

3.3 Product and Quotient Laws of Logarithms Worksheet with solutions

This document was composed and formatted by Annie Bernatchez.

The course material originated from www.jensenmath.ca

1) Simplify using laws of logarithms and then evaluate.

a) $\log 9 + \log 6$

$$\begin{aligned}\log 9 + \log 6 &= \log(9 \times 6) \\ &= \log(54) \\ &= 1.732\end{aligned}$$

b) $\log 48 - \log 6$

$$\begin{aligned}\log 48 - \log 6 &= \log\left(\frac{48}{6}\right) \\ &= \log(8) \\ &= 0.903\end{aligned}$$

c) $\log_3 7 + \log_3 3$

$$\begin{aligned}\log_3 7 + \log_3 3 &= \log_3(7 \times 3) \\ &= \log_3(21) \\ &= \frac{\log 21}{\log 3} \\ &= 2.771\end{aligned}$$

2) Simplify each algebraic expression.

a) $\log x + \log y + \log(2z)$

$$= \log(2xyz)$$

b) $\log_2 a + \log_2(3b) - \log_2(2c)$

$$= \log_2\left(\frac{3ab}{2c}\right)$$

c) $2\log m + 3\log n - 4\log y$

$$\begin{aligned}2\log m + 3\log n - 4\log y &= \log m^2 + \log n^3 - \log y^4 \\ &= \log\left(\frac{m^2 n^3}{y^4}\right)\end{aligned}$$

3) Evaluate using the product law of logarithms.

a) $\log_6 18 + \log_6 2$

$$\begin{aligned}\log_6 18 + \log_6 2 &= \log_6(18 \times 2) \\ &= \log_6(36) \\ &= \frac{\log 36}{\log 6} \\ &= 2\end{aligned}$$

b) $\log 40 + \log 2.5$

$$\begin{aligned}\log 40 + \log 2.5 &= \log(40 \times 2.5) \\ &= \log(100) \\ &= 2\end{aligned}$$

c) $\log_{12} 8 + \log_{12} 2 + \log_{12} 9$

$$\begin{aligned}\log_{12} 8 + \log_{12} 2 + \log_{12} 9 &= \log_{12}(8 \times 2 \times 9) \\ &= \log_{12}(144) \\ &= \frac{\log 144}{\log 12} \\ &= 2\end{aligned}$$

4) Evaluate using the quotient law of logarithms.

a) $\log_3 54 - \log_3 2$

$$\begin{aligned}\log_3 54 - \log_3 2 &= \log_3\left(\frac{54}{2}\right) \\ &= \log_3(27) \\ &= 3\end{aligned}$$

b) $\log 50\,000 - \log 5$

$$\begin{aligned}\log 50\,000 - \log 5 &= \log\left(\frac{50\,000}{5}\right) \\ &= \log(10\,000) \\ &= 4\end{aligned}$$

c) $\log_4 320 - \log_4 5$

$$\begin{aligned}\log_4 320 - \log_4 5 &= \log_4 \left(\frac{320}{5} \right) \\ &= \log_4 (64) \\ &= 3\end{aligned}$$

5) Evaluate, using the laws of logarithms

a) $3\log_{16} 2 + 2\log_{16} 8 - \log_{16} 2$

$$\begin{aligned}3\log_{16} 2 + 2\log_{16} 8 - \log_{16} 2 &= \log_{16} (2^3) + \log_{16} (8^2) - \log_{16} (2) \\ &= \log_{16} (8) + \log_{16} (64) - \log_{16} (2) \\ &= \log_{16} (256) \\ &= 2\end{aligned}$$

b) $\log 20 + \log 2 + \frac{1}{3} \log 125$

$$\begin{aligned}\log 20 + \log 2 + \frac{1}{3} \log 125 &= \log (20 \times 2) + \log \left(125^{\frac{1}{3}} \right) \\ &= \log (40) + \log (5) \\ &= \log (40 \times 5) \\ &= \log (200) \\ &\cong 2.301\end{aligned}$$

6) Write as a sum or difference of logarithms. Simplify, if possible.

a) $\log_7 (cd)$

$$= \log_7 c + \log_7 d$$

b) $\log_3 \left(\frac{m}{n} \right)$

$$= \log_3 m - \log_3 n$$

c) $\log (uv^3)$

$$\begin{aligned}\log (uv^3) &= \log u + \log v^3 \\ &= \log u + 3\log v\end{aligned}$$

d) $\log \left(\frac{a\sqrt{b}}{c^2} \right)$

$$\begin{aligned}\log \left(\frac{a\sqrt{b}}{c^2} \right) &= \log a + \log b^{\frac{1}{2}} - \log c^2 \\ &= \log a + \frac{1}{2} \log b - 2\log c\end{aligned}$$

e) $\log_2 10$

$$\begin{aligned}\log_2 10 &= \log_2 (2 \times 5) \\ &= \log_2 2 + \log_2 5 \\ &= 1 + \log_2 5\end{aligned}$$

7) Simplify

a) $\log \left(\frac{x^2}{\sqrt{x}} \right)$

$$\begin{aligned}\log \left(\frac{x^2}{\sqrt{x}} \right) &= \log x^2 - \log x^{\frac{1}{2}} \\ &= 2\log x - \frac{1}{2} \log x \\ &= \frac{3}{2} \log x\end{aligned}$$

b) $\log \sqrt{k} + \log (\sqrt{k})^3 + \log \sqrt[3]{k^2}$

$$\begin{aligned}\log \sqrt{k} + \log (\sqrt{k})^3 + \log \sqrt[3]{k^2} &= \log k^{\frac{1}{2}} + \log k^{\frac{3}{2}} + \log k^{\frac{2}{3}} \\ &= \frac{1}{2} \log k + \frac{3}{2} \log k + \frac{2}{3} \log k \\ &= \frac{3}{6} \log k + \frac{9}{6} \log k + \frac{4}{6} \log k \\ &= \frac{16}{6} \log k \\ &= \frac{8}{3} \log k\end{aligned}$$

c) $\log (x^2 - 4) - \log (x - 2)$

$$\begin{aligned}\log (x^2 - 4) - \log (x - 2) &= \log \left(\frac{x^2 - 4}{x - 2} \right) \\ &= \log \left[\frac{(x - 2)(x + 2)}{x - 2} \right] \\ &= \log (x + 2)\end{aligned}$$

d) $\log(x^2 - x - 6) - \log(2x - 6)$

$$\begin{aligned}\log(x^2 - x - 6) - \log(2x - 6) &= \log\left(\frac{x^2 - x - 6}{2x - 6}\right) \\ &= \log\left[\frac{(x - 3)(x + 2)}{2(x - 3)}\right] \\ &= \log\left(\frac{x + 2}{2}\right)\end{aligned}$$