

## 3.6 Applications of Logarithms and Exponentials in Physical Sciences Worksheet with solutions

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### Exponential Formulas

| General                                | Half-life                                          | Doubling                      |
|----------------------------------------|----------------------------------------------------|-------------------------------|
| $A(t) = A_0(1 + i)^t$                  | $A(t) = A_0\left(\frac{1}{2}\right)^{\frac{t}{H}}$ | $A(t) = A_0(2)^{\frac{t}{D}}$ |
| $i$ is percent growth (+) or decay (-) | $H$ is the half-life period                        | $D$ is the doubling period    |

### Logarithmic Formulas

| pH                                                                      | Decibel                                                            | Richter                                                 |
|-------------------------------------------------------------------------|--------------------------------------------------------------------|---------------------------------------------------------|
| $pH = -\log[H^+]$                                                       | $\beta_2 - \beta_1 = 10\log\left(\frac{I_2}{I_1}\right)$           | $M = \log\left(\frac{I}{I_0}\right)$                    |
| $pH$ is acidity and $[H^+]$ is concentration of hydronium ions in mol/L | $\beta$ is loudness in dB and $I$ is intensity of sound in $W/m^2$ | $M$ is magnitude measured by richters, $I$ is intensity |

1) The half-life of a radioactive form of tritium is about 2 years. How much of a 5-kg sample of this material would remain after ...

a) 8 years

$$A(8) = 5\left(\frac{1}{2}\right)^{\frac{8}{2}}$$

$$A(8) = 5\left(\frac{1}{2}\right)^4$$

$$A(8) = \frac{5}{16}$$

$$A(8) = 0.3125 \text{ kg}$$

b) 12 months

12 months = 1 year

$$A(1) = 5\left(\frac{1}{2}\right)^{\frac{1}{2}}$$

$$A(1) \approx 3.536 \text{ kg}$$

2) The population of Littleton is currently (2014) 23 000, and is increasing exponentially with a growth rate of 2% per year. Estimate when Littleton will have a population of 30 000.

$$30\,000 = 23\,000(1.02)^t$$

$$\frac{30}{23} = (1.02)^t$$

$$t = \log_{1.02}\left(\frac{30}{23}\right)$$

$$t = \frac{\log\left(\frac{30}{23}\right)}{\log 1.02}$$

$$t \approx 13.42 \text{ years}$$

3) The population of purple martins in Algonquin park was estimated to be 35 000 in 1992. Ten years later, in 2002, the population had risen to 44 400.

a) What is the annual growth rate for the purple martin population?

$$44\,400 = 35\,000(b)^{10}$$

$$\frac{444}{350} = (b)^{10}$$

$$\sqrt[10]{\frac{444}{350}} = b$$

$$b \approx 1.024$$

The growth rate is about 2.4% per year.

b) Estimate the population for 2010 to the nearest hundred.

$$A(8) = 44\,400(1 + 0.024)^8$$

$$A(8) = 53\,676.3$$

$$A(8) \cong 53\,700$$

4) After an accident at a nuclear plant, the radiation level in the plant was 950 R (roentgens). Five hours later the level was 800 R. How long will it take before safe levels of radiation are reached, which is less than 0.01 R?

$$800 = 950(b)^5$$

$$b = \sqrt[5]{\frac{800}{950}}$$

$$b \cong 0.9662$$

$$0.01 = 950(0.9662)^t$$

$$\frac{0.01}{950} = 0.9662^t$$

$$t = \frac{\log\left(\frac{0.01}{950}\right)}{\log(0.9662)}$$

$$t \cong 333.3 \text{ hours}$$

5) The value of a new minivan drops 40% after the first year, and then decreases exponentially at a rate of 12% per year after that. When will a minivan that cost \$35 000 new be worth less than \$10 000?

