

5.1/5.2 Graphing Trig Functions Lesson with solutions

This document was composed and formatted by Annie Bernatchez.

The course material originated from www.jensenmath.ca

Part 1: Remember the Unit Circle

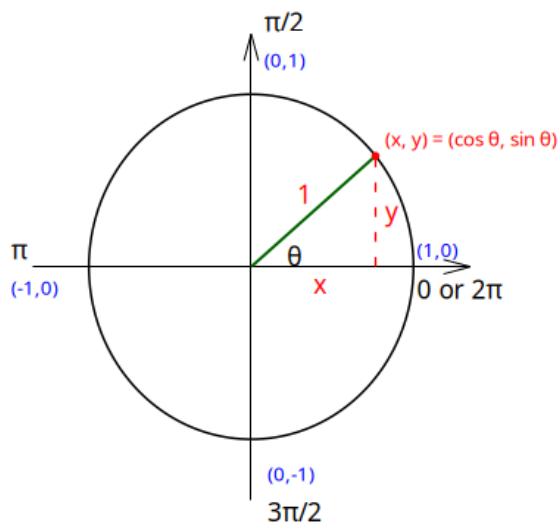
The unit circle is a circle that is centered at the origin and has a radius of **1 unit**. On the unit circle, the sine and cosine functions take a simple form:

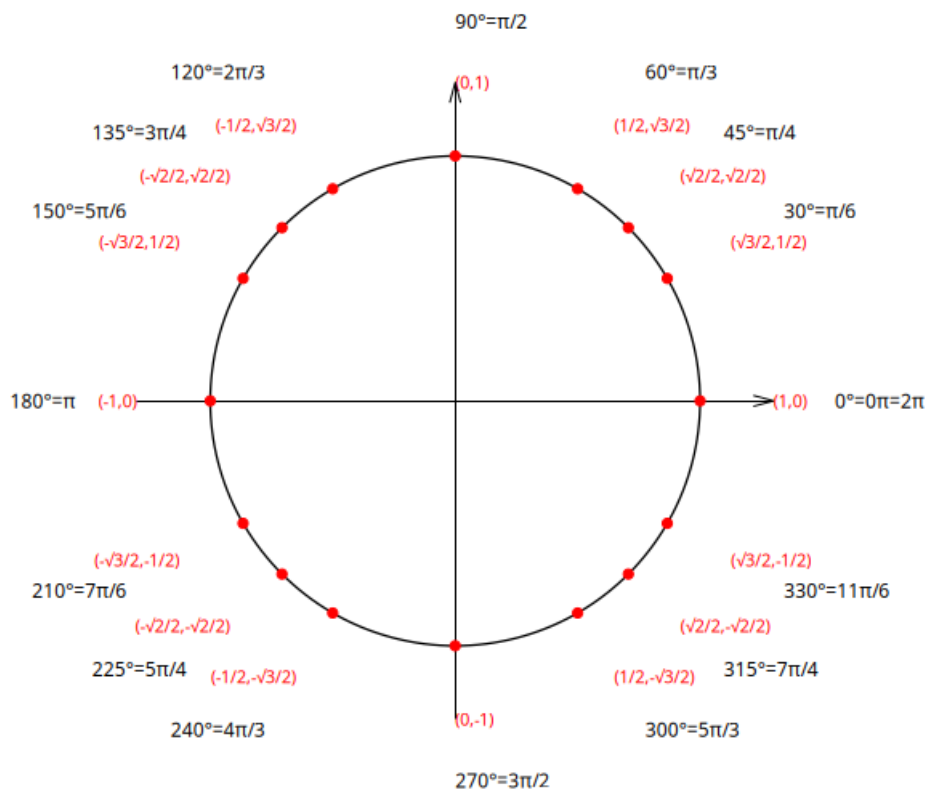
$$\sin \theta = \frac{y}{r} = \frac{y}{1} = y$$

$$\cos \theta = \frac{x}{r} = \frac{x}{1} = x$$

The value of $\sin \theta$ is the **y-coordinate** of each point on the unit circle

The value of $\cos \theta$ is the **x-coordinate** of each point on the unit circle





