

3.6 Applications of Logarithms and Exponentials in Physical Sciences Worksheet

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Exponential Formulas

General	Half-life	Doubling
$A(t) = A_0(1 + i)^t$	$A(t) = A_0\left(\frac{1}{2}\right)^{\frac{t}{H}}$	$A(t) = A_0(2)^{\frac{t}{D}}$
i is percent growth (+) or decay (-)	H is the half-life period	D is the doubling period

Logarithmic Formulas

pH	Decibel	Richter
$pH = -\log[H^+]$	$\beta_2 - \beta_1 = 10\log\left(\frac{I_2}{I_1}\right)$	$M = \log\left(\frac{I}{I_0}\right)$
pH is acidity and $[H^+]$ is concentration of hydronium ions in mol/L	β is loudness in dB and I is intensity of sound in W/m^2	M is magnitude measured by richters, I is intensity

1) The half-life of a radioactive form of tritium is about 2 years. How much of a 5-kg sample of this material would remain after ...

a) 8 years

b) 12 months

2) The population of Littleton is currently (2014) 23 000, and is increasing exponentially with a growth rate of 2% per year. Estimate when Littleton will have a population of 30 000.

3) The population of purple martins in Algonquin park was estimated to be 35 000 in 1992. Ten years later, in 2002, the population had risen to 44 400.

a) What is the annual growth rate for the purple martin population?

b) Estimate the population for 2010 to the nearest hundred.

4) After an accident at a nuclear plant, the radiation level in the plant was 950 R (roentgens). Five hours later the level was 800 R. How long will it take before safe levels of radiation are reached, which is less than 0.01 R?

5) The value of a new minivan drops 40% after the first year, and then decreases exponentially at a rate of 12% per year after that. When will a minivan that cost \$35 000 new be worth less than \$10 000?

6) A crab fossil contains 38.6% of its original Carbon-14 isotope, which has a half life of 5370 years. Approximately how old is the crab fossil?

7) A Trimark mutual fund has a track record of 4.2% growth per year. What is the doubling period for this investment?

8) A treatment to help a patient stop smoking involves chewing nicotine gum. Each gum introduces 1.5 mg of nicotine into the patient's system. Nicotine has a half life of 3 hours. The patient will feel the urge to smoke when the level of nicotine drops below 0.45 mg in her system. If she chewed a gum at 8:00 a.m., and another at 10:00 a.m., at what time will she next feel the urge to smoke?

9) Determine the pH of a solution with hydronium ion concentration:

a) 0.01

b) 1.5×10^{-10}

10) Determine the hydronium concentration, in moles per litre, of a solution with pH:

a) 8.5

b) 3

11) How many times as intense is the sound of a shout as the sound of a whisper?

12) A loud car stereo has a decibel level of 110 dB. How many times as intense as the sound of a loud car stereo is the sound of a rock concert speaker?

13) The sound intensity of a pin drop is about $1/30\,000$ of the sound intensity of a normal conversation. What is the decibel level of a pin drop?

14) On September 26, 2001, an earthquake in North Bay, Ontario, occurred that was 10 000 times as intense as I_0 . What was the measure of this earthquake on the Richter scale?

15) On February 10, 2000, an earthquake happened in Welland, Ontario, that measured 2.3 on the Richter scale.

a) How many times as intense was this as a standard low-level earthquake?

b) On July 22, 2001, an earthquake in St. Catharines measured 1.1 on the Richter scale. How many times as intense as the St. Catharines earthquake was the Welland earthquake?

16) The stellar magnitude scale compares the brightness of stars using the equation $m_2 - m_1 = \log\left(\frac{b_1}{b_2}\right)$, where m_2 and m_1 are the apparent magnitude of the two stars being compared (how bright they appear in the sky) and b_1 and b_2 are their brightness (how much light they actually emit). This relationship does not factor in how far from Earth the stars are.

a) Sirius is the brightest-appearing star in our sky, with an apparent magnitude of -1.5. How much brighter does Sirius appear than Betelgeuse, whose apparent magnitude is 0.12?

b) The Sun appears about 1.3×10^{10} times as brightly in our sky as does Sirius. What is the apparent magnitude of the Sun?

Answer Key

1) a) 0.3125 kg b) 3.536 kg

2) $\cong 13.42$ years

3) a) growth rate of 2.4% per year b) approx. 53 700 birds

4) approx. 333 hours

5) approx. 6.8 years (about 7 years)

6) approx. 7375 years

7) approx. 16.85 years

8) $t \cong 7.3$ hours, so about 5:18 p.m.

9) a) 2 b) 9.8

10) a) 3.2×10^{-9} mol/L b) 0.001 mol/L

11) a shout is 100 000 times more intense than a whisper

12) a rock concert speaker is 10 000 times more intense than a loud car speaker

13) a pin drop is about 15.23 dB

14) level 4 on the Richter scale

15) a) about 200 times more intense b) 15.85 times more intense

16) a) 41.69 times brighter b) apparent magnitude is -11.61