

3.8 The Natural Logarithm Lesson

This document was composed and formatted by Annie Bernatchez.

The course material originated from www.jensenmath.ca

Part 1: What is 'e'?

Example 1: Suppose you invest \$1 at 100% interest for 1 year at various compounding levels. What is the highest amount of money you can have after 1 year?

Note: the formula used for compound interest of \$1 at 100% interest annually compounded n times during the year is:

$$A = 1\left(1 + \frac{1}{n}\right)^n$$

Compounding Level, n	Amount, A in dollars
Annually (once a year)	_____
Semi-annually (2-times)	_____
Quarterly (4-times)	_____
Monthly (12-times)	_____
Daily (365-times)	_____
Secondly (31 536 000-times)	_____
Continuously (1 000 000 000-times)	_____

Properties of e :

- $e = \lim_{n \rightarrow \infty} \left(1 + \frac{1}{n}\right)^n \cong$ _____
- e is an _____ number, similar to π . They are non-terminating and non-repeating.
- $\log_e x$ is known as the _____ and can be written as _____
- Many naturally occurring phenomena can be modelled using base- e exponential and logarithmic functions.
- $\log_e e = \ln e =$ _____

Part 2: Reminder of Log Rules

Rule	Formula
Power Law of Logarithms	$\log_b x^n = n \log_b x$ for $b > 0, b \neq 1, x > 0$
Product Law of Logarithms	$\log_b(mn) = \log_b m + \log_b n$ for $b > 0, b \neq 1, m > 0, n > 0$
Quotient Law of Logarithms	$\log_b\left(\frac{m}{n}\right) = \log_b m - \log_b n$ for $b > 0, b \neq 1, m > 0, n > 0$
Change of Base Formula	$\log_b m = \frac{\log m}{\log b}, m > 0, b > 0, b \neq 1$
Exponential to Logarithmic	$y = b^x \rightarrow x = \log_b y$
Logarithmic to Exponential	$y = \log_b x \rightarrow x = b^y$
Other useful tips	$\log_a(a^b) = b$ $\log a = \log_{10} a$ $\log_b b = 1$

Part 3: Solving Problems Involving e

Example 2: Evaluate each of the following

- a) e^3

- b) $\ln 10$

- c) $\ln e$

Example 3: Solve each of the following equations

- a) $20 = 3e^x$
- b) $e^{1-2x} = 55$
- c) $2\ln(x - 3) - 7 = 3$
- d) $\ln(4e^x) = 2$

Part 4: Graphing Functions Involving e

Example 4: Graph the functions $y = e^x$ and $y = \ln x$

Note: $y = \ln x$ is the inverse of $y = e^x$

Table 1. $y = e^x$

x	y
_____	_____
_____	_____
_____	_____
HA	_____

Table 2. $y = \ln x$

x	y
_____	_____
_____	_____
_____	_____
VA	_____