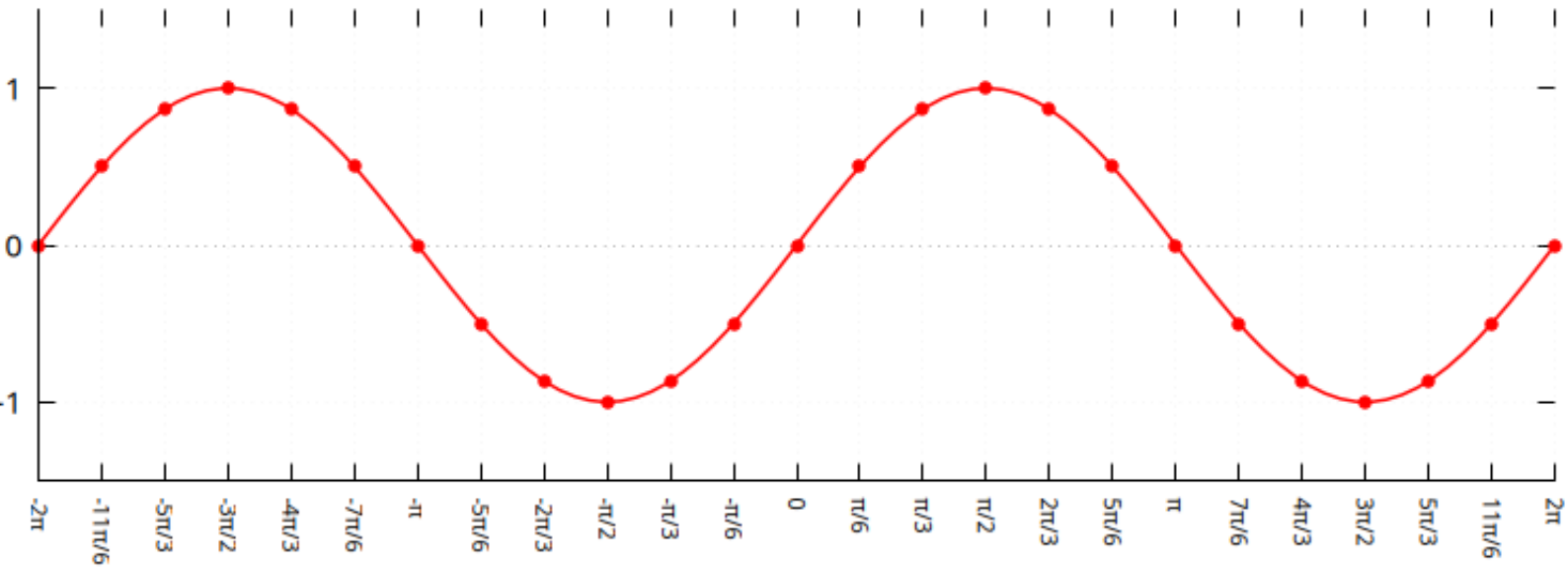
Part 2: Graphing Sine and Cosine

To graph sine and cosine, we will be using a Cartesian plane that has angles for x values.

Example 1: Complete the following table of values for the function $f(x) = \sin(x)$. Use special triangles, the unit circle, or a calculator to find values for the function at $30^\circ = \frac{\pi}{6}$ radian intervals.

