

3.3 Product and Quotient Laws of Logarithms Lesson with solutions

This document was composed and formatted by Annie Bernatchez.
The course material originated from www.jensenmath.ca

Part 1: Proof of Product Law of Logarithms

Let $x = \log_b m$ and $y = \log_b n$
Written in exponential form:
 $b^x = m$ and $b^y = n$

$$\begin{aligned} mn &= b^x b^y \\ mn &= b^{x+y} \\ \log_b(mn) &= x + y \\ \log_b(mn) &= \log_b m + \log_b n \end{aligned}$$

Part 2: Summary of Log Rules

Rule	Formula
Power Law of Logarithms	$\log_b x^n = n \log_b x$ for $b > 0, b \neq 1, x > 0$
Product Law of Logarithms	$\log_b(mn) = \log_b m + \log_b n$ for $b > 0, b \neq 1, m > 0, n > 0$
Quotient Law of Logarithms	$\log_b\left(\frac{m}{n}\right) = \log_b m - \log_b n$ for $b > 0, b \neq 1, m > 0, n > 0$
Change of Base Formula	$\log_b m = \frac{\log m}{\log b}, m > 0, b > 0, b \neq 1$
Exponential to Logarithmic	$y = b^x \rightarrow x = \log_b y$
Logarithmic to Exponential	$y = \log_b x \rightarrow x = b^y$
Other useful tips	$\log_a(a^b) = b$ $\log a = \log_{10} a$ $\log_b b = 1$

Part 3: Practice Using Log Rules

Example 1: Write as a single logarithm

a) $\log_5 6 + \log_5 8 - \log_5 16$

$$\begin{aligned} \log_5 6 + \log_5 8 - \log_5 16 &= \log_5 \left(\frac{6 \times 8}{16} \right) \\ &= \log_5 3 \end{aligned}$$

b) $\log x + \log y + \log(3x) - \log y$
 $= \log x + \log(3x)$

Note: Started by collecting like terms. Must have the same base and argument.

$$\begin{aligned} &= \log [(x)(3x)] \\ &= \log(3x^2) \end{aligned}$$

Note: Can't use the power law because the exponent 2 applies only to x , not $3x$.

c) $\frac{\log_2 7}{\log_2 5}$
 $= \log_5 7$

Note: Used the change of base formula.

d) $\log 12 - 3\log 2 + 2\log 3$

$$\begin{aligned}\log 12 - 3\log 2 + 2\log 3 &= \log 12 - \log 2^3 + \log 3^2 \\ &= \log 12 - \log 8 + \log 9 \\ &= \log \left(\frac{12 \times 9}{8} \right) \\ &= \log \left(\frac{27}{2} \right)\end{aligned}$$

Example 2: Write as a single logarithm and then evaluate

a) $\log_8 4 + \log_8 16$

$$\begin{aligned}\log_8 4 + \log_8 16 &= \log_8 (4 \times 16) \\ &= \log_8 64 \\ &= \frac{\log 64}{\log 8} \\ &= 2\end{aligned}$$

b) $\log_3 405 - \log_3 5$

$$\begin{aligned}\log_3 405 - \log_3 5 &= \log_3 \left(\frac{405}{5} \right) \\ &= \log_3 81 \\ &= \frac{\log 81}{\log 3} \\ &= 4\end{aligned}$$

c) $2\log 5 + \frac{1}{2} \log 16$

$$\begin{aligned}2\log 5 + \frac{1}{2} \log 16 &= \log 5^2 + \log \sqrt{16} \\ &= \log 25 + \log 4 \\ &= \log (25 \times 4) \\ &= \log 100 \\ &= 2\end{aligned}$$

Example 3: Write the Logarithm as a Sum or Difference of Logarithms

a) $\log_3(xy)$

$$= \log_3 x + \log_3 y$$

b) $\log 20$

$$= \log 4 + \log 5$$

c) $\log(ab^2c)$

$$\begin{aligned}\log(ab^2c) &= \log a + \log b^2 + \log c \\ &= \log a + 2\log b + \log c\end{aligned}$$

Example 4: Simplify the Following Algebraic Expressions

a) $\log \left(\frac{\sqrt{x}}{x^2} \right)$

$$\begin{aligned}\log \left(\frac{\sqrt{x}}{x^2} \right) &= \log \left(\frac{x^{\frac{1}{2}}}{x^2} \right) \\ &= \log x^{-\frac{3}{2}} \\ &= -\frac{3}{2} \log x\end{aligned}$$

b) $\log(\sqrt{x})^3 + \log x^2 - \log \sqrt{x}$

$$\begin{aligned}\log(\sqrt{x})^3 + \log x^2 - \log \sqrt{x} &= \log x^{\frac{3}{2}} + \log x^2 - \log x^{\frac{1}{2}} \\ &= \frac{3}{2} \log x + 2 \log x - \frac{1}{2} \log x \\ &= \frac{3}{2} \log x + \frac{4}{2} \log x - \frac{1}{2} \log x \\ &= 3 \log x\end{aligned}$$

c) $\log(2x - 2) - \log(x^2 - 1)$

$$\begin{aligned}\log(2x - 2) - \log(x^2 - 1) &= \log \left(\frac{2x - 2}{x^2 - 1} \right) \\ &= \log \left[\frac{2(x - 1)}{(x - 1)(x + 1)} \right] \\ &= \log \left(\frac{2}{x + 1} \right)\end{aligned}$$