

## 3.5 Solving Logarithmic Equations Worksheet with solutions

This document was composed and formatted by Annie Bernatchez.

The course material originated from [www.jensenmath.ca](http://www.jensenmath.ca)

1) Find the roots of each equation

a)  $2 = \log(x + 25)$

$$\begin{aligned} 10^2 &= x + 25 \\ 100 &= x + 25 \\ x &= 75 \end{aligned}$$

b)  $1 = \log(w - 7) = 0$

$$\begin{aligned} 1 &= \log(w - 7) \\ 10^1 &= w - 7 \\ 10 &= w - 7 \\ w &= 17 \end{aligned}$$

c)  $6 = 3\log(2n) = 0$

$$\begin{aligned} 6 &= 3\log(2n) \\ 2 &= \log(2n) \\ 10^2 &= 2n \\ 100 &= 2n \\ n &= 50 \end{aligned}$$

2) Solve

a)  $5 = \log_2(2x - 10)$

$$\begin{aligned} 2^5 &= 2x - 10 \\ 32 &= 2x - 10 \\ 42 &= 2x \\ x &= 21 \end{aligned}$$

b)  $9 = \log_5(x + 100) + 6$

$$\begin{aligned} 3 &= \log_5(x + 100) \\ 5^3 &= x + 100 \\ 125 &= x + 100 \\ x &= 25 \end{aligned}$$

c)  $\log_3(n^2 - 3n + 5) = 2$

$$\begin{aligned} 3^2 &= n^2 - 3n + 5 \\ 9 &= n^2 - 3n + 5 \\ 0 &= n^2 - 3n - 4 \\ 0 &= (n - 4)(n + 1) \\ n &= 4 \text{ or } n = -1 \end{aligned}$$

Both values check: in each case  $n^2 - 3n + 5 = 9 > 0$ , so both are valid solutions.

3) Solve. Make sure to reject any extraneous roots.

a)  $\log x + \log(x - 4) = 1$

$$\begin{aligned} \log[x(x - 4)] &= 1 \\ 10^1 &= x(x - 4) \\ 10 &= x^2 - 4x \\ 0 &= x^2 - 4x - 10 \\ x &= \frac{4 \pm \sqrt{(-4)^2 - 4(1)(-10)}}{2(1)} \\ x &= \frac{4 \pm \sqrt{56}}{2} \\ x &= \frac{4 \pm 2\sqrt{14}}{2} \\ x &= 2 \pm \sqrt{14} \end{aligned}$$

Reject  $x = 2 - \sqrt{14}$  (negative, so  $x - 4$  is undefined).

The only solution is  $x = 2 + \sqrt{14}$

b)  $\log x^3 - \log 2 = \log(2x^2)$

$$\begin{aligned} \log x^3 &= \log 2 + \log(2x^2) \\ \log x^3 &= \log(4x^2) \\ x^3 &= 4x^2 \\ x^3 - 4x^2 &= 0 \\ x^2(x - 4) &= 0 \\ x &= 0 \text{ or } x = 4 \end{aligned}$$

Reject  $x = 0$  because  $\log x^3$  is undefined for this value.

The only solution is  $x = 4$

c)  $\log(v - 1) = 2 + \log(v - 16)$

$$\begin{aligned}\log(v - 1) - \log(v - 16) &= 2 \\ \log\left(\frac{v - 1}{v - 16}\right) &= 2 \\ \frac{v - 1}{v - 16} &= 10^2 \\ v - 1 &= 100(v - 16) \\ v - 1 &= 100v - 1600 \\ 1599 &= 99v \\ v &= \frac{533}{33}\end{aligned}$$

d)  $\log(k + 2) + \log(k - 1) = 1$

$$\begin{aligned}\log[(k + 2)(k - 1)] &= 1 \\ 10^1 &= (k + 2)(k - 1) \\ 10 &= k^2 + k - 2 \\ 0 &= k^2 + k - 12 \\ 0 &= (k + 4)(k - 3)\end{aligned}$$

Reject  $k = -4$  (makes  $k + 2$  negative, so the logarithm is undefined).

The only solution is  $k = 3$

4) Solve. Check for extraneous roots.

a)  $\log\sqrt{x^2 - 3x} = \frac{1}{2}$

$$\begin{aligned}\log(x^2 - 3x)^{\frac{1}{2}} &= \frac{1}{2} \\ \frac{1}{2} \log(x^2 - 3x) &= \frac{1}{2} \\ \log(x^2 - 3x) &= 1 \\ 10^1 &= x^2 - 3x \\ 0 &= x^2 - 3x - 10 \\ 0 &= (x - 5)(x + 2) \\ x &= 5 \text{ or } x = -2\end{aligned}$$

Both values check: in each case  $x^2 - 3x = 10 > 0$ , so both are valid solutions.

b)  $\log\sqrt{x^2 + 48x} = 1$

$$\begin{aligned}\log(x^2 + 48x)^{\frac{1}{2}} &= 1 \\ \frac{1}{2} \log(x^2 + 48x) &= 1 \\ \log(x^2 + 48x) &= 2 \\ 10^2 &= x^2 + 48x \\ 0 &= x^2 + 48x - 100 \\ 0 &= (x + 50)(x - 2) \\ x &= -50 \text{ or } x = 2\end{aligned}$$

Both values check: in each case  $x^2 + 48x = 100 > 0$ , so both are valid solutions.

5) Solve. Check for extraneous roots.

a)  $\log_2(x + 5) - \log_2(2x) = 8$

$$\begin{aligned}\log_2\left(\frac{x + 5}{2x}\right) &= 8 \\ 2^8 &= \frac{x + 5}{2x} \\ 256 &= \frac{x + 5}{2x} \\ 512x &= x + 5 \\ 511x &= 5 \\ x &= \frac{5}{511}\end{aligned}$$

This checks:  $x = \frac{5}{511} > 0$ , so both  $x + 5$  and  $2x$  are positive.

b)  $\log(2k + 4) = 1 + \log k$

$$\begin{aligned}\log(2k + 4) - \log k &= 1 \\ \log\left(\frac{2k + 4}{k}\right) &= 1 \\ \frac{2k + 4}{k} &= 10 \\ 2k + 4 &= 10k \\ 4 &= 8k \\ k &= \frac{1}{2}\end{aligned}$$

This checks:  $k = \frac{1}{2} > 0$ , so  $2k + 4$  and  $k$  are both positive.