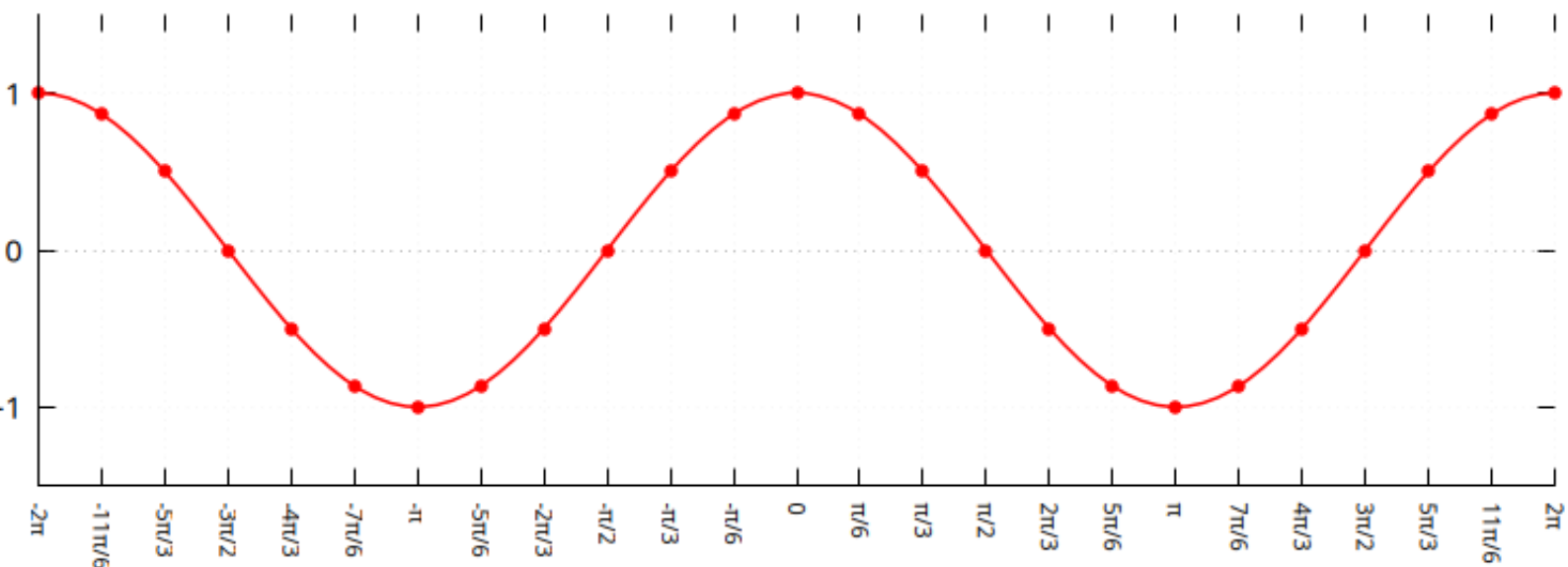
x	$\sin x$
0	0
$\frac{\pi}{6}$	$\frac{1}{2}$
$\frac{2\pi}{6} = \frac{\pi}{3}$	$\frac{\sqrt{3}}{2} \cong 0.87$
$\frac{3\pi}{6} = \frac{\pi}{2}$	1
$\frac{4\pi}{6} = \frac{2\pi}{3}$	$\frac{\sqrt{3}}{2} \cong 0.87$
$\frac{5\pi}{6}$	$\frac{1}{2}$
$\frac{6\pi}{6} = \pi$	0
$\frac{7\pi}{6}$	$-\frac{1}{2}$
$\frac{8\pi}{6} = \frac{4\pi}{3}$	$-\frac{\sqrt{3}}{2} \cong -0.87$
$\frac{9\pi}{6} = \frac{3\pi}{2}$	-1
$\frac{10\pi}{6} = \frac{5\pi}{3}$	$-\frac{\sqrt{3}}{2} \cong -0.87$

$\frac{11\pi}{6}$	$-\frac{1}{2}$
$\frac{12\pi}{6} = 2\pi$	0



Example 2: Complete the following table of values for the function $f(x) = \cos(x)$. Use special triangles, the unit circle, or a calculator to find values for the function at $30^\circ = \frac{\pi}{6}$ radian intervals.

x	$\cos x$
0	1
$\frac{\pi}{6}$	$\frac{\sqrt{3}}{2} \cong 0.87$
$\frac{2\pi}{6} = \frac{\pi}{3}$	$\frac{1}{2}$
$\frac{3\pi}{6} = \frac{\pi}{2}$	0
$\frac{4\pi}{6} = \frac{2\pi}{3}$	$-\frac{1}{2}$
$\frac{5\pi}{6}$	$-\frac{\sqrt{3}}{2} \cong -0.87$
$\frac{6\pi}{6} = \pi$	-1
$\frac{7\pi}{6}$	$-\frac{\sqrt{3}}{2} \cong -0.87$
$\frac{8\pi}{6} = \frac{4\pi}{3}$	$-\frac{1}{2}$
$\frac{9\pi}{6} = \frac{3\pi}{2}$	0
$\frac{10\pi}{6} = \frac{5\pi}{3}$	$\frac{1}{2}$
$\frac{11\pi}{6}$	$\frac{\sqrt{3}}{2} \cong 0.87$
$\frac{12\pi}{6} = 2\pi$	1



