

3.4 Solving Exponential Equations Worksheet with solutions

This document was composed and formatted by Annie Bernatchez.

The course material originated from www.jensenmath.ca

1) Write each expression with base 2.

a) 4^6

$$\begin{aligned} 4^6 &= (2^2)^6 \\ &= 2^{12} \end{aligned}$$

b) 8^3

$$\begin{aligned} 8^3 &= (2^3)^3 \\ &= 2^9 \end{aligned}$$

c) $\left(\frac{1}{8}\right)^2$

$$\begin{aligned} \left(\frac{1}{8}\right)^2 &= (2^{-3})^2 \\ &= 2^{-6} \end{aligned}$$

d) 14

$$\begin{aligned} 2^x &= 14 \\ x &= \log_2 14 \end{aligned}$$

$$\therefore 14 = 2^{\frac{\log 14}{\log 2}}$$

2) Write each expression as a power of 4.

a) $(\sqrt{16})^3$

$$= 4^3$$

b) $\sqrt[3]{16}$

$$\begin{aligned} \sqrt[3]{16} &= (16)^{\frac{1}{3}} \\ &= (4^2)^{\frac{1}{3}} \\ &= 4^{\frac{2}{3}} \end{aligned}$$

c) $\sqrt{64} \times (\sqrt[4]{128})^3$

$$\begin{aligned} \sqrt{64} \times (\sqrt[4]{128})^3 &= 64^{\frac{1}{2}} \times 128^{\frac{3}{4}} \\ &= (4^3)^{\frac{1}{2}} \times \left[(4^2)^7\right]^{\frac{3}{4}} \\ &= 4^{\frac{3}{2}} \times 4^{\frac{21}{8}} \\ &= 4^{\frac{12}{8}} \times 4^{\frac{21}{8}} \\ &= 4^{\frac{33}{8}} \end{aligned}$$

3) Solve each equation

a) $2^{4x} = 4^{x+3}$

$$\begin{aligned} 2^{4x} &= (2^2)^{x+3} \\ 2^{4x} &= 2^{2x+6} \\ 4x &= 2x + 6 \\ 2x &= 6 \\ x &= 3 \end{aligned}$$

b) $3^{w+1} = 9^{w-1}$

$$\begin{aligned} 3^{w+1} &= (3^2)^{w-1} \\ 3^{w+1} &= 3^{2w-2} \\ w+1 &= 2w-2 \\ 3 &= w \end{aligned}$$

c) $4^{3x} = 8^{x-3}$

$$\begin{aligned} (2^2)^{3x} &= (2^3)^{x-3} \\ 2^{6x} &= 2^{3x-9} \\ 6x &= 3x-9 \\ 3x &= -9 \\ x &= -3 \end{aligned}$$

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d) $125^{2y-1} = 25^{y+4}$

$$\begin{aligned}(5^3)^{2y-1} &= (5^2)^{y+4} \\ 5^{6y-3} &= 5^{2y+8} \\ 6y-3 &= 2y+8 \\ 4y &= 11 \\ y &= \frac{11}{4}\end{aligned}$$

4) Consider the equation $10^{2x} = 100^{2x-5}$

a) Solve this equation by expressing both sides as powers of a common base.

$$\begin{aligned}10^{2x} &= (10^2)^{2x-5} \\ 10^{2x} &= 10^{4x-10} \\ 2x &= 4x-10 \\ 10 &= 2x \\ x &= 5\end{aligned}$$

b) Solve the same equation by taking the common logarithm of both sides.

$$\begin{aligned}\log(10^{2x}) &= \log(100^{2x-5}) \\ 2x \log(10) &= (2x-5) \log(100) \\ 2x(1) &= (2x-5)(2) \\ 2x &= 4x-10 \\ 10 &= 2x \\ x &= 5\end{aligned}$$

5) Solve $2^{3x} > 4^{x+1}$

$$\begin{aligned}2^{3x} &> (2^2)^{x+1} \\ 2^{3x} &> 2^{2x+2} \\ 3x &> 2x+2 \\ x &> 2\end{aligned}$$

6) Solve for t . Round answers to 2 decimal places.

a) $2 = 1.07^t$

$$\begin{aligned}t &= \log_{1.07} 2 \\ t &\cong 10.24\end{aligned}$$

b) $100 = 10(1.04)^t$

$$\begin{aligned}10 &= 1.04^t \\ t &= \log_{1.04} 10 \\ t &\cong 58.71\end{aligned}$$

c) $15 = \left(\frac{1}{2}\right)^{\frac{t}{4}}$

$$\begin{aligned}\frac{t}{4} &= \log_{\frac{1}{2}}(15) \\ t &= 4 \log_{\frac{1}{2}}(15) \\ t &\cong -15.63\end{aligned}$$

7) Solve each equation. Round answers to 3 decimal places.

