

3.2 Power Law of Logarithms Worksheet with solutions

This document was composed and formatted by Annie Bernatchez.

The course material originated from www.jensenmath.ca

1) Evaluate.

a) $\log_2 16^3$

$$\begin{aligned}\log_2 16^3 &= 3 \log_2 (2^4) \\ &= 3(4) \\ &= 12\end{aligned}$$

b) $\log_4 8^2$

$$\begin{aligned}\log_4 8^2 &= \log_4 64 \\ &= \log_4 (4^3) \\ &= 3\end{aligned}$$

c) $\log 100^{-4}$

$$\begin{aligned}\log 100^{-4} &= -4 \log (10^2) \\ &= -4(2) \\ &= -8\end{aligned}$$

d) $\log 0.12^{\frac{1}{2}}$

$$\begin{aligned}\log 0.12^{\frac{1}{2}} &= \frac{1}{2} \log (10^{-1}) \\ &= \frac{1}{2} (-1) \\ &= -\frac{1}{2}\end{aligned}$$

e) $\log_2 \sqrt{8}$

$$\begin{aligned}\log_2 \sqrt{8} &= \frac{1}{2} \log_2 (2^3) \\ &= \frac{1}{2} (3) \\ &= \frac{3}{2}\end{aligned}$$

f) $\log_3 (\sqrt[3]{81})^6$

$$\begin{aligned}\log_3 (\sqrt[3]{81})^6 &= \log_3 (81)^2 \\ &= 2 \log_3 (3^4) \\ &= 2(4) \\ &= 8\end{aligned}$$

2) Solve for t to two decimal places.

a) $10 = 4^t$

$$\begin{aligned}\log_4 10 &= t \\ \frac{\log 10}{\log 4} &= t \\ t &= 1.66\end{aligned}$$

b) $5^t = 250$

$$\begin{aligned}\log_5 250 &= t \\ \frac{\log 250}{\log 5} &= t \\ t &= 3.43\end{aligned}$$

c) $2 = 1.08^t$

$$\begin{aligned}\log_{1.08} 2 &= t \\ \frac{\log 2}{\log 1.08} &= t \\ t &= 9.01\end{aligned}$$

d) $500 = 100(1.06)^t$

$$\begin{aligned}5 &= 1.06^t \\ \log_{1.06} 5 &= t \\ \frac{\log 5}{\log 1.06} &= t \\ t &= 27.62\end{aligned}$$

3) An investment earns 7% interest, compounded annually. The amount, A , that the investment is worth as a function of time, t , in years, is given by $A(t) = 500(1.07)^t$.

a) Use the equation to determine the value of the investment after 4 years.

$$\begin{aligned}A(4) &= 500(1.07)^4 \\ &= \$655.40\end{aligned}$$

b) How long will it take for the investment to double in value?

$$\begin{aligned} 1000 &= 500(1.07)^t \\ 2 &= 1.07^t \\ \log_{1.07} 2 &= t \\ \frac{\log 2}{\log 1.07} &= t \\ t &= 10.2 \text{ years} \end{aligned}$$

4) Use the change of base formula to evaluate each of the following. Round to 3 decimal places.

a) $\log_3 23$

$$\frac{\log 23}{\log 3} \cong 2.854$$

b) $\log_6 20$

$$\frac{\log 20}{\log 6} \cong 1.672$$

c) $-\log_{12} 4$

$$-\frac{\log 4}{\log 12} \cong -0.558$$

d) $\log_{\frac{1}{2}} 30$

$$\frac{\log 30}{\log \left(\frac{1}{2}\right)} \cong -4.907$$

5) Write each as a single logarithm

a) $\frac{\log 8}{\log 5}$

$$\log_5 8$$

b) $\frac{\log 17}{\log 9}$

$$\log_9 17$$

c) $\frac{\log \left(\frac{1}{2}\right)}{\log \left(\frac{2}{3}\right)}$

$$\log_{\frac{2}{3}} \left(\frac{1}{2}\right)$$

d) $\frac{\log(x+1)}{\log(x-1)}$

$$\log_{x-1}(x+1)$$

6)a) Evaluate $\log_2 8^5$ without using the power law of logarithms.

$$\begin{aligned} \log_2 8^5 &= \log_2 (2^3)^5 \\ &= \log_2 (2^{15}) \\ &= 15 \end{aligned}$$

b) Evaluate the same expression by applying the power law of logarithms.

$$\begin{aligned} \log_2 8^5 &= 5 \log_2 (2^3) \\ &= 5(3) \\ &= 15 \end{aligned}$$

c) Which method do you prefer?

Answers will vary.

7) Solve for x , correct to 3 decimal places.

a) $2 = \log 3^x$

$$\begin{aligned} 2 &= x \log 3 \\ x &= \frac{2}{\log 3} \\ x &= 4.192 \end{aligned}$$

b) $100 = 10 \log 1000^x$

$$\begin{aligned} 10 &= \log 1000^x \\ 10 &= x \log 1000 \\ x &= \frac{10}{\log 1000} \\ x &= 3.333 \end{aligned}$$

c) $4 = \log_3 15^x$

$$\begin{aligned}3^4 &= 15^x \\ \log 81 &= x \log 15 \\ x &= \frac{\log 81}{\log 15} \\ x &= 1.623\end{aligned}$$