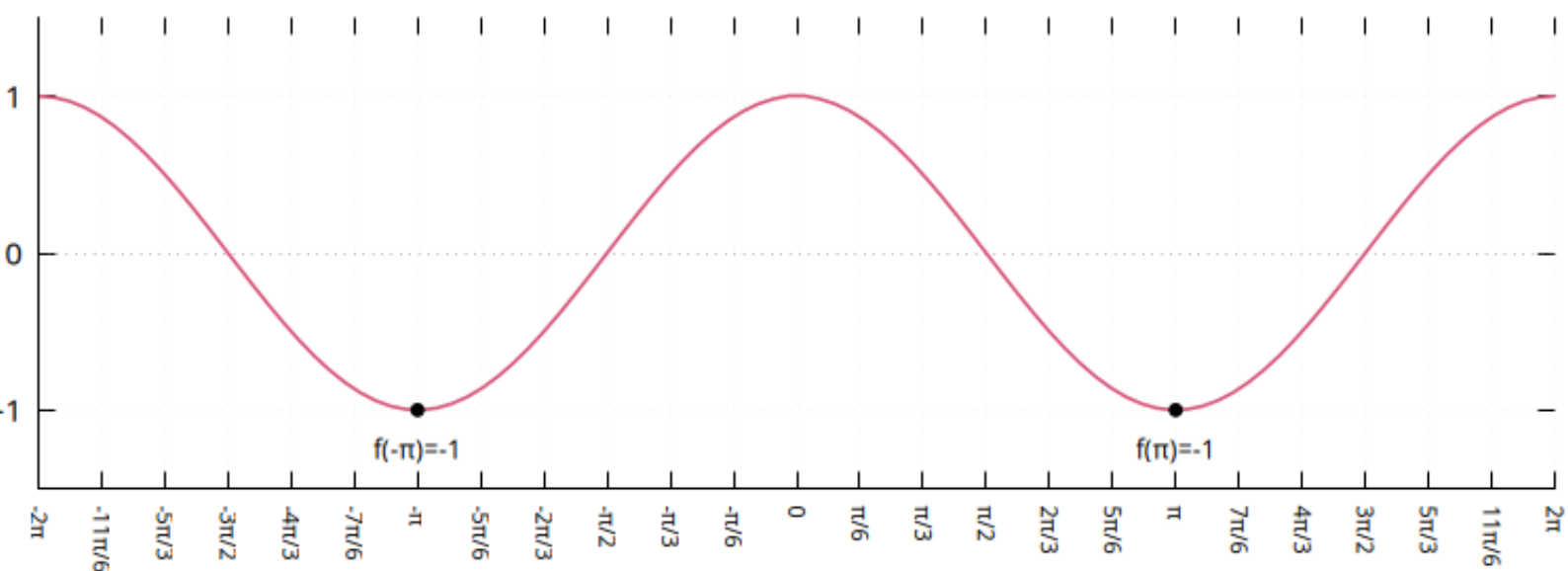
Even Functions	Odd Functions
Line symmetry over the y-axis	Point symmetry about the origin (0,0)

Rule: $f(-x) = f(x)$

Rule: $-f(x) = f(-x)$



Example: $y = \cos(x)$ is an even function.

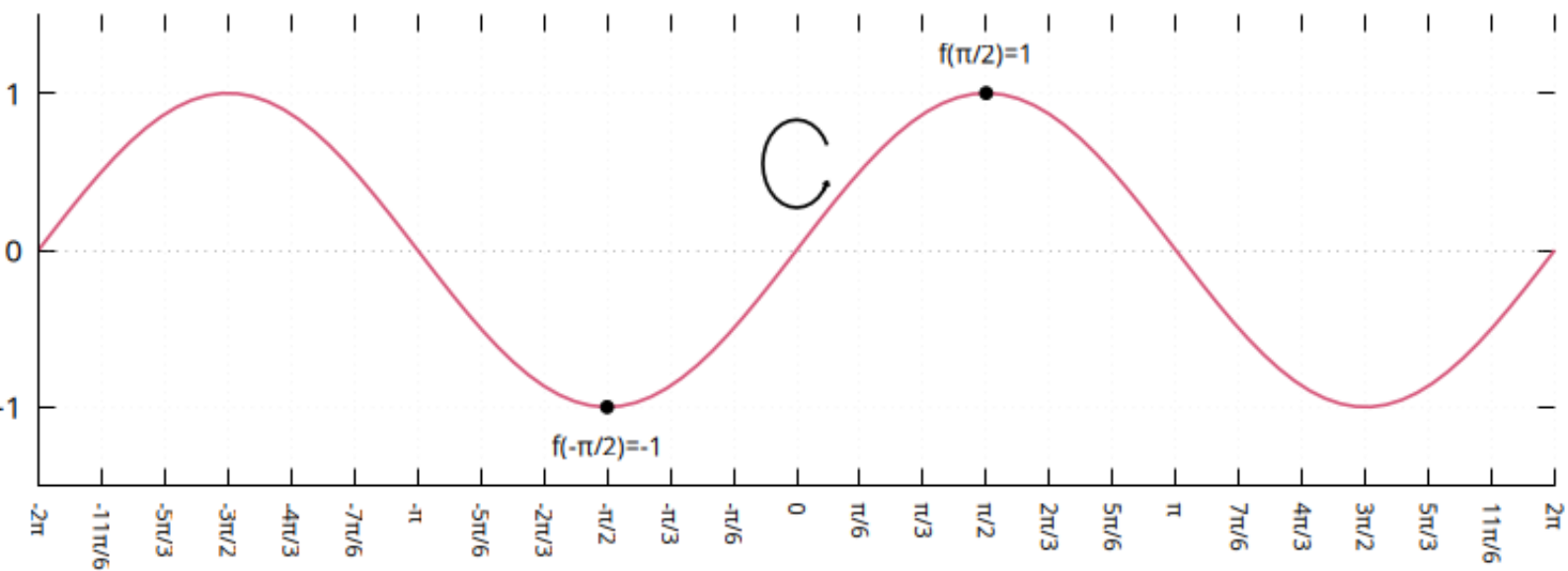


$$f(\pi) = -1$$

$$f(-\pi) = -1$$

Therefore, $f(\pi) = f(-\pi)$

Example: $y = \sin(x)$ is an odd function.



$$f\left(\frac{\pi}{2}\right) = 1$$

$$f\left(-\frac{\pi}{2}\right) = -1$$

Therefore, $-f\left(\frac{\pi}{2}\right) = f\left(-\frac{\pi}{2}\right)$

$y = \tan(x)$ is also an odd function.

