

3.5 Solving Logarithmic Equations Lesson with solutions

This document was composed and formatted by Annie Bernatchez.

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Part 1: Try and Solve a Logarithmic Equation

Solve the equation $\log(x + 5) = 2\log(x - 1)$

Hint: apply the power law of logarithms to the right side of the equation

$$\begin{aligned}\log(x + 5) &= \log(x - 1)^2 \\ x + 5 &= (x - 1)^2 \\ x + 5 &= x^2 - 2x + 1 \\ 0 &= x^2 - 3x - 4 \\ 0 &= (x - 4)(x + 1) \\ x &= 4 \text{ or } x = -1\end{aligned}$$

Reject $x = -1$ because $\log(x - 1)$ is undefined for this value of x .

Therefore, the only solution is $x = 4$

Note: If $\log_m a = \log_m b$, then $a = b$.

Part 2: Solve Simple Logarithmic Equations

Reminder: to complete this lesson, you will need to remember how to change from logarithmic to exponential form:

$$y = \log_b x \rightarrow x = b^y$$

Example 2: Solve each of the following equations

a) $\log(x + 4) = 1$

Method 1: re-write in exponential form

$$\begin{aligned}x + 4 &= 10^1 \\ x + 4 &= 10 \\ x &= 6\end{aligned}$$

Method 2: express both sides as a logarithm of the same base

$$\begin{aligned}\log(x + 4) &= \log(10) \\ x + 4 &= 10 \\ x &= 6\end{aligned}$$

b) $\log_5(2x - 3) = 2$

$$\begin{aligned}5^2 &= 2x - 3 \\ 25 &= 2x - 3 \\ 28 &= 2x \\ 14 &= x\end{aligned}$$

Part 3: Apply Factoring Strategies to Solve Equations

Example 3: Solve each equation and reject any extraneous roots

a) $\log(x - 1) - 1 = -\log(x + 2)$

$$\begin{aligned}\log(x - 1) + \log(x + 2) &= 1 \\ \log[(x - 1)(x + 2)] &= 1 \\ \log(x^2 + x - 2) &= 1 \\ x^2 + x - 2 &= 10^1 \\ x^2 + x - 12 &= 0 \\ (x + 4)(x - 3) &= 0 \\ x &= -4 \text{ or } x = 3\end{aligned}$$

Reject $x = -4$ because both of the original expressions are undefined for this value.

The only solution is $x = 3$

b) $\log \sqrt[3]{x^2 + 48x} = \frac{2}{3}$

$$\begin{aligned}\log(x^2 + 48x)^{\frac{1}{3}} &= \frac{2}{3} \\ \frac{1}{3} \log(x^2 + 48x) &= \frac{2}{3} \\ 3 \left[\frac{1}{3} \log(x^2 + 48x) \right] &= 3 \left(\frac{2}{3} \right) \\ \log(x^2 + 48x) &= 2 \\ x^2 + 48x &= 10^2 \\ x^2 + 48x - 100 &= 0 \\ (x + 50)(x - 2) &= 0 \\ x &= -50 \text{ or } x = 2\end{aligned}$$

Both are valid solutions because they both make the argument of the logarithm positive.

c) $\log_3 x - \log_3(x - 4) = 2$

$$\begin{aligned}\log_3 \left(\frac{x}{x-4} \right) &= 2 \\ \frac{x}{x-4} &= 3^2 \\ \frac{x}{x-4} &= 9 \\ x &= 9(x-4) \\ x &= 9x - 36 \\ 36 &= 8x \\ \frac{9}{2} &= x\end{aligned}$$

Example 4: If $\log_a b = 3$, then use log rules to find the value of...

a) $\log_a ab^2$

$$\begin{aligned}\log_a ab^2 &= \log_a a + \log_a b^2 \\ &= \log_a a + 2\log_a b \\ &= 1 + 2(3) \\ &= 7\end{aligned}$$

b) $\log_b a$

Hint: need to change the base: $\log_b m = \frac{\log_a m}{\log_a b}$

$$\begin{aligned}\log_b a &= \frac{\log_a a}{\log_a b} \\ &= \frac{1}{3}\end{aligned}$$