Properties of both Sine and Cosine Functions

Domain: $\{x \in \mathbb{R}\}$

Range: $\{y \in \mathbb{R} \mid -1 \leq y \leq 1\}$

Period: 2π radians

Amplitude: $\frac{\max - \min}{2} = \frac{1 - (-1)}{2} = 1$

PERIOD: the horizontal length of one cycle on a graph.

AMPLITUDE: half the distance between the maximum and minimum values of a periodic function.

Part 3: Graphing the Tangent Function

Recall: $\tan \theta = \frac{\sin \theta}{\cos \theta}$

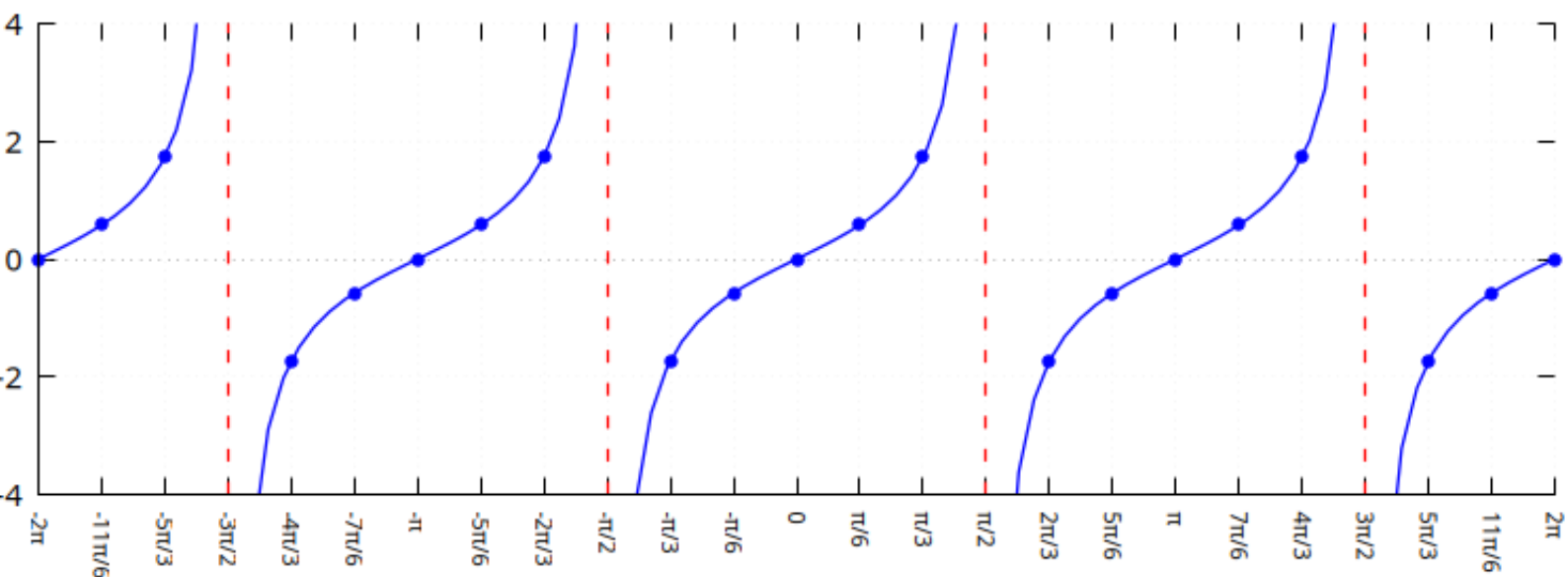
Note: Since $\cos \theta$ is in the denominator, any time $\cos \theta = 0$, $\tan \theta$ will be undefined which will lead to a vertical asymptote.