a) $2^x = 3^{x-1}$

$$\begin{aligned}\log(2^x) &= \log(3^{x-1}) \\ x \log 2 &= (x-1) \log 3 \\ \log 3 &= x \log 3 - x \log 2 \\ \log 3 &= x(\log 3 - \log 2) \\ \frac{\log 3}{\log\left(\frac{3}{2}\right)} &= x \\ x &= \log_{\frac{3}{2}}(3) \\ x &\cong 2.710\end{aligned}$$

b) $5^{x-2} = 4^x$

$$\begin{aligned}\log(5^{x-2}) &= \log(4^x) \\ (x-2) \log 5 &= x \log 4 \\ x \log 5 - 2 \log 5 &= x \log 4 \\ x \log 5 - x \log 4 &= 2 \log 5 \\ x[\log 5 - \log 4] &= \log 25 \\ x &= \frac{\log 25}{\log\left(\frac{5}{4}\right)} \\ x &= \log_{\frac{5}{4}}(25) \\ x &\cong 14.425\end{aligned}$$

c) $7^{2x+1} = 4^{x-2}$

$$\begin{aligned}\log(7^{2x+1}) &= \log(4^{x-2}) \\ (2x+1)\log 7 &= (x-2)\log 4 \\ 2x\log 7 + \log 7 &= x\log 4 - 2\log 4 \\ \log 7 + 2\log 4 &= x\log 4 - 2x\log 7 \\ \log 7 + \log 16 &= x[\log 4 - 2\log 7] \\ \log(112) &= x\log\left(\frac{4}{49}\right) \\ x &= \frac{\log(112)}{\log\left(\frac{4}{49}\right)} \\ x &= \log_{\frac{4}{49}}(112) \\ x &\cong -1.883\end{aligned}$$

8) Solve $2^{2x} + 2^x - 6 = 0$ using the quadratic formula (or by factoring). Clearly identify any extraneous roots.

$$(2^x)^2 + (2^x) - 6 = 0$$

Let $k = 2^x$

$$\begin{aligned}k^2 + k - 6 &= 0 \\ (k+3)(k-2) &= 0\end{aligned}$$

$k = -3$ or $k = 2$

Case 1

$$2^x = -3$$

$$x = \frac{\log(-3)}{\log 2}$$

This is an extraneous root — the argument of a logarithm cannot be negative.

Case 2

$$\begin{aligned}2^x &= 2 \\ x &= 1\end{aligned}$$

This is the only solution.

9) Solve $8^{2x} - 2(8^x) - 5 = 0$ using the quadratic formula. Clearly identify any extraneous roots.

$$(8^x)^2 - 2(8^x) - 5 = 0$$

Let $k = 8^x$

$$\begin{aligned}k^2 - 2k - 5 &= 0 \\ k &= \frac{2 \pm \sqrt{(-2)^2 - 4(1)(-5)}}{2(1)} \\ k &= \frac{2 \pm \sqrt{24}}{2} \\ k &= \frac{2 \pm 2\sqrt{6}}{2} \\ k &= 1 \pm \sqrt{6}\end{aligned}$$

Case 1

$$\begin{aligned}8^x &= 1 + \sqrt{6} \\ x &= \log_8(1 + \sqrt{6}) \\ x &\cong 0.6\end{aligned}$$

This is the only solution.

Case 2

$$8^x = 1 - \sqrt{6}$$

No real solution. The argument of a logarithm must be greater than zero.

10) Use the decay equation for polonium-218, $A(t) = A_0\left(\frac{1}{2}\right)^{\frac{t}{3.1}}$, A is the amount remaining after t minutes and A_0 is the initial amount.

a) How much will remain after 90 seconds from an initial sample of 50 mg?

$$\begin{aligned}A(1.5) &= 50\left(\frac{1}{2}\right)^{\frac{1.5}{3.1}} \\ &\cong 35.75 \text{ mg}\end{aligned}$$

b) How long will it take for this sample to decay to 10% of its initial amount of 50 mg?

$$\begin{aligned}5 &= 50\left(\frac{1}{2}\right)^{\frac{t}{3.1}} \\0.1 &= (0.5)^{\frac{t}{3.1}} \\\frac{t}{3.1} &= \log_{0.5}(0.1) \\t &= 3.1 \log_{0.5}(0.1) \\t &\cong 10.3 \text{ minutes}\end{aligned}$$

11) A 20-mg sample of thorium-233 decays to 17 mg after 5 minutes.

a) What is the half-life of thorium-233?

$$\begin{aligned}17 &= 20\left(\frac{1}{2}\right)^{\frac{5}{H}} \\0.85 &= 0.5^{\frac{5}{H}} \\\frac{5}{H} &= \log_{0.5}(0.85) \\H &= \frac{5}{\log_{0.5}(0.85)} \\H &\cong 21.3 \text{ minutes}\end{aligned}$$

b) How long will it take this sample to decay to 1 mg?

$$\begin{aligned}1 &= 20\left(\frac{1}{2}\right)^{\frac{t}{21.3}} \\0.05 &= 0.5^{\frac{t}{21.3}} \\\frac{t}{21.3} &= \log_{0.5}(0.05) \\t &= 21.3 \log_{0.5}(0.05) \\t &\cong 92.06 \text{ minutes}\end{aligned}$$