

3.6 Applications of Logarithms in Physical Sciences Lesson with solutions

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Part 1: Review of Solving Logarithmic Equations

Example 1: Solve for x in the following equation

$$\log_2(x - 6) = 4 - \log_2 x$$

$$\begin{aligned}\log_2(x - 6) + \log_2 x &= 4 \\ \log_2[(x - 6)(x)] &= 4 \\ 2^4 &= (x - 6)(x) \\ 16 &= x^2 - 6x \\ 0 &= x^2 - 6x - 16 \\ 0 &= (x - 8)(x + 2) \\ x &= 8\end{aligned}$$

Reject $x = -2$ as both original logarithmic expressions are undefined for this value.

Part 2: pH Scale

The pH scale is used to measure the acidity or alkalinity of a chemical solution. It is defined as:

$$pH = -\log[H^+]$$

where $[H^+]$ is the concentration of hydronium ions, measured in moles per liter.

pH	Concentration (mol/L)	Classification
1.0	10^{-1}	Acidic
7.0	10^{-7}	Neutral
14.0	10^{-14}	Alkaline

pH	Example substance
0	battery acid, strong hydrofluoric acid
1	hydrochloric acid secreted by stomach lining
2	lemon juice, gastric acid, vinegar
3	grapefruit, orange juice, soda
4	tomato juice, acid rain
5	soft drinking water, black coffee
6	urine, saliva
7	"pure" water
8	seawater
9	baking soda
10	Great Salt Lake, milk of magnesia
11	ammonia solution
12	soapy water
13	bleaches, oven cleaner
14	liquid drain cleaner

Example 2: Answer the following pH scale questions

a) Tomato juice has a hydronium ion concentration of approximately 0.0001 mol/L. What is its pH?

$$\begin{aligned}pH &= -\log 0.0001 \\ pH &= -(-4) \\ pH &= 4\end{aligned}$$

b) Blood has a hydronium ion concentration of approximately 4×10^{-7} mol/L. Is blood acidic or alkaline?

$$\begin{aligned} pH &= -\log(4 \times 10^{-7}) \\ pH &\approx 6.4 \end{aligned}$$

Since this is below the neutral value of 7, blood is acidic.

c) Orange juice has a pH of approximately 3. What is the concentration of hydronium ions in orange juice?

$$\begin{aligned} 3 &= -\log[H^+] \\ -3 &= \log[H^+] \\ 10^{-3} &= [H^+] \\ [H^+] &= 0.001 \text{ mol/L} \end{aligned}$$

Part 3: Decibel Scale

Some common sound levels are indicated on the decibel scale below. The difference in sound levels, in decibels, can be found using the equation:

$$\beta_2 - \beta_1 = 10 \log \left(\frac{I_2}{I_1} \right)$$

where $\beta_2 - \beta_1$ is the difference in sound levels, in decibels, and $\frac{I_2}{I_1}$ is the ratio of their sound intensities, where I is measured in watts per square meter (W/m^2)

Sound	Level (dB)
Rocket	200
Jet engine	160
Rock concert speaker	150
Symphony (peak)	120
Maximum stereo output	100
Niagara Falls	90
Normal city traffic	85
Shout	80
Normal conversation	60
Whisper	30
Rustle of leaves	10

Example 3: Answer the following questions about decibels

a) How many times as intense as a whisper is the sound of a normal conversation?

$$\begin{aligned} 60 - 30 &= 10 \log \left(\frac{I_2}{I_1} \right) \\ 30 &= 10 \log \left(\frac{I_2}{I_1} \right) \\ 3 &= \log \left(\frac{I_2}{I_1} \right) \\ 10^3 &= \frac{I_2}{I_1} \\ \frac{I_2}{I_1} &= 1000 \end{aligned}$$

A conversation sounds 1000 times as intense as a whisper.

b) The sound level in normal city traffic is approximately 85 dB. The sound level while riding a snowmobile is about 32 times as intense. What is the sound level while riding a snowmobile, in decibels?

$$\begin{aligned} \beta_2 - 85 &= 10 \log(32) \\ \beta_2 &= 10 \log(32) + 85 \\ \beta_2 &\approx 100 \text{ dB} \end{aligned}$$

Part 4: Richter Scale

The magnitude, M , of an earthquake is measured using the Richter scale, which is defined as:

$$M = \log\left(\frac{I}{I_0}\right)$$

where I is the intensity of the earthquake being measured and I_0 is the intensity of a standard, low-level earthquake.

Example 4: Answer the following questions about the Richter Scale

a) How many times as intense as a standard earthquake is an earthquake measuring 2.4 on the Richter scale?

$$\begin{aligned}2.4 &= \log\left(\frac{I}{I_0}\right) \\10^{2.4} &= \frac{I}{I_0} \\\frac{I}{I_0} &\cong 251.19\end{aligned}$$

It is about 251 times as intense as a standard earthquake.

b) What is the magnitude of an earthquake 1000 times as intense as a standard earthquake?

$$\begin{aligned}M &= \log 1000 \\M &= 3\end{aligned}$$