Since $\sin \theta$ is in the numerator, any time $\sin \theta = 0$, $\tan \theta$ will equal 0 which will be an x -intercept.

Example 3: Complete the following table of values for the function $f(x) = \tan(x)$. Use the quotient identity to find y -values.

x	$\tan x$
0	0
$\frac{\pi}{6}$	$\frac{1}{\sqrt{3}} \cong 0.58$
$\frac{2\pi}{6} = \frac{\pi}{3}$	$\sqrt{3} \cong 1.73$
$\frac{3\pi}{6} = \frac{\pi}{2}$	und.
$\frac{4\pi}{6} = \frac{2\pi}{3}$	$-\sqrt{3} \cong -1.73$
$\frac{5\pi}{6}$	$-\frac{1}{\sqrt{3}} \cong -0.58$
$\frac{6\pi}{6} = \pi$	0
$\frac{7\pi}{6}$	$\frac{1}{\sqrt{3}} \cong 0.58$
$\frac{8\pi}{6} = \frac{4\pi}{3}$	$\sqrt{3} \cong 1.73$

$\frac{9\pi}{6} = \frac{3\pi}{2}$	und.
$\frac{10\pi}{6} = \frac{5\pi}{3}$	$-\sqrt{3} \cong -1.73$
$\frac{11\pi}{6}$	$-\frac{1}{\sqrt{3}} \cong -0.58$
$\frac{12\pi}{6} = 2\pi$	0



Properties of the Tangent Function

Domain: $\{x \in \mathbb{R} \mid x \neq \frac{\pi + 2k\pi}{2}\}$ where $k \in \mathbb{Z}$

Range: $\{y \in \mathbb{R}\}$

Period: π radians

Amplitude: none (no max or min)

Part 4: Graphing Reciprocal Trig Functions

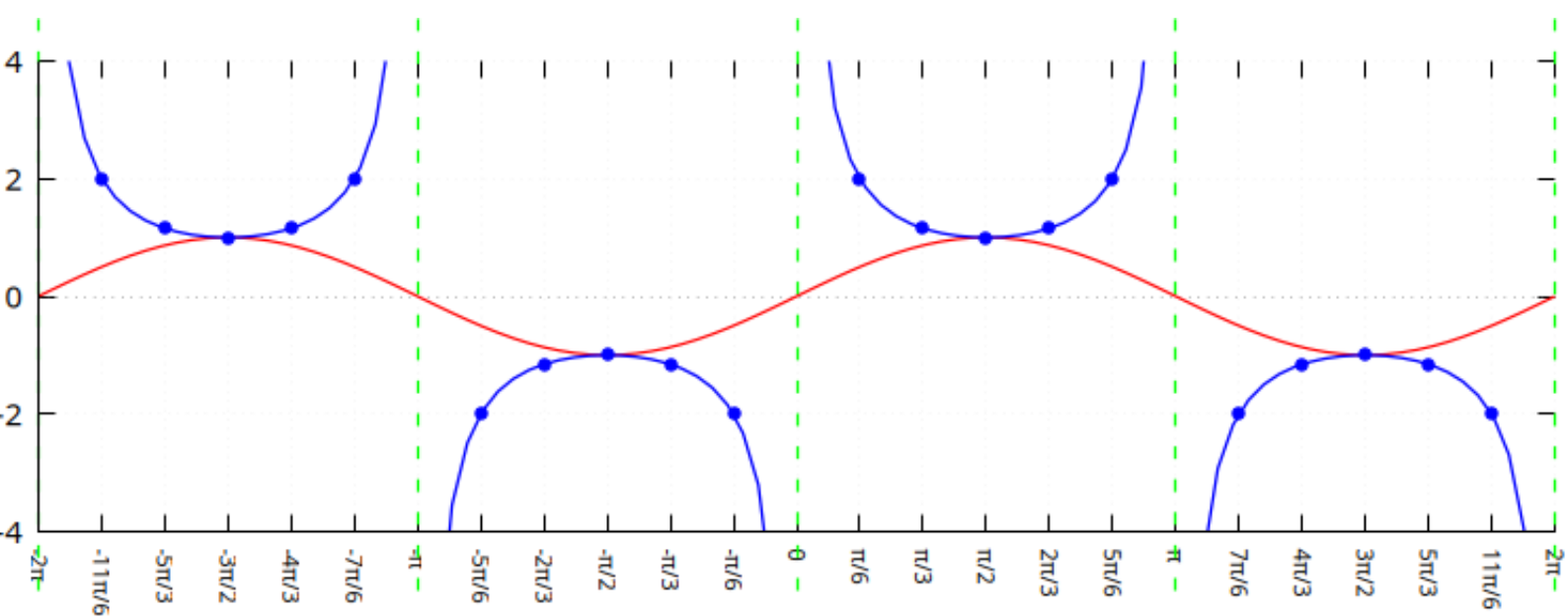
The graph of a reciprocal trig function is related to the graph of its corresponding primary trig function in the following ways:

