

3.1 Introduction to Logarithms and Review of Exponentials Lesson

This document was composed and formatted by Annie Bernatchez.

The course material originated from www.jensenmath.ca

In this section you will learn about how a logarithmic function is the inverse of an exponential function. You will also learn how to express exponential equations in logarithmic form.

Part 1: Review of Exponential Functions

Equation: $y = a(b)^x$

- a = initial amount
- b = growth ($b > 1$) or decay ($0 < b < 1$) factor
- y = future amount
- x = number of times a has increased or decreased

To calculate x , use the equation:

$$x = \frac{\text{total time}}{\text{time it takes for one growth or decay period}}$$

Example 1: An insect colony has a current population of 50 insects. Its population doubles every 3 days.

a) What is the population after 12 days?

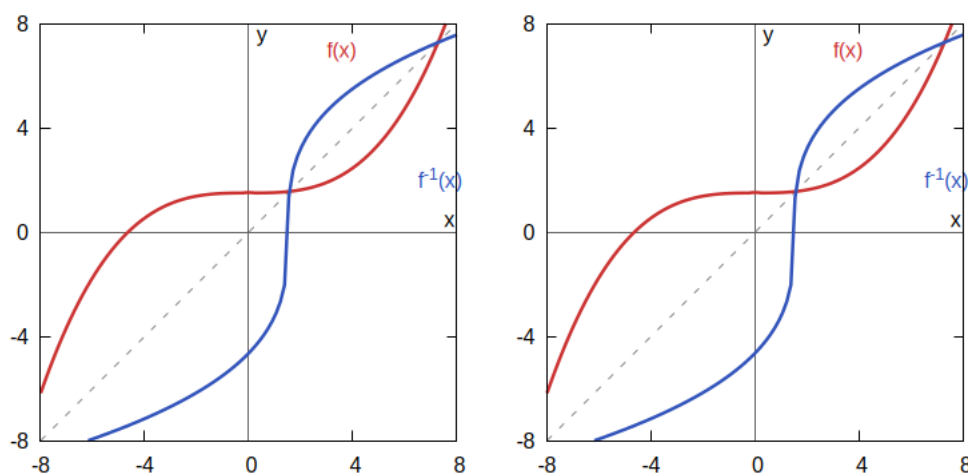
b) How long until the population reaches 25 600?

Part 2: Review of Inverse Functions

Inverse of a function:

- The inverse of a function f is denoted as f^{-1}
- The function and its inverse have the property that if $f(a) = b$, then $f^{-1}(b) = a$
- So if $f(5) = 13$, then $f^{-1}(13) = 5$
- More simply put: the inverse of a function has all the same points as the original function, except that the x 's and y 's have been reversed.

The graph of $f^{-1}(x)$ is the graph of $f(x)$ reflected in the line $y = x$. This is true for all functions and their inverses.



Example 2: Determine the equation of the inverse of the function $f(x) = 3(x - 5)^2 + 1$

Method for finding the inverse algebraically:

1. Replace $f(x)$ with y
2. Switch the x and y variables
3. Isolate for y
4. Replace y with $f^{-1}(x)$

Equation of inverse: _____

Part 3: Review of Exponent Laws

Name	Rule
Product Rule	$x^a \cdot x^b = \underline{\hspace{2cm}}$
Quotient Rule	$\frac{x^a}{x^b} = \underline{\hspace{2cm}}$
Power of a Power Rule	$(x^a)^b = \underline{\hspace{2cm}}$
Negative Exponent Rule	$x^{-a} = \underline{\hspace{2cm}}$
Exponent of Zero	$x^0 = \underline{\hspace{2cm}}$

Part 4: Inverse of an Exponential Function

Example 3:

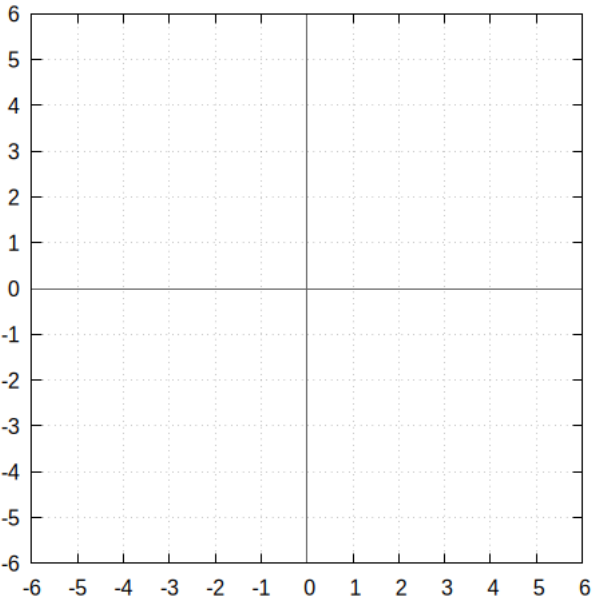
a) Find the equation of the inverse of $f(x) = 2^x$.

Note: this step uses the 'change of base' formula that we will cover later in the unit: $\log_b m = \frac{\log m}{\log b}$

b) Graph both $f(x)$ and $f^{-1}(x)$.

Note: just swap the x and y coordinates to get key points for the inverse of a function. The graph should appear to be a reflection across the line $y = x$.

$f(x) = 2^x$		$f^{-1}(x) = \log_2 x$	
x	y	x	y
_____	_____	_____	_____
_____	_____	_____	_____
_____	_____	_____	_____
_____	_____	_____	_____
_____	_____	_____	_____



c) Complete the chart of key properties for both functions

$y = 2^x$	$y = \log_2 x$
x -int: _____	x -int: _____
y -int: _____	y -int: _____
Domain: _____	Domain: _____
Range: _____	Range: _____
Asymptote: _____	Asymptote: _____

Part 5: What is a Logarithmic Function?

The logarithmic function is the _____ of the exponential function with the same base.

The **logarithmic function** is defined as $y = \log_b x$, or y equals the logarithm of x to the base b .

The function is defined only for _____

In this notation, _____ is the exponent to which the base, _____, must be raised to give the value of _____.

In other words, the solution to a logarithm is always an _____.

The logarithmic function is most useful for solving for unknown _____.

_____ are logarithms with a base of 10. It is not necessary to write the base for common logarithms: $\log x$ means the same as $\log_{10} x$

Part 6: Writing Equivalent Exponential and Logarithmic Expressions

Exponential equations can be written in logarithmic form, and vice versa:

- $y = b^x \rightarrow$ _____
- $y = \log_b x \rightarrow$ _____

Example 4: Rewrite each equation in logarithmic form

a) $16 = 2^4$

b) $m = n^3$

c) $3^{-2} = \frac{1}{9}$

Example 5: Write each logarithmic equation in exponential form

a) $\log_4 64 = 3$

b) $y = \log x$

Note: because there is no base written, this is understood to be the common logarithm of x .

Part 7: Evaluate a Logarithm

Example 6: Evaluate each logarithm without a calculator

Note: either of the two rules below is appropriate for evaluating logarithmic expressions.

Rule: if $x^a = x^b$, then $a = b$

a) $y = \log_3 81$

b) $y = \log \left(\frac{1}{100} \right)$

c) $y = \log_2 \left(\frac{1}{8} \right)$

Rule: $\log_a (a^b) = b$

d) $y = \log_4 64$