Value after 1 year:  $35\,000(0.6) = 21\,000$

$$A(t) = A_0(1 - i)^t$$

$$10\,000 = 21\,000(1 - 0.12)^t$$

$$\frac{10}{21} = (0.88)^t$$

$$t = \log_{0.88}\left(\frac{10}{21}\right)$$

$$t = \frac{\log\left(\frac{10}{21}\right)}{\log(0.88)}$$

$$t \cong 5.8 \text{ years}$$

1 year at 40%, and then about 5.8 years at 12%; this means it would take about 6.8 years total for the minivan to be worth less than \$10 000.

6) A crab fossil contains 38.6% of its original Carbon-14 isotope, which has a half life of 5370 years. Approximately how old is the crab fossil?

$$A(t) = A_0\left(\frac{1}{2}\right)^{\frac{t}{H}}$$

$$0.386(A_0) = A_0\left(\frac{1}{2}\right)^{\frac{t}{5370}}$$

$$0.386 = \left(\frac{1}{2}\right)^{\frac{t}{5370}}$$

$$\log(0.386) = \frac{t}{5370} \log\left(\frac{1}{2}\right)$$

$$\frac{\log(0.386)}{\log(0.5)} = \frac{t}{5370}$$

$$t = \frac{5370 \log(0.386)}{\log(0.5)}$$

$$t \cong 7375 \text{ years old}$$

7) A Trimark mutual fund has a track record of 4.2% growth per year. What is the doubling period for this investment?

$$2 = 1(1.042)^t$$

$$\log 2 = \log(1.042)^t$$

$$\log 2 = t \log(1.042)$$

$$t = \frac{\log 2}{\log(1.042)}$$

$$t \cong 16.85 \text{ years}$$

8) A treatment to help a patient stop smoking involves chewing nicotine gum. Each gum introduces 1.5 mg of nicotine into the patient's system. Nicotine has a half life of 3 hours. The patient will feel the urge to smoke when the level of nicotine drops below 0.45 mg in her system. If she chewed a gum at 8:00 a.m., and another at 10:00 a.m., at what time will she next feel the urge to smoke?

Concentration at 10:00 a.m.:

$$A(2) = 1.5\left(\frac{1}{2}\right)^{\frac{2}{3}}$$

$$A(2) \cong 0.94494 \text{ mg}$$

Then add 1.5 mg for the new piece:  $0.94494 + 1.5 = 2.44494 \text{ mg}$

When will 2.44494 mg reduce to 0.45 mg?

$$\begin{aligned}
 0.45 &= 2.44494 \left( \frac{1}{2} \right)^{\frac{t}{3}} \\
 0.1840535964 &= \left( \frac{1}{2} \right)^{\frac{t}{3}} \\
 \frac{t}{3} &= \frac{\log(0.1840535964)}{\log\left(\frac{1}{2}\right)} \\
 t &= \frac{3 \log(0.1840535964)}{\log(0.5)} \\
 t &\cong 7.3 \text{ hours, or about 5:18 p.m.}
 \end{aligned}$$

9) Determine the pH of a solution with hydronium ion concentration:

a) 0.01

$$\begin{aligned}
 pH &= -\log(0.01) \\
 pH &= -(-2) \\
 pH &= 2
 \end{aligned}$$

b)  $1.5 \times 10^{-10}$

$$\begin{aligned}
 pH &= -\log(1.5 \times 10^{-10}) \\
 pH &\cong 9.8
 \end{aligned}$$

10) Determine the hydronium concentration, in moles per litre, of a solution with pH:

a) 8.5

$$\begin{aligned}
 8.5 &= -\log[H^+] \\
 -8.5 &= \log[H^+] \\
 [H^+] &= 10^{-8.5} \\
 [H^+] &\cong 3.2 \times 10^{-9} \text{ mol/L}
 \end{aligned}$$

b) 3

$$\begin{aligned}
 3 &= -\log[H^+] \\
 -3 &= \log[H^+] \\
 [H^+] &= 10^{-3} \\
 [H^+] &= 0.001 \text{ mol/L}
 \end{aligned}$$

11) How many times as intense is the sound of a shout as the sound of a whisper?

$$\begin{aligned}
 \beta_2 - \beta_1 &= 10 \log \left( \frac{I_2}{I_1} \right) \\
 80 - 30 &= 10 \log \left( \frac{I_2}{I_1} \right) \\
 5 &= \log \left( \frac{I_2}{I_1} \right) \\
 \frac{I_2}{I_1} &= 10^5
 \end{aligned}$$

A shout is 100 000 times as intense as a whisper.