- Reciprocal has a vertical asymptote at each zero of its primary trig function
- Reciprocal has a zero at each vertical asymptote of its primary trig function
- Has the same positive/negative intervals but intervals of increasing/decreasing are reversed
- y -values of 1 and -1 do not change and therefore this is where the reciprocal and primary intersect
- Local min points of the primary become local max of the reciprocal and vice versa.

Example 4: Complete the following table of values for the function $f(x) = \csc(x)$. Use the reciprocal identity to find y -values.

x	$\csc x$
0	und.
$\frac{\pi}{6}$	2
$\frac{2\pi}{6} = \frac{\pi}{3}$	$\frac{2}{\sqrt{3}} \cong 1.15$

$\frac{3\pi}{6} = \frac{\pi}{2}$	1
$\frac{4\pi}{6} = \frac{2\pi}{3}$	$\frac{2}{\sqrt{3}} \cong 1.15$
$\frac{5\pi}{6}$	2
$\frac{6\pi}{6} = \pi$	UNDEFINED
$\frac{7\pi}{6}$	-2
$\frac{8\pi}{6} = \frac{4\pi}{3}$	$-\frac{2}{\sqrt{3}} \cong -1.15$
$\frac{9\pi}{6} = \frac{3\pi}{2}$	-1
$\frac{10\pi}{6} = \frac{5\pi}{3}$	$-\frac{2}{\sqrt{3}} \cong -1.15$
$\frac{11\pi}{6}$	-2
$\frac{12\pi}{6} = 2\pi$	und.



Properties of the Cosecant Function

Domain: $\{x \in \mathbb{R} \mid x \neq k\pi\}$ where $k \in \mathbb{Z}$

Range: $\{y \in \mathbb{R} \mid y \leq -1 \text{ or } y \geq 1\}$

Period: 2π radians

Amplitude: none (no max or min)

