

3.8 The Natural Logarithm Worksheet with solutions

This document was composed and formatted by Annie Bernatchez.

The course material originated from www.jensenmath.ca

1) Use a calculator to approximate each to the nearest thousandth

a) $\ln 6.2$

$$\cong 1.825$$

b) $\ln 2.1$

$$\cong 0.742$$

c) $\ln e$

$$= 1$$

d) e^5

$$\cong 148.413$$

2) Expand each logarithm

a) $\ln x^2$

$$= 2\ln x$$

b) $\ln \sqrt[3]{x}$

$$\begin{aligned}\ln \sqrt[3]{x} &= \ln \left(x^{\frac{1}{3}}\right) \\ &= \frac{1}{3} \ln x\end{aligned}$$

c) $\ln \frac{u^3}{wv^4}$

$$\begin{aligned}\ln \frac{u^3}{wv^4} &= \ln(u^3) - \ln(wv^4) \\ &= 3\ln(u) - [\ln(w) + \ln(v^4)] \\ &= 3\ln(u) - \ln(w) - 4\ln(v)\end{aligned}$$

3) Condense each expression to a single logarithm

a) $4\ln 2$

$$\begin{aligned}4\ln 2 &= \ln(2^4) \\ &= \ln(16)\end{aligned}$$

b) $\ln 10 - 5\ln 7$

$$\begin{aligned}\ln 10 - 5\ln 7 &= \ln(10) - \ln(7^5) \\ &= \ln\left(\frac{10}{7^5}\right) \\ &= \ln\left(\frac{10}{16807}\right)\end{aligned}$$

c) $3\ln x + 3\ln y$

$$\begin{aligned}3\ln x + 3\ln y &= \ln(x^3) + \ln(y^3) \\ &= \ln(x^3y^3)\end{aligned}$$

4) Solve each equation. Round your answer to 4 decimal places if necessary.

a) $e^x = 2$

$$\begin{aligned}x &= \ln(2) \\ x &\cong 0.6931\end{aligned}$$

b) $e^{-3n} = 83$

$$\begin{aligned}-3n &= \ln(83) \\ n &= \frac{\ln(83)}{-3} \\ n &\cong -1.4729\end{aligned}$$

c) $e^{k+7} = 26$

$$\begin{aligned}k + 7 &= \ln(26) \\ k &= \ln(26) - 7 \\ k &\cong -3.7419\end{aligned}$$

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d) $9e^{1.4p-10} - 10 = 17$

$$\begin{aligned}9e^{1.4p-10} &= 27 \\e^{1.4p-10} &= 3 \\1.4p - 10 &= \ln(3) \\p &= \frac{\ln(3) + 10}{1.4} \\p &\cong 7.9276\end{aligned}$$

e) $\ln x = -5$

$$\begin{aligned}e^{-5} &= x \\x &\cong 0.0067\end{aligned}$$

f) $7.316 = e^{\ln(2x)}$

Rule: $b^{\log_b(x)} = x$

$$\begin{aligned}7.316 &= 2x \\x &= \frac{7.316}{2} \\x &\cong 3.658\end{aligned}$$

g) $\ln(-m) = \ln(m+10)$

$$\begin{aligned}-m &= m+10 \\-2m &= 10 \\m &= -5\end{aligned}$$

h) $\ln(9x+1) = \ln(x^2+9)$

$$\begin{aligned}9x+1 &= x^2+9 \\0 &= x^2-9x+8 \\0 &= (x-8)(x-1) \\x &= 8, 1\end{aligned}$$

i) $\ln(1-8x) - 10 = -7$

$$\begin{aligned}\ln(1-8x) &= 3 \\e^3 &= 1-8x \\8x &= 1-e^3 \\x &= \frac{1-e^3}{8} \\x &\cong -2.3857\end{aligned}$$

j) $\ln(5-2x^2) + \ln 9 = \ln 43$

$$\begin{aligned}\ln(5-2x^2) &= \ln(43) - \ln(9) \\\ln(5-2x^2) &= \ln\left(\frac{43}{9}\right) \\5-2x^2 &= \frac{43}{9} \\5 - \frac{43}{9} &= 2x^2 \\\frac{2}{9} &= 2x^2 \\\frac{1}{9} &= x^2 \\x &= \pm \frac{1}{3}\end{aligned}$$