12) A loud car stereo has a decibel level of 110 dB. How many times as intense as the sound of a loud car stereo is the sound of a rock concert speaker?

$$\begin{aligned}
 150 - 110 &= 10 \log \left( \frac{I_2}{I_1} \right) \\
 4 &= \log \left( \frac{I_2}{I_1} \right) \\
 10^4 &= \frac{I_2}{I_1} \\
 \frac{I_2}{I_1} &= 10\,000
 \end{aligned}$$

A rock concert speaker is 10 000 times as intense as a loud car stereo.

13) The sound intensity of a pin drop is about 1/30 000 of the sound intensity of a normal conversation. What is the decibel level of a pin drop?

$$\begin{aligned}
 \beta_2 - 60 &= 10 \log \left( \frac{1}{30\,000} \right) \\
 \beta_2 &= 10 \log \left( \frac{1}{30\,000} \right) + 60 \\
 \beta_2 &\cong 15.23 \text{ dB}
 \end{aligned}$$

The decibel level of a pin drop is about 15.23 dB.

14) On September 26, 2001, an earthquake in North Bay, Ontario, occurred that was 10 000 times as intense as  $I_0$ . What was the measure of this earthquake on the Richter scale?

$$\begin{aligned}M &= \log\left(\frac{I}{I_0}\right) \\M &= \log(10\,000) \\M &= 4\end{aligned}$$

15) On February 10, 2000, an earthquake happened in Welland, Ontario, that measured 2.3 on the Richter scale.

a) How many times as intense was this as a standard low-level earthquake?

$$\begin{aligned}2.3 &= \log\left(\frac{I}{I_0}\right) \\10^{2.3} &= \frac{I}{I_0} \\\frac{I}{I_0} &\cong 199.53\end{aligned}$$

About 200 times as intense.

b) On July 22, 2001, an earthquake in St. Catharines measured 1.1 on the Richter scale. How many times as intense as the St. Catharines earthquake was the Welland earthquake?

$$\begin{aligned}2.3 - 1.1 &= 1.2 \\1.2 &= \log\left(\frac{I}{I_0}\right) \\10^{1.2} &= \frac{I}{I_0} \\\frac{I}{I_0} &\cong 15.85\end{aligned}$$

About 16 times (15.85 times) as intense.

16) The stellar magnitude scale compares the brightness of stars using the equation  $m_2 - m_1 = \log\left(\frac{b_1}{b_2}\right)$ , where  $m_2$  and  $m_1$  are the apparent magnitude of the two stars being compared (how bright they appear in the sky) and  $b_1$  and  $b_2$  are their brightness (how much light they actually emit). This relationship does not factor in how far from Earth the stars are.

a) Sirius is the brightest-appearing star in our sky, with an apparent magnitude of -1.5. How much brighter does Sirius appear than Betelgeuse, whose apparent magnitude is 0.12?

$$\begin{aligned}0.12 - (-1.5) &= \log\left(\frac{b_1}{b_2}\right) \\1.62 &= \log\left(\frac{b_1}{b_2}\right) \\10^{1.62} &= \frac{b_1}{b_2} \\\frac{b_1}{b_2} &\cong 41.69\end{aligned}$$

Sirius is about 41.69 times brighter than Betelgeuse.

b) The Sun appears about  $1.3 \times 10^{10}$  times as brightly in our sky as does Sirius. What is the apparent magnitude of the Sun?

$$\begin{aligned}-1.5 - m_{\text{sun}} &= \log(1.3 \times 10^{10}) \\m_{\text{sun}} &= -1.5 - \log(1.3 \times 10^{10}) \\m_{\text{sun}} &\cong -11.61\end{aligned}$$