

3.7 Transformations of Exponential and Logarithmic Functions Lesson with solutions

This document was composed and formatted by Annie Bernatchez.

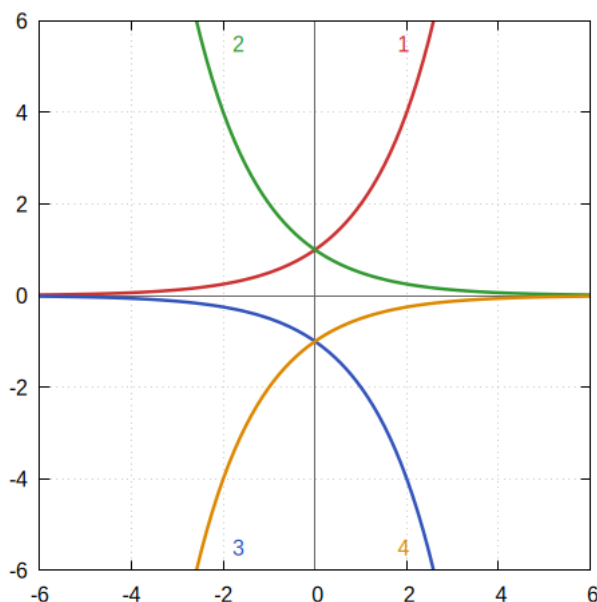
The course material originated from www.jensenmath.ca

Part 1: Properties of Exponential Functions

General Equation: $y = a(b)^{k(x-d)} + c$ where the base function is $y = b^x$

There are 4 possible shapes for an exponential function

- 1) $a > 0$ and $b > 1$ (ex. $y = 2^x$)
- 2) $a > 0$ and $0 < b < 1$ (ex. $y = \left(\frac{1}{2}\right)^x$)
- 3) $a < 0$ and $b > 1$ (ex. $y = -1(2)^x$)
- 4) $a < 0$ and $0 < b < 1$ (ex. $y = -1\left(\frac{1}{2}\right)^x$)



To graph the base function $y = b^x$, find the following key features:

- Horizontal asymptote
 - Starts at $y = 0$ and can be shifted by c
- y-intercept
 - Set $x = 0$ and solve
- At least one other point to be sure of shape
 - Common to choose $x = 1$ and solve for y

You can then use transformational properties of a , k , d , and c to graph a transformed function

Part 2: Transformations of Exponential Functions

Example 1: Sketch the graph of $f(x) = 2(3)^{x+4} - 5$ and $g(x) = -\frac{1}{32}x + 4$ using transformations

Table 1. $y = 3^x$

x	y
-1	0.33
0	1
1	3
HA	$y = 0$

Table 2. $f(x) = 2(3)^{x+4} - 5$

$x - 4$	$2y - 5$
-5	-4.33
-4	-3
-3	1
HA	$y = -5$

Table 3. $g(x) = -\frac{1}{32}x + 4$

$2x$	$-1y + 4$
-2	3.67
0	3
2	1
HA	$y = 4$

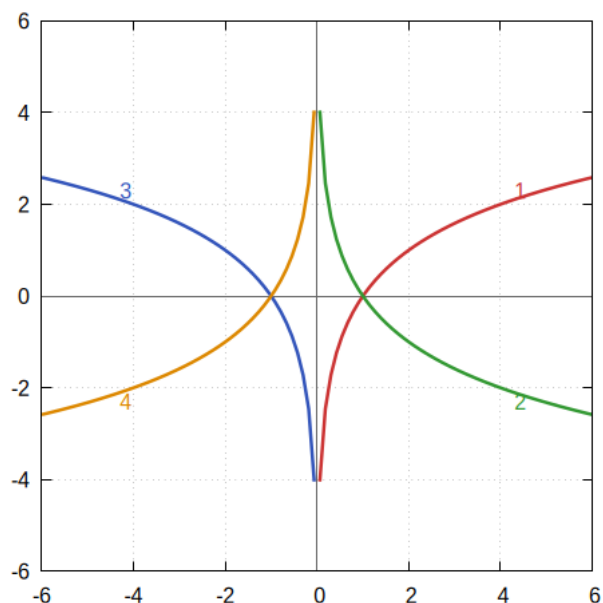
Part 3: Properties of Logarithmic Functions

General Equation: $y = a \log_b[k(x - d)] + c$ where the base function is $y = \log_b x$

Remember that $y = \log_b x$ is the inverse of the exponential function $y = b^x$

There are 4 possible shapes for a logarithmic function

- 1) $k > 0$ and $b > 1$ (ex. $y = \log_2(x)$)
- 2) $k > 0$ and $0 < b < 1$ (ex. $y = \log_{0.5}(x)$)
- 3) $k < 0$ and $b > 1$ (ex. $y = \log_2(-x)$)
- 4) $k < 0$ and $0 < b < 1$ (ex. $y = \log_{0.5}(-x)$)



To graph the base function $y = \log_b x$, find the following key features:

- Vertical asymptote
 - Starts at $x = 0$ and can be shifted by d
- x -intercept
 - Set $y = 0$ and solve
- At least one other point to be sure of shape
 - Common to choose $y = 1$ and solve for x

Part 4: Transformations of Logarithmic Functions

Example 2: Sketch the graph of $f(x) = -4\log_3(x) + 2$ and $g(x) = \log_3[-(x + 2)] - 4$ using transformations

Table 4. $y = \log_3(x)$

x	y
0.33	-1
1	0
3	1
VA	$x = 0$

Table 5. $f(x) = -4\log_3(x) + 2$

x	$-4y + 2$
0.33	6
1	2
3	-2
VA	$x = 0$

Table 6. $g(x) = \log_3[-(x + 2)] - 4$

$-x - 2$	$y - 4$
-2.33	-5
-3	-4

-5	-3
VA	$x = -2$