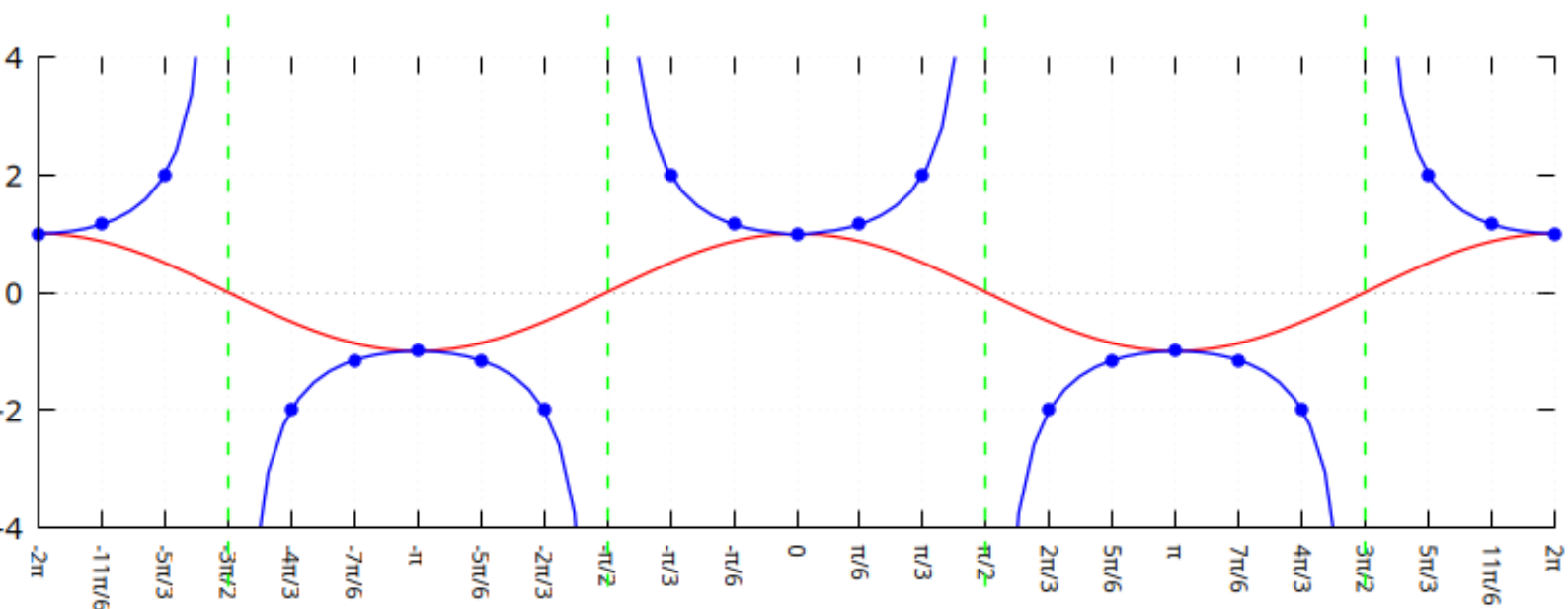
Reciprocal Identities

$$\csc \theta = \frac{1}{\sin \theta} \quad \sec \theta = \frac{1}{\cos \theta} \quad \cot \theta = \frac{1}{\tan \theta}$$

Example 5: Complete the following table of values for the function $f(x) = \sec(x)$. Use the reciprocal identity to find y-values.

x	$\sec x$
0	1
$\frac{\pi}{6}$	$\frac{2}{\sqrt{3}} \cong 1.15$
$\frac{2\pi}{6} = \frac{\pi}{3}$	2
$\frac{3\pi}{6} = \frac{\pi}{2}$	und.

$\frac{4\pi}{6} = \frac{2\pi}{3}$	-2
$\frac{5\pi}{6}$	$-\frac{2}{\sqrt{3}} \cong -1.15$
$\frac{6\pi}{6} = \pi$	-1
$\frac{7\pi}{6}$	$-\frac{2}{\sqrt{3}} \cong -1.15$
$\frac{8\pi}{6} = \frac{4\pi}{3}$	-2
$\frac{9\pi}{6} = \frac{3\pi}{2}$	und.
$\frac{10\pi}{6} = \frac{5\pi}{3}$	2
$\frac{11\pi}{6}$	$\frac{2}{\sqrt{3}} \cong 1.15$
$\frac{12\pi}{6} = 2\pi$	1



Properties of the Secant Function

Domain: $\{x \in \mathbb{R} \mid x \neq \frac{\pi + 2k\pi}{2}\}$ where $k \in \mathbb{Z}$

Range: $\{y \in \mathbb{R} \mid y \leq -1 \text{ or } y \geq 1\}$

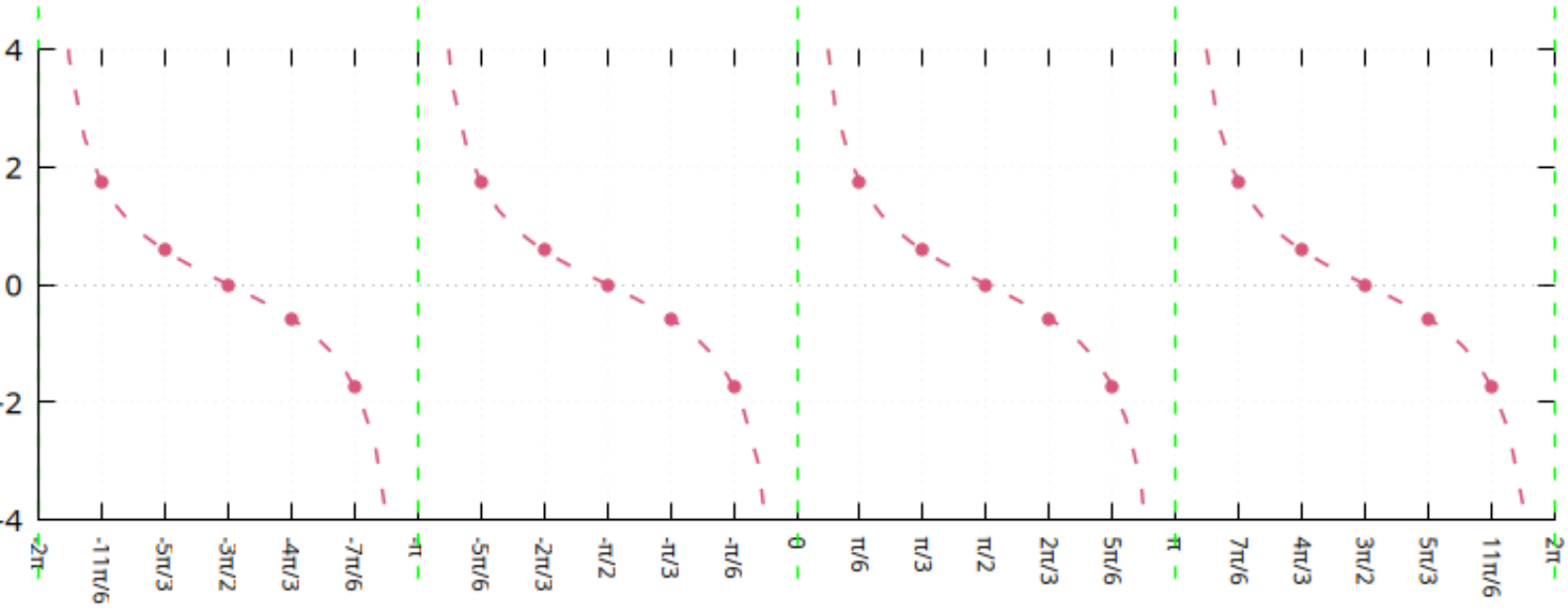
Period: 2π radians

Amplitude: none (no max or min)

Example 6: Complete the following table of values for the function $f(x) = \cot(x)$. Use the reciprocal identity to find y-values.

x	$\cot x$
0	und.
$\frac{\pi}{6}$	$\sqrt{3} \cong 1.73$
$\frac{2\pi}{6} = \frac{\pi}{3}$	$\frac{1}{\sqrt{3}} \cong 0.58$
$\frac{3\pi}{6} = \frac{\pi}{2}$	0
$\frac{4\pi}{6} = \frac{2\pi}{3}$	$-\frac{1}{\sqrt{3}} \cong -0.58$
$\frac{5\pi}{6}$	$-\sqrt{3} \cong -1.73$
$\frac{6\pi}{6} = \pi$	und.

$\frac{7\pi}{6}$	$\sqrt{3} \cong 1.73$
$\frac{8\pi}{6} = \frac{4\pi}{3}$	$\frac{1}{\sqrt{3}} \cong 0.58$
$\frac{9\pi}{6} = \frac{3\pi}{2}$	0
$\frac{10\pi}{6} = \frac{5\pi}{3}$	$-\frac{1}{\sqrt{3}} \cong -0.58$
$\frac{11\pi}{6}$	$-\sqrt{3} \cong -1.73$
$\frac{12\pi}{6} = 2\pi$	und.



Properties of the Cotangent Function

Domain: $\{x \in \mathbb{R} \mid x \neq k\pi\}$ where $k \in \mathbb{Z}$

Range: $\{y \in \mathbb{R}\}$

Period: π radians

Amplitude: none (